On the Spectral Efficiency of Space-Time Spreading Schemes for Multiple Antenna CDMA Systems

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Abstract

We have recently introduced a particular space-time spreading (STS) scheme that can be used in order to provide important diversity gains in CDMA systems that use multiple antenna transceivers. While such simple STS schemes are simpler to implement (especially in terms of error-correction coding) than the recently proposed BLAST (Bell-labs LAyered Space-Time) architecture, it is of interest to quantify the expected penalties paid in terms of spectral efficiency. In this paper we derive capacity expressions for such STS systems and compare them to the corresponding expressions of related BLAST systems. Interestingly enough, it turns out that for small numbers of antennas, the deviations in performance are not so high. These results suggest that, when small numbers of antennas are available, space-time spreading is a viable option that provides a good tradeoff between ease of implementation and increased spectral efficiency of direct-spreading CDMA systems.

1 Introduction

The use of multiple antenna transceivers in wireless communication systems is a field that has been receiving growing attention in the last years. Whereas system capacity in wireless networks is affected by a number of issues such as error correction coding, voice activity, frequency reuse patterns, multiple access schemes, power control, handoff etc., as well as on the propagation environment itself, multiple antenna transceivers are widely regarded as an extra dimension that can offer additional benefits. As current cellular systems begin reaching saturation in the number of allowable active subscribers per sector, antenna arrays are seen as an important asset that allows for extra system capacity. Moreoever, wireless data will begin playing an increasingly important role in next generation wireless standards (3G and beyond). With the anticipated increase in demand for higher data rates, especially in the wireless downlink (base-to-subscriber), the potential gains offered by antenna arrays will not be easily neglected.

Traditionally, multiple antennas which are employed at the base station for receive processing offer three types of gains: (1) array gain; (2) diversity gain and (3) interference reduction.

Array gain is the SNR improvement which is achieved (even with static channels and in the absence of interference) when the antenna elements are spaced closely enough (in the order of half a wavelength or so) and are combined in an optimal fashion. Diversity gain is important when the channel is time-varying (i.e. in the presence of fading). It is achieved when the antenna elements are spaced far enough so that their received signals fade almost independently: coherent combining then provides higher robustness to fading. Finally, interference reduction is important when the received signal is interfered by other co-channel signals: by properly shaping the antenna pattern, it is possible to attenuate the signals from the interfering directions. In some sense, all the above gains attempt to improve the wireless uplink (subscriber-to-base) in different noise/channel/interference scenarios. Similar link gains are possible with antenna arrays in the downlink. These can be achieved either with transmit processing (such as downlink beamforming and transmit diversity) with a base-located array or with receiver processing if the subscriber can afford the use of more than one antenna for reception.

More recently, antenna arrays were shown to offer yet another important benefit. By using multiple antennas at both sides of the communication link, it is possible, with the synergy of a rich scattering environment, to create multiple virtually parallel wire-

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less links, which carry each a different information sequence [1]. By doing so, these techniques (known as either BLAST transceivers, MIMO systems, or antenna multiplexing systems) are able of increasing the spectral efficiency of the link by a factor roughly proportional to the number of antenna elements on each side of the link. Systems that employ such techniques have already been built, and were shown to achieve uncommonly high spectral efficiencies (in the order of several tens of bits/second/Hz).

Hence, it is expected that, beyond the traditional benefits of antenna arrays, their potential to offer increased spectral efficiency in wireless links will make them crucial to the employment of (especially high data rate) future wireless systems. However, BLAST-type systems only see their full potential when a large number of elements (such as in the order of about four or more) are used on *each* side of the link. While this constraint may not be prohibitive for some applications (such as, e.g., fixed wireless systems or mobile systems with large-size terminals), it is clearly limiting in cases where the subscriber unit cannot employ more than a very small number of antennas (such as one or at most two), as in today's cell-phones. Moreover, as these techniques only emerged very recently, the advanced space-time coding methods that they require in order to approach Shannon capacity have not yet reached the maturity of purely temporal coding (such as Turbo codes) which can be employed in more conventional antenna systems.

