TRANSMIT DIVERSITY AS A COMBINATION OF SPATIAL AND DELAY DIVERSITY

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ABSTRACT

We document here performance results of some of the algebraic space-time (ST) block codes based on the combination of Hadamard spreading sequences with rotated constellations. Given a vector $\boldsymbol{x}$ in a rotated constellation in dimension n that has full spatial diversity, we spread each component x_j of $\boldsymbol{x}$ by a different Hadamard sequence of length n and send the resulted sequence over n antennas at the time j. These codes transmit at a rate of 1 symbol/sec and have a maximum transmit diversity gain and near maximum coding gain but they only exist for $n = 1, 2$ and n multiple of 4. We also propose new schemes for the other dimensions to obtain maximum transmit diversity and near maximum coding gain based on the combination of delay and the spatial diversity of rotated constellations. We compare the proposed codes with some existing ST block codes and comment on the large improvement in performance at the detrimental of moderate increase of decoding complexity.

1. INTRODUCTION

Recently, signal processing over multiple transmit-receive antennas has received a lot of attention [9, 7, 1, 8, 3, 4]. This is due to the large capacity of the multi-antenna system and the constant need to transmit at higher data rate with better performance over the wireless channel. The theory of space-time (ST) codes has been established in [9] where hand constructed trellis ST codes were presented. In [8] ST block codes from orthogonal design have been explored, where ST codes over real constellations were given till dimension $n = 8$ verifying the full transmit diversity criteria and transmitting at 1 symbol/sec. Over complex constellations, ST block codes of rate 1/2 symbol/sec have been presented for $n = 3, 4$. In [4] we have introduced a general framework to build algebraic ST codes of a rate of 1 symbol/sec with full transmit diversity over real and complex constellations.

In this paper we document some performance results of these codes and give further analysis of their good performance. Furthermore, we propose new simple schemes of algebraic ST codes with maximum transmit diversity and near maximum coding gain for any n in the absence of multi-path.

2. SYSTEM MODEL

In a system of n transmit and m receive antennas, we transmit n symbols simultaneously from all the transmit antennas at each time. The launched power is normalized by n. The transmit diversity is obtained by the joint coding of the transmitted symbols over n antennas and l symbol periods; its maximum value equals to n. A block ST code is defined by an $n \times l$ matrix $\boldsymbol{B}$ where one transmits b_{jt} from the transmitter $1 \leq j \leq n$, at time $1 \leq t \leq l$. In a quasi-static fading model the received signal is given by

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{B} + \boldsymbol{\nu}, \tag{1}$$

where $\boldsymbol{H}$ denotes the channel transfer $(m \times n)$ matrix, with h_{kj} the fade between transmitter j and receiver k. We assume that the transmit and receive antennas are spaced enough such that we can model h_{kj} by independent complex Gaussian random variables of variance 0.5 per real dimension. $\boldsymbol{\nu}$ is $m \times l$ complex AWGN matrix component-wise independent.

Over a quasi-static fading, for a block ST code $\boldsymbol{B}$ one has [9]

- *The rank criterion:* In order to achieve the maximum transmit-receive diversity nm, the matrix $\boldsymbol{B}(\boldsymbol{x}, \boldsymbol{e})$ taken on the difference $\boldsymbol{x} - \boldsymbol{e}$, has to be full rank for any pair of distinct codewords $\boldsymbol{x}$, $\boldsymbol{e}$. The rank of $\boldsymbol{B}(\boldsymbol{x}, \boldsymbol{e})$, r, is called the diversity gain.
- *The determinant criterion:* The minimum of the r-th root of the sum of determinants of all $r \times r$ principal cofactors of $\boldsymbol{A}(\boldsymbol{x}, \boldsymbol{e}) \triangleq \boldsymbol{B}\boldsymbol{B}^H$ taken[1]

[1] The superscript ^{H} denotes the transpose conjugate.

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over all the codewords pairs $(\boldsymbol{x}, \boldsymbol{e})$ must be maximized (coding gain).

