Comparison of Optimal Design of Different Spiral Inductors

G. Stojanovic and Lj. Zivanov

Abstract - One of the key components in many RF ICs applications is the inductor. It is very important that inductor has optimal design, that means, optimal geometry with the best possible characteristics. In this work optimal design of square, octagonal, and circular spiral inductors are compared. Our optimization algorithm is based on compact model, where the physical behavior is described through analytical expressions in geometric programming (GP) form. Globally optimal trade-off curves between maximum Q-factor or self-resonance frequency and inductance are presented. Results of our software tool show that the best layout of spiral inductor is circular then octagonal and square spiral.

I. Introduction

The interest in the realization of low-cost RF integrated circuits is very high and many researchers from companies and universities worldwide are investigating how to improve silicon integrated inductors [1]-[4]. Most of them have used measurements to construct models. While this technique is most practical, it does not permit optimization. Researchers have used commercial 3D electromagnetic simulators to design and analyze inductors. While this approach is accurate, it can be computationally very expensive and time-consuming. A computer-aided optimization technique using GP has been recently used [5], [6] to find the optimum design for square spiral inductors. The algorithm for comparison between different layouts of inductor optimization over a wide frequency range has not been reported yet. In this paper, the goal is selection of the best geometry of inductors, and then, to find optimal values of inductor parameters (the number of turns and layout dimensions). Based on our algorithm, a compact computer program INOPT (**IN**ductor **OPT**imization) with its graphical user interface has been developed as CAD tool for fast optimization of planar spiral inductors.

II. Geometric Programming

Geometric programming (GP) is one of the more recent developments in optimization theory. GP is a particularly powerful method for engineering design and optimization problems.

A generalized GP problem with N variables and M constraints has the form:

$$\text{minimize} \qquad y_o(x) = \sum_{t=1}^{T_0} \sigma_{ot} \cdot c_{ot} \prod_{n=1}^{N} x_n^{a_{otn}}, \qquad (1)$$

subject to constraints

$$\sum_{t=1}^{T_m} \sigma_{mt} \cdot c_{mt} \prod_{n=1}^{N} x_n^{a_{mtn}} \le \sigma_m, \text{ for m=1, 2, …, M} \qquad (2)$$

where σ_{ot} and $\sigma_{mt} = \pm 1$ are the sign of each term in the objective function and m^{th} constraint, respectively, c_{ot} and $c_{mt} > 0$ are the coefficients of each term in the objective function and m^{th} constraint, respectively. $x_n > 0$ is the independent variables, $\sigma_m = \pm 1$ is the constant bound of the m^{th} constraint, a_{otn} and a_{mtn} are the exponents of the n^{th} independent variable of the t^{th} term of objective function and m^{th} constraint, respectively, M is the number of constraints, N is the number of variables, T_o is the number of terms in the objective function, $T_1, T_2, \ldots T_M$ are the number of terms in each constraint, 1 to M, respectively, $\sigma = \pm 1$ assumed sign of the objective function. If $T_o = 1$, the objective function is called a monomial function. The sign of each term in the objective function or the constraint functions determines whether the polynomials is a posynomial (if σ_{ot} or $\sigma_{mt} = +1$) or a signomial (if σ_{ot} or $\sigma_{mt} = -1$).

III. Modeling of Different Spiral Inductors

The computer-based algorithm for optimization of square spiral inductors is presented in [5]. No octagonal or circular inductor optimization tools exist to help with the design of RF integrated circuits. The layout optimization parameters are the number of turns – n, the width of conductor – w, the spacing between adjacent conductor – s, the outer diameter – d_{out}, and the average diameter – d_{avg}, which can be expressed as $d_{avg} = 0.5 \cdot (d_{out} + d_{in})$. These five variables are optimization variables. Some of these variables are shown in Fig. 1. Other geometry parameters of interest are the inductor length and the inductor area. The total length of the inductor can be expressed as $l = n \cdot d_{avg} \cdot N \cdot \tan(\pi / N)$ and the inductor area as $A = d_{out}^2$ [7]. For a square inductor N=4 and $l = 4 \cdot n \cdot d_{avg}$, for octagonal N=8 and $l = 3.31 \cdot n \cdot d_{avg}$, for circular spiral N is the great number and after simple mathematical calculations can be obtained $l = 3.14 \cdot n \cdot d_{avg}$.