As a result of the above, in systems that make use of only a very small number of antennas at the receive terminal, it makes sense to resort to simpler alternatives for increased data rates. One promising approach in this direction is the use of transmit diversity systems: these techniques typically send the same information from all the antenna elements, but multiplexed in a different way on each antenna (such as time multiplexing or code multiplexing in TDMA and CDMA systems, respectively). By doing so, these methods provide increased diversity gain at the receiver terminal. Combined to their relatively simple processing at both sides of the link, as well as to the fact that, similarly to BLAST, they rely on the richness of the channel and typically do not require good channel estimation (as opposed to advanced downlink beamforming), these techniques show good promise for feasible increased data rates at the downlink. In TDMA systems, a recent interesting transmit diversity technique was proposed in [2], using space-time block codes. As a counterpart for CDMA systems, a technique called Space-Time-Spreading (STS) was recently introduced

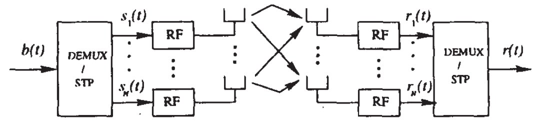

Figure 1: A generic (M, N) transmit diversity system

in [3], [4] and was chosen as an optional transmission mode in phase A of the IS-2000 standard [5].

In this paper, we are interested in studying the spectral efficiency of STS schemes for CDMA systems, as a measure of their potential to offer increased data rates in a given bandwidth. We will derive theoretical expressions for the spectral efficiency of these systems in different cases and we will evaluate them numerically and compare them to those of other schemes. Our main point will be the potential of STS techniques to provide higher data rates with little need for extra processing, resources (including spreading codes, power, complexity), or elaborate code design, in CDMA systems that cannot afford large numbers of antennas, especially at the subscriber unit.

2 Space-Time Spreading

Figure 1 shows a generic transmit diversity system that employs M transmitter and N receiver antennas. We will denote this as an (M, N) system. In transmit diversity, the information-bearing input signal $b(t)$ is demultiplexed in $s_m(t)$, $m = 1, \ldots, M$, each of which is transmitted (after up-conversion) by the m-th antenna. In DS-CDMA, each symbol of the information-bearing sequence $\{b(i)\}$ of a particular user is spread by a code sequence $\mathbf{c}_i = [c_i(1) \ \cdots \ c_i(N)]^T$ where N is the spreading factor and T denotes vector transpose. This spreading operation can be represented mathematically simply as $\mathbf{s}(i) = b(i)\mathbf{c}_i$, where $\mathbf{s}(i)$ is the $N \times 1$ vector that contains a block of N consecutive (chip-rate) samples corresponding to the user's i-th symbol period. In the forward link, $\mathbf{c}_i$ typically repeats itself in each symbol period ($\mathbf{c}_i = \mathbf{c}$), however, for the purpose of presenting transmit diversity schemes, we will not impose this restriction. When multiple antennas are available at the transmitter, we instantly face the challenge of choosing the way that the user's information sequence $\{b(i)\}$ is spread on each antenna. One naive choice would be to spread the user's signal in the same exactly fashion on each antenna, i.e. $\mathbf{s}_m(i) = \mathbf{c}_i b(i)$. However, such a scheme would offer no diversity gain at the receiver, since the system would be equivalent to transmitting from a single antenna with the same total

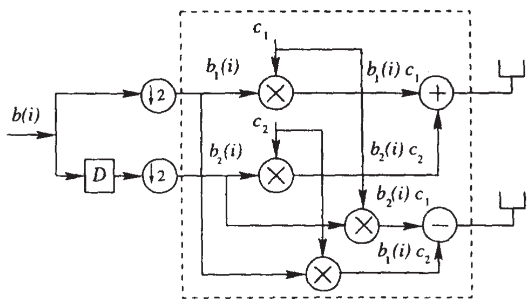

Figure 2: $(2,1)$ STS transmit diversity scheme

transmitted power. An interesting alternative would be to use a different code to spread the user's signal on each different antenna, i.e. $\mathbf{s}_m(i) = \mathbf{c}_i^m b(i)$, where $\{\mathbf{c}_i^m,\ m = 1,\ldots,M\}$ is a set of orthogonal codes ($\mathbf{c}_i^{mT}\mathbf{c}_i^n = \delta_{mn}$) where δ is the Kronecker delta. Such a scheme provides an important diversity advantage at the receiver, however it requires M times more orthogonal codes.