In [4] we proposed the following ST coding scheme: $l = n$, $\boldsymbol{B} = \Xi_n$ with

$$\Xi_n \triangleq \mathcal{H}_n \mathrm{diag}(x_1, \ldots, x_n), \qquad (2)$$

where $\mathcal{H}_n$ denotes the Hadamard transform in dimension n and $\boldsymbol{x} = (x_1, \ldots, x_n)^T = \boldsymbol{M}_n \boldsymbol{a}^T$, with $\boldsymbol{a}$ the transmitted symbols vector[2]; $\boldsymbol{M}_n$ is a full spatial diversity rotation [2]. These codes have a transmit diversity of n and a coding gain[3] $\delta_n = (dp_{n,min})^{2/n}$, with $dp_{n,min}$ the minimum product distance [2] of the rotated constellation given by the quantity

$$dp_{n,min} \triangleq \min_{\boldsymbol{y}=\boldsymbol{x}_1-\boldsymbol{x}_2, \boldsymbol{x}_1 \neq \boldsymbol{x}_2 \in Q} \prod_{j=1}^{n} |y_j|, \qquad (3)$$

where Q is the considered rotated constellation in dimension n.

Here, we propose a ST schemes for the dimensions n for which there exist no Hadamard transform.

3. COMBINATION OF DELAY AND SPATIAL DIVERSITY

3.1. Coding Algorithm

Given a column vector $\boldsymbol{x} = \boldsymbol{M}_n \boldsymbol{a}^T$ in a rotated constellation in dimension n with $dp_{n,min} > 0$, we propose to send x_j at the antenna j at time j and zeros elsewhere. In other words, we propose to choose $l = n$, $\boldsymbol{B} = \boldsymbol{\Upsilon}_n$ with

$$\boldsymbol{\Upsilon}_n \triangleq \mathrm{diag}(x_1, \ldots, x_n). \qquad (4)$$

Proposition 1 *Given a full spatial diversity rotation $\boldsymbol{M}_n$ with $dp_{n,min} > 0$, the ST block code $\boldsymbol{\Upsilon}_n$ has a transmit diversity of n and a coding gain*

$$\delta_n = (dp_{n,min})^{2/n}. \qquad (5)$$

Proof: Let $\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{e} = \boldsymbol{M}_n(\boldsymbol{a} - \boldsymbol{b})^T$ such that $\boldsymbol{a} \neq \boldsymbol{b}$. Consider the ST code $\boldsymbol{\Upsilon}_n$ (4) at the word $\boldsymbol{y}$. Since $\boldsymbol{M}_n$ generates a full spatial diversity lattice, one has $y_j \neq 0 \ \forall j = 1 \ldots n$ taken over all the vectors $\boldsymbol{a} \neq \boldsymbol{b}$ in the considered constellation. It follows that the matrix $\mathrm{diag}(y_1, \ldots, y_n)$ has a full rank, and also $\boldsymbol{\Upsilon}_n$ has a full rank over all the differences of codewords.
For the coding gain compute

$$\det(\boldsymbol{\Upsilon}_n \boldsymbol{\Upsilon}_n^H) = \prod_{j=1}^{n} |y_j|^2. \qquad (6)$$

[2]The superscript ^{T} denotes the transpose.

[3]Over normalized constellations and after the power normalization by the number of transmit antennas.

n	2	3	4	6
δ_n	0.4472	0.2733	0.1581	0.1222

Table 1: Coding gains of $\boldsymbol{\Upsilon}_n$ codes for $n = 2, 3, 4, 6$.

Taking the minimum over $\boldsymbol{y}$ of the determinant above and then taking the n-th root one obtains the coding gain of the $\boldsymbol{\Upsilon}_n$ ST code (5). □

Table 1 lists the coding gain of $\boldsymbol{\Upsilon}_n$ codes over normalized constellations for $n = 2, 3, 4, 6$ where the rotations are taken from [2].

For the dimensions $n = 2$ and n multiple of 4, Ξ_n and $\boldsymbol{\Upsilon}_n$ have the same performance. The inconvenience of $\boldsymbol{\Upsilon}_n$ compared to Ξ_n [4] is the high peak to average values of the launched power over n periods and n transmit antennas. Also $\boldsymbol{\Upsilon}_n$ are more sensitive to peak power limitation at the transmitter.