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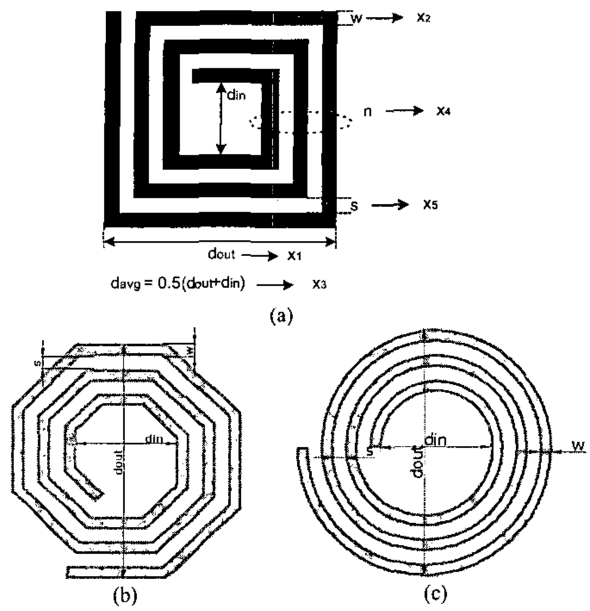

Fig. 1. Layout of planar inductors: square (a), octagonal (b), and circular (c).

All the results in this paper are based on a relatively simple, resonably accurate one-port inductor model shown in Fig. 2 (a). A simplified model is shown in Fig. 2 (b) [1].

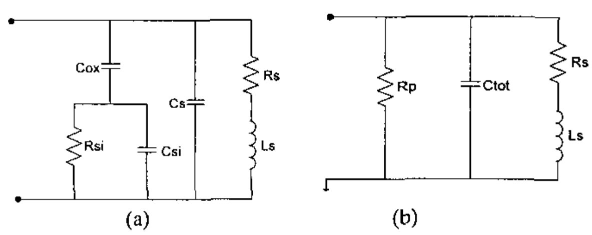

Fig. 2. One-port model of a planar spiral inductor (a). Simplified model of planar spiral inductor (b).

In this subsection, we give expressions for elements of inductor equivalent model in the GP form.

Inductance Ls. The formula from [7]

$$L_S = \beta \cdot d_{out}^{\alpha_1} \cdot w^{\alpha_2} \cdot d_{avg}^{\alpha_3} \cdot n^{\alpha_4} \cdot s^{\alpha_5}, \quad (3)$$

is a monomial expression, obtained by fitting technique, with six fitting factors. For a square inductor values of fitting factors are $\beta = 1.66 \cdot 10^{-3}$, $\alpha_1 = -1.33$, $\alpha_2 = -0.125$, $\alpha_3 = 2.5$, $\alpha_4 = 1.83$, $\alpha_5 = -0.022$, and for octagonal inductor are $\beta = 1.33 \cdot 10^{-3}$, $\alpha_1 = -1.21$, $\alpha_2 = -0.163$, $\alpha_3 = 2.43$, $\alpha_4 = 1.75$, $\alpha_5 = -0.049$. For simplicity, we introduce the following notation: $d_{out} = x_1$, $w = x_2$, $d_{avg} = x_3$, $n = x_4$, and $s = x_5$.

Used the fitting techniques, we obtained expresion for inductance of circular inductor in GP form ($\mu_o = 4 \cdot \pi \cdot 10^{-7}$ H/m)

$$L_S = 12.128 \cdot \mu_o \cdot x_3 \cdot x_4^2 - 19.575 \cdot \mu_o \cdot x_1 \cdot x_4^2 + \\ + 12.589 \cdot \mu_o \cdot x_1^2 \cdot x_3^{-1} \cdot x_4^2 - 3.640 \cdot \mu_o \cdot x_1^3 \cdot x_3^{-2} \cdot x_4^2 + \\ + 0.396 \cdot \mu_o \cdot x_1^4 \cdot x_3^{-1} \cdot x_4^2 \quad (4)$$

The monomial expresion (3), and signomial expression (4) are useful because, they can be used for optimal design of planar spiral inductors with GP.

Another parameters of inductor model (Fig. 2 (a) and Fig. 2 (b)) have the following GP form:

series resistance $R_S = K_1 \cdot x_2^{-1} \cdot x_3 \cdot x_4$;

oxide capacitance $C_{ox} = K_2 \cdot x_2 \cdot x_3 \cdot x_4$;

series capacitance $C_S = K_3 \cdot x_2^2 \cdot x_4$;

substrate capacitance $C_{Si} = K_4 \cdot x_2 \cdot x_3 \cdot x_4$;

substrate resistance $R_{Si} = K_5 \cdot x_2^{-1} \cdot x_3^{-1} \cdot x_4^{-1}$;

parallel resistance $R_p = K_6 \cdot x_2^{-1} \cdot x_3^{-1} \cdot x_4^{-1}$;

parallel capacitance

$$C_{tot} = K_3 \cdot x_2^2 \cdot x_4 + K_7 \cdot x_2 \cdot x_3 \cdot x_4 + K_8 \cdot x_2^2 \cdot x_3^2 \cdot x_4^2$$

Coefficients K_i, i = 1,2,...8 in above expressions depend on technology and frequency.