Consider now that each user's data is split into K sub-streams $\{b_1\},\cdots,\{b_K\}$, such that $b_k(i)=b((i-1)K+k)$, $k = 1,\ldots,K$. Then we will allow each sub-stream $b_k(i)$ to be spread by a different code $\mathbf{c}_k$ (same for all symbols in that sub-stream). Space-time spreading (STS), as introduced in [3], [4], relies on the following observation: each user's transmitted signal from each antenna $\mathbf{s}_m(i)$ can in general be a different combination of the user's sub-streams and corresponding spreading codes, without necessarily using more than 1 code / sub-stream, i.e.

$$\mathbf{s}_m(i) = \mathbf{f}_m\left(b_1(i),\cdots,b_K(i),\mathbf{c}_1,\cdots,\mathbf{c}_K\right) \qquad (1)$$

The design of functions $\{\mathbf{f}_m(\cdot)\}$ that allow for a "clever" re-use of code signatures across the different antennas so as to provide maximal diversity gain at the receiver with relatively simple processing then becomes an interesting problem. We will begin our study by describing a $(2,1)$ STS scheme and evaluating its spectral efficiency.

3 The $(2,1)$ STS scheme

In the (2,1) STS scheme, first introduced publicly in [3], the transmitted signals from the two antennas are as shown in Figure 2, i.e.,

$$\begin{aligned} \mathbf{s}_1(i) &= 1/\sqrt{2}\left(b_1(i)\mathbf{c}_1 + b_2(i)\mathbf{c}_2\right) \\ \mathbf{s}_2(i) &= 1/\sqrt{2}\left(b_2(i)\mathbf{c}_1 - b_1(i)\mathbf{c}_2\right) \end{aligned} \qquad (2)$$

where $\{b_1(i)\}$, $\{b_2(i)\}$ represent the (assumed real) odd and even user information-bearing sequences and the normalization by $\sqrt{2}$ is used to normalize the total transmit average power to unity. Assuming single-path propagation, the channel from the m-th antenna to the single receive antenna is represented by the complex scalar h_m. The received signal can then be written as $\mathbf{r}(i)=\frac{1}{\sqrt{2}}\left(h_1(b_1(i)\mathbf{c}_1+b_2(i)\mathbf{c}_2)+h_2(b_2(i)\mathbf{c}_1-b_1(i)\mathbf{c}_2)\right)+\mathbf{n}(i)$, where $\mathbf{n}^T(i)=[n_1(i)\ n_2(i)]$ is the additive noise vector. The receiver despreads the received signal separately with $\mathbf{c}_1$ and $\mathbf{c}_2$ to yield:

$$\begin{aligned} d_1(i)&=\mathbf{c}_1^T\mathbf{r}(i)=(1/\sqrt{2})\left(h_1b_1(i)+h_2b_2(i)\right)+v_1(i) \\ d_2(i)&=\mathbf{c}_2^T\mathbf{r}(i)=(1/\sqrt{2})\left(-h_2b_1(i)+h_1b_2(i)\right)+v_2(i) \end{aligned} \qquad (3)$$

where $v_m(i) = \mathbf{c}_m^T\mathbf{n}(i)$, $m = 1,2$. Eq. (3) can be equivalently written as

$$\begin{aligned} \mathbf{d}(i) &= (1/\sqrt{2})\begin{bmatrix} h_1 & h_2 \\ -h_2 & h_1 \end{bmatrix}\begin{bmatrix} b_1(i) \\ b_2(i) \end{bmatrix} + \mathbf{v}(i) \\ &= (1/\sqrt{2})\mathbf{H}\mathbf{b}(i) + \mathbf{v}(i) \end{aligned} \qquad (4)$$

where $\mathbf{v}^T(i) = [v_1(i)\ v_2(i)]$. Notice that $\mathbf{H}$ is a unitary matrix. Simple matched-filter processing then yields

$$\mathrm{Re}\{\mathbf{H}^H\mathbf{d}(i)\} = (1\sqrt{2})(|h_1|^2 + |h_2|^2)\mathbf{b}(i) + \mathbf{v}'(i) \quad (5)$$

hence two-branch diversity combining is achieved at the receiver. Notice that this is being done by using no extra resources (such as excessive codes or transmit power) and no feedback from the subscriber unit.