3.2. Decoding Algorithm

A perfect channel state-information (CSI) is supposed available at the receiver. For the ST codes $\boldsymbol{\Upsilon}_n$, the received signal in (1) could be written as

$$\begin{aligned} \boldsymbol{r}_1 = Vec(\boldsymbol{r}^T) &= \begin{bmatrix} \mathrm{diag}(\sqrt{n}\boldsymbol{H}_1) \\ \vdots \\ \mathrm{diag}(\sqrt{n}\boldsymbol{H}_m) \end{bmatrix} \boldsymbol{x} + Vec(\boldsymbol{\nu}^T) \\ &= \boldsymbol{\mathcal{A}} \boldsymbol{M}_n \boldsymbol{a}^T + \boldsymbol{\nu}_1, \end{aligned} \qquad (7)$$

where $\boldsymbol{H}_j$ denotes the jth row of $\boldsymbol{H}$ representing the jth receiver. $Vec(\boldsymbol{r})$ arranges the matrix $\boldsymbol{r}$ in one column vector by putting its columns one after the other.

Let $\boldsymbol{\mathcal{A}}_p$ denote the $mn \times mn$ matrix of the inverse of the phase of the matrix $\boldsymbol{\mathcal{A}}$, i.e.,

$$\boldsymbol{\mathcal{A}}_p = \mathrm{diag}\left(e^{-\mathrm{phase}(h_{11})}, \ldots, e^{-\mathrm{phase}(h_{1n})}, \ldots \right.$$
$$\left. \ldots, e^{-\mathrm{phase}(h_{m1})}, \ldots, e^{-\mathrm{phase}(h_{mn})}\right).$$

The receiver performs a coherence detection by correcting the phase of the new channel transfer matrix $\boldsymbol{\mathcal{A}}$

$$\begin{aligned} \boldsymbol{r}_2 &= \boldsymbol{\mathcal{A}}_p \boldsymbol{r}_1 = \boldsymbol{\mathcal{A}}_p \boldsymbol{\mathcal{A}} \boldsymbol{M}_n \boldsymbol{a}^T + \boldsymbol{\mathcal{A}}_p \boldsymbol{\nu}_1 \\ &= \boldsymbol{\mathcal{A}}' \boldsymbol{M}_n \boldsymbol{a}^T + \boldsymbol{\nu}_2, \end{aligned} \qquad (8)$$

where $\boldsymbol{\nu}_2$ is a complex $mn \times 1$ Gaussian noise componentwise independent with variance σ^2 per real dimension: $E[\boldsymbol{\nu}_2 \boldsymbol{\nu}_2^H] = 2\sigma^2 \boldsymbol{I}_{mn}$. If the number of receivers equals one then one can apply directly the sphere decoder (SD) [10, 6] on (8). when $m > 1$, we propose the following treatment based on single values decomposition (SVD) [5]

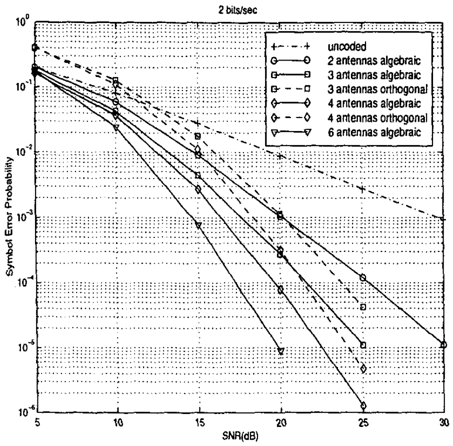

Figure 1: Symbol error rate as a function of SNR, one receive antenna.

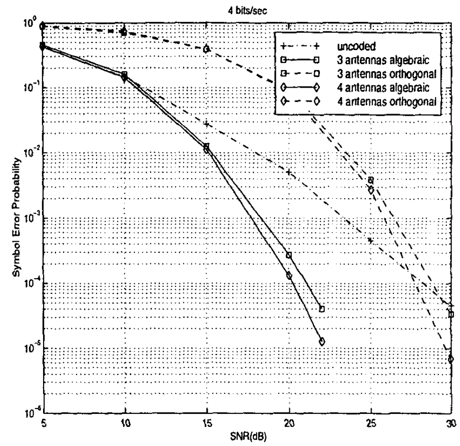

Figure 2: Symbol error rate as a function of SNR, two receive antennas.