IV. Constraints and Improvement for Optimal Design of Spiral Inductors

A geometric programming requires each of the constraint functions (or our objective function) to be a monomial, posynomial or a signomial for a difference to [5],[6] which is restricted to be either monomial or posynomial.

Constraint for Ls. The our software tool requires that the inductance to be equal some specific value, Ls = Lreq.

Geometry constraints. Since, width of conductor and spacing between adjacent conductors to belong to optimization variables, very simple, it can be constrained. For example, $x_2 \geq w_{min}$ or $x_5 \geq s_{min}$.

Quality factor. Q-factor of an inductor, via elements of the one-port model shown in Fig. 2 (b), can be defined as follows

$$Q_L = \frac{\omega \cdot L_S}{R_S} \cdot \frac{R_P \left(1 - \frac{R_S^2 \cdot C_{tot}}{L_S} - \omega^2 \cdot L_S \cdot C_{tot}\right)}{R_P + \left[\left(\frac{\omega \cdot L_S}{R_S}\right)^2 + 1\right] \cdot R_S}. \quad (5)$$

However, the problem is that because of equation (5) is not writen in GP form. In the reported paper [5], this problem is solved thus is introduced one new variable Q_{Lmin}

or x_6 and in the objective function this variable is maximized. Then, an additive constraint must be written as $Q_L \ge Q_{Lmin}$. This approach is correct, but introduced one variable more and one new constraint more. For difference to this, in our software tool INOPT we suggest the following approach. Let us find the best possible expression for Q-factor, thus it can be written in GP form. This approach is enabled type of our implemented algorithm in which objective function and constraints functions can be written in monomial, posynomial or signomial form (for difference to program in [5]). This difference is crucial in the context to reduce the CPU time for optimization. We found the following expression for the Q-factor in the GP form

$$Q_L = \frac{\omega \cdot L_S}{R_S} - \omega \cdot R_S \cdot C_{tot} - \frac{\omega^3 \cdot L_S^2 \cdot C_{tot}}{R_S}. \qquad (6)$$

Constraint for minimum self-resonance frequency. We want that the self-resonance frequency is greater than or equal to exactly specific value that means $\omega_{sr} \ge \omega_{sr,\,min}$. This condition can be written as follows

$$\omega_{sr,\min}^2 \cdot L_S \cdot C_{tot} + \frac{R_S^2 \cdot C_{tot}}{L_S} \le 1. \qquad (7)$$

V. Comparison of Results for Optimal Design of Spiral Inductors

One much used figure of merit for the inductor characterization is the Q-factor, which we have already introduced in the previous section. The Q-factor of an inductor is a figure of merit of usefulness of the component. With practical aspect we want to maximize Q-factor for a specific value inductance. In order to demonstrate our optimization program we represented GP problem as:

$$\text{maximize} \quad Q_L, \qquad (8)$$

subject to

$$L_s = L_{req},\ s \ge s_{min},\ \omega_{sr} \ge \omega_{sr,min}, \qquad (9)$$

In Fig. 3 is shown maximum Q-factor at 2.5GHz versus inductance value for the square, octagonal and circular inductor with patterned ground shield - PGS. The specific values for inductance of these spiral inductors are changed from 2nH to 22nH and constraint is $s \ge 1.9\mu m$. For curves depicted in Fig. 4 are added constraint for *minimum self–resonance frequency as* $\omega_{sr} \ge 7GHz$. *It is* clear, that circular inductor, then octagonal and square inductor have the best characteristics. When the self-resonance frequency is not constrained one obtains a higher values of Q-factor. The results of our program presented in Fig. 3, and Fig. 4 show good agreement when they compared with the measured data in the open literature [5]. *Our program gives graphical view for trade-off curves in* an approximately one second (important advantage in compare to another programs in the published literature).

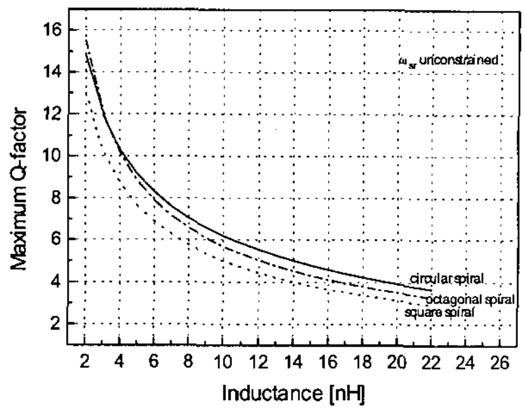

Fig. 3. Maximum Q-factor of inductor versus inductance.