Spectral efficiency

The SNR of the processed signals in (5) equals

$$\rho_{2,1} = \rho(|h_1|^2+|h_2|^2) \qquad (6)$$

where ρ is the SNR of each of the received signals after despreading. The Shannon capacity of this system then equals

$$C_{2,1} = 1/2\log_2\left(1 + \rho(|h_1|^2 + |h_2|^2)\right) \qquad (7)$$

bits/second/Hz. This should be compared to the single-antenna expression given by

$$C_{1,1} = 1/2\log_2\left(1 + 2\rho|h_1|^2\right) \qquad (8)$$

(the factor of 2 appearing in front of ρ in (8) is due to the fact that we assume all systems to have the same total average transmitted power, independently of the number of transmitter antennas). Notice that these capacities are random variables, since the channel coefficients are random variables themselves.

If the input $b(i)$ is complex instead of real, then a similar to the above-described STS scheme can be designed (see [4] for more details) that yields again the same gains that are described in Eq. (5) (even though with slightly different receive processing). In that case, the Shannon capacity will be given by

$$C^c_{2,1} = \log_2 \left(1 + \frac{\rho(|h_1|^2 + |h_2|^2)}{2}\right) \qquad (9)$$

i.e. there is a 3 dB hit in SNR (since noise comes in from both the I and Q branches) but a two-fold increase in the number of independently transmitted data streams. Again, $C^c_{2,1}$ should be compared to

$$C^c_{1,1} = \log_2 \left(1 + \rho|h_1|^2\right) \qquad (10)$$

4 Other STS schemes

The design of STS schemes for different $(M, 1)$ configurations is closely related to the theory of orthogonal designs, as described in [6]. A $(4, 1)$ scheme that achieves a 4-branch diversity gain with no redundancy (i.e. 1 code/ sub-stream and 1 substream/ antenna) was described in [3] for real inputs. For values of M other than 2, 4 and 8 (real case) and other than 2 (complex case) it can be shown that no non-redundant schemes exist. However, redundant schemes can be still designed. In order to acquire a feeling of how capacity scales as the number of transmitter antennas grows, we will assume that for each $(M, 1)$ system we have a scheme that provides an M-branch diversity gain at the receiver. The resulting SNR is then

$$\rho_{M,1} = 2\rho \sum_{m=1}^{M} |h_m|^2/M \qquad (11)$$

(assuming again real inputs) and (7) generalizes to

$$C_{M,1} = 1/2 \log_2 \left(1 + \frac{2\rho}{M} \sum_{m=1}^{M} |h_m|^2\right) \qquad (12)$$

Notice that the expression in (12) is the same as that of a degenerate $(M, 1)$ BLAST system for CDMA. This should not be surprising, as BLAST sees its real potential only when multiple antenna elements are available on both sides of the link. In order to get an idea of the asymptotic limitations of an $(M, 1)$ transmit system, we can evaluate the expression in (12) for the case of an infinity of transmit elements (keeping the total transmit power finite and fixed). The capacity of such a system would be

$$C_{\infty,1} = 1/2 \log_2 (1 + 2\rho) \qquad (13)$$

since $\lim_{M\to\infty} \frac{1}{M} \sum_{m=1}^{M} |h_m|^2 = 1$. It is clear from (13) that, as the number of antenna elements grows, the incremental capacity gains drop and eventually vanish. This reflects the fact that, in the limit of a large number of antenna elements, the channel becomes deterministic.

In [3] it was shown how the above-desribed $(2, 1)$ system can be used to achieve 4-branch diversity gain if 2 antennas are used at the receiver. In the general case of M transmit and M receiver antennas, the receiver output samples would take the form

$$z(i) = \frac{1}{\sqrt{M}} \left(\sum_{m=1}^{M} \sum_{n=1}^{M} |h_{mn}|^2\right) b(i) + n(i) \qquad (14)$$

or, by normalizing,

$$\hat{b}(i) = b(i) + n'(i) \qquad (15)$$

corresponding to an SNR of

$$\rho_{M,M} = 2\rho \sum_{m=1}^{M} \sum_{n=1}^{M} |h_{mn}|^2/M \qquad (16)$$

The resulting capacity expression is

$$C_{M,M} = 1/2 \log_2 \left(1 + \frac{2\rho}{M} \sum_{m=1}^{M} \sum_{n=1}^{M} |h_{mn}|^2\right) \qquad (17)$$