- $[\boldsymbol{U}, \boldsymbol{\Sigma}, \boldsymbol{V}] = \mathrm{SVD}(\mathcal{A}')$: $\mathcal{A}' = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$, with $\boldsymbol{\Sigma}$ $n \times n$ diagonal matrix, $\boldsymbol{U}$ is an $mn \times n$ matrix and $\boldsymbol{V}$ is an $n \times n$ matrix with $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{V}\boldsymbol{V}^T = \boldsymbol{I}_n$.
- $\boldsymbol{r}_3 \longleftarrow \boldsymbol{U}^T\boldsymbol{r}_2 = \boldsymbol{\Sigma}\boldsymbol{V}^T\boldsymbol{M}_n\boldsymbol{a}^T + \boldsymbol{\nu}_3$, where $\boldsymbol{\nu}_3 = \boldsymbol{U}^T\boldsymbol{\nu}_2$ with $E[\boldsymbol{\nu}_3\boldsymbol{\nu}_3^H] = \boldsymbol{U}^T E[\boldsymbol{\nu}_2\boldsymbol{\nu}_2^H]\boldsymbol{U} = 2\sigma^2\boldsymbol{I}_n$.
- Apply the SD on $\boldsymbol{r}_3$.

The SD has a polynomial complexity in n and independent of the size of the used constellation [10, 6, 5].

Writing the received signal as in (7) allows one to understand how the ST $\boldsymbol{\Upsilon}_n$ codes exploit the transmit diversity. Using a ST $\boldsymbol{\Upsilon}_n$ code over n transmitters and m receivers is equivalent to sending the word $(x_1, \ldots, x_n)$ over one transmitter and m receivers during n periods of time, where the channel changes randomly every time instant. The latter scheme has a diversity of mn [2] since the lattice from which we transmit the words has a full spatial diversity.

4. SIMULATION RESULTS

We normalize the used constellations such that the average energy per symbol $\bar{E}_s = 1$. The transfer matrix $\boldsymbol{H}$ is modeled as in (1); the AWGN has a variance $\sigma^2 = 1/(2\mathrm{SNR})$ per real dimension. Comparisons with the ST block codes from orthogonal design [8] (denoted by $\mathcal{G}_n$) are done for $n = 3, 4$, at the same spectral efficiency.

Figure 1 shows the performance of the codes $\boldsymbol{\Upsilon}_n$ for $n = 2, 3, 4, 6$ with the normalized 4-QAM modulation and one receive antenna. It also shows the performance of the codes $\mathcal{G}_3$ and $\mathcal{G}_4$ with the normalized 16-QAM modulation. At a symbol error probability of 10^{-5}, $\boldsymbol{\Upsilon}_6$ shows a gain of 10 dB over $\boldsymbol{\Upsilon}_2$. At the spectral efficiency of 2 bits/sec, and at the same diversity gain $\boldsymbol{\Upsilon}_3$ shows a coding gain of more than 2 dB over $\mathcal{G}_3$; $\boldsymbol{\Upsilon}_4$ shows a gain of more than 1 dB over $\mathcal{G}_4$.

Figure 2 shows the performance of the codes $\boldsymbol{\Upsilon}_n$ and $\mathcal{G}_n$ for $n = 3, 4$ at the spectral efficiency of 4 bits/sec with two receive antennas. We notice that the improvement over the ST codes from orthogonal design is enhanced when the spectral efficiency increases. The latter observation is explained as follows: For a normalized constellation, the ST codes $\boldsymbol{\Upsilon}_n$ and $\mathcal{G}_n$ preserve their diversity and coding gains. On the other hand, when increasing the size of the modulation one looses approximately 3 dB per each added bit. Since at the same SNR $\boldsymbol{\Upsilon}_n$ transmits twice times the number of bits transmitted by $\mathcal{G}_n$, the difference in performance is enlarged when the size of the constellation increases. For example, at the spectral efficiency of 4 bits/sec and two receive antennas, $\boldsymbol{\Upsilon}_n$ has a gain of more than 7 dB over $\mathcal{G}_n$ for $n = 3, 4$.

5. CONCLUSION

We presented in this paper new schemes for exploiting the transmit diversity in a multi-antenna environment. Either by the use of Hadamard spreading sequences for n multiple of 4, or the combination of delay and spatial diversity for the other dimensions, we have shown that by the use of full spatial diversity rotated constellations one can transmit at 1 symbol/sec with maxi-

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mum transmit diversity and near maximum coding gain along with moderate decoding complexity.

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