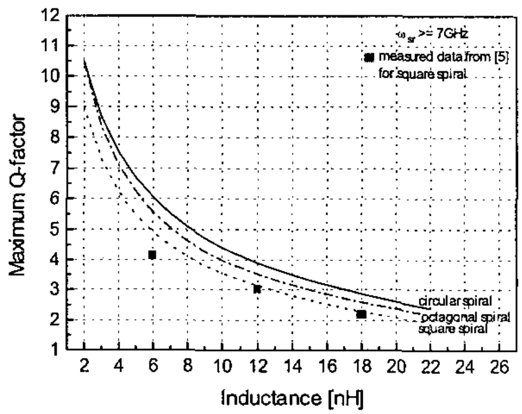

Fig. 4. Maximum Q-factor of inductor versus inductance.

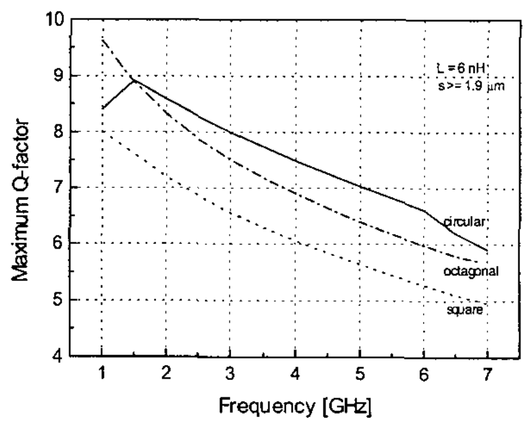

Fig. 5. Maximum Q-factor of different planar inductor versus frequency.

The corresponding geometrical dimensions are all in a feasible technical range. It means that an optimal design of the all analyzed inductors is possible in the considered cases.

It's necessary to emphasize, again, coefficients Ki, i = 1,2 ...8 in expressions for elements of equivalent model of inductors are dependent on technological parameters and frequency. In Fig. 5 are depicted dependence maximum Q-factor and frequency (in the range of interest for practical applications) for fixed inductance value and for different geometry of spiral inductors, with $s \geq 1.9\mu m$.

VI. Optimization Self-Resonance Frequency ω_{SR} via GP

In addition to maximized Q-factor, very important is maximized self-resonance frequency, in the context of practical application an inductor. In this section we represent results of our simulation for maximize self-resonance frequency via GP. Thus, a design problem can be posed as

$$\text{maximize} \quad \omega_{srmin}, \tag{10}$$

subject to:

$$\omega_{sr} \geq \omega_{srmin}, Q = 3, Ls = Lreq, s \geq 5\ \mu m. \tag{11}$$

We can maximize the self-resonance frequency by maximizing ω_{srmin} subject to constraint. Results of our simulations are depicted in Fig. 6, for different geometry of planar spiral inductors at operating frequency 1GHz.

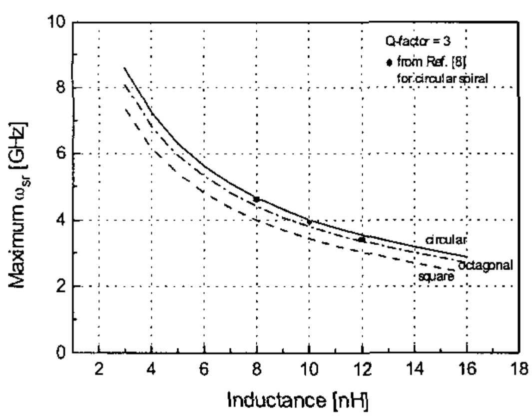

Fig. 6. Maximum the self-resonance frequency versus inductance.

As it can be seen the circular inductor has a little greater value of maximum ω_{sr} than octagonal and square inductor. We compared maximum self-resonance frequency predicted by our program INOPT with values from previously published paper [8] (for circular inductor on silicon), and we found excellent agreement, as can be seen in Fig. 6.

VII. Conclusion

We presented a procedure to obtain optimal design of spiral inductor on silicon substrate. The procedure has been implemented in a numerical code that performs a fast optimization of the inductor. Thanks to its flexibility and accuracy, our algorithm is especially useful for comparing alternatives during design optimization. Using this approach we performed a comparison of optimum characteristics between square, octagonal, and circular inductors, in order to select the best suited solution to improve the performance of planar inductors and then RF ICs. The designer has to introduce technological parameters, the frequency and the desired inductance, and then the results are the optimal geometry parameters of inductor and maximum Q-factor. The accuracy of our program is established through comparisons between some measurement and simulation results for various inductor structures from published papers.

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