Notice that, compared to (12), where the SNR is χ^2-distributed with $2M$ (real) degrees of freedom, in (17), the SNR is χ^2 with $2M^2$ degrees of freedom. This reflects an extra diversity gain due to the extra $M-1$ receive antenna elements. As the number of receive and transmit elements grows, the expression (17) converges to:

$$C_{M,M} = 1/2 \log_2 (1 + 2M\rho) \simeq 1/2 \log_2 (2M\rho) \quad (18)$$

Notice the different way in which capacity scales in the symmetrical (M, M) case, as compared to (13): now the capacity increases logarithmically with the number of antennas. This is clearly better than the $(M, 1)$ case, it is however, as expected, inferior to an (M, M) BLAST scheme, whose capacity grows roughly linearly with M (the corresponding BLAST-type capacity would be $C = M/2 \log_2(1 + 2\rho/M)$). For complex inputs, the corresponding capacities can be similarly obtained and the same tradeoffs hold.

5 Numerical results

In Figure 3 we have evaluated some of the derived capacity expressions for different numbers of antenna elements as a function of ρ. We assume that each channel coefficient is an independent Rayleigh fading

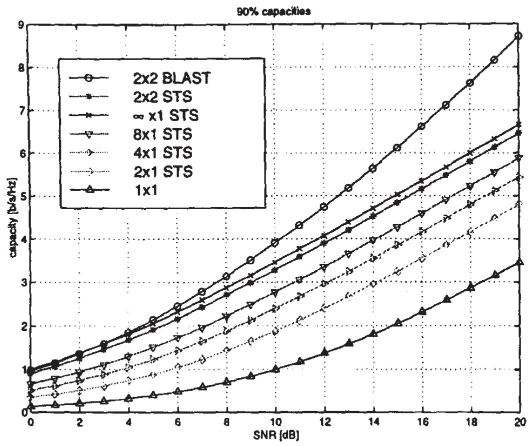

Figure 3: Multiple Tx/ Rx antenna capacities

random variable ($\mathcal{N}(0,1)$) and that the input is complex in all cases. The plotted capacities are for a 10^{-1} outage probability (i.e. we plot the performance of the worse among the top performing 90% cases), out of a total of 10^4 independent runs each time. Notice how, as expected, the throughput gain is large when transitioning from a conventional (1,1) to a (2,1) STS system. Also notice how additional transmitter antennas in the $(M,1)$ case begin offering diminishing returns after some point, approaching the deterministic capacity of the hypothetical $(\infty,1)$ system. It is also worth noticing that the (2,2) STS capacity is roughly the same as that of the $(\infty,1)$ system. This shows the high potential of adding extra antennas at the subscriber unit. Another important observation is that the capacity of the 2×2 STS scheme is very close to that of the 2×2 BLAST at low SNR's, and only modestly lower than it at higher SNR's. We should emphasize at this point that the STS scheme can use state-of-the-art (such as Turbo) temporal coding in an overlay fashion in order to approach its Shannon limit, whereas full BLAST capacity is very hard to reach with today's space-time codes. Moreover, the more practical vertical-BLAST (V-BLAST) transceiver (which allows, similarly to STS, for overlay coding) has a lower Shannon capacity than the optimal BLAST capacity shown in the figure. The STS capacity would hence be even closer to the V-BLAST capacity for $M=2$. This makes STS a very good transmission scheme when small numbers of antennas (such as two) are available on each side of the link.

6 Conclusions

We have studied the problem of spectral efficiency of CDMA transmit diversity systems that employ a recently proposed transmit diversity technique called Space-Time-Spreading. Our results show that a simple (2,1) STS system can offer important capacity gains over a conventional single-antenna system. Moreover, for symmetrical systems with small numbers of antennas (such as 2), we showed that STS offers capacity gains that are comparable with those of corresponding BLAST systems. Combined with the fact that STS can benefit from powerful error correction codes to approach closely its predicted capacities, these findings make STS a good transceiver approach for wireless CDMA systems that use small numbers of antenna elements. Current work is focused on the evaluation of the impact of multipath channels, as well as on the impact of STS in system capacity for cellular networks.

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