shaping results in an opposite tendency. Hence, when a notch filter is used in connection with a finite or variable load, the *RC* line should be shaped as much in the form of Fig. 1*c* of Reference 1 as possible, to reduce the effect of load resistance.

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References

1 SU, K. L.: 'Hyperbolic *RC* transmission line', *Electronics Letters*, 1965, **1**, p. 59
2 SU, K. L.: 'Selectivity of notch filters using non-uniform *RC* lines', *ibid.*, **1**, p. 204

STABILITY OF CONTROL SYSTEMS WITH TIME DELAY

To find the approximate stability limit on the forward gain in control systems with small time delay, this note suggests approximating the exponential in the characteristic equation by the first few terms of its series and using the Routh–Hurwitz criterion. This approximation avoids all the time-consuming graphical work and gives a somewhat pessimistic maximum bound for the gain constant.

There are many control systems in which the corrective action applied at any particular time is dependent on the error T seconds earlier. The block diagram of such a system is shown in Fig. 1, where $G(s)$ is the transfer function of the plant given by

$$G(s) = \frac{s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

with $n > m$, and K is the gain constant of the system, which is assumed to be positive. The characteristic equation of this system is given by

$$1 + K\,G(s)\,\varepsilon^{-sT} = 0 \quad \ldots \quad (1)$$

or

$$G(s) = -K^{-1}\,\varepsilon^{sT}$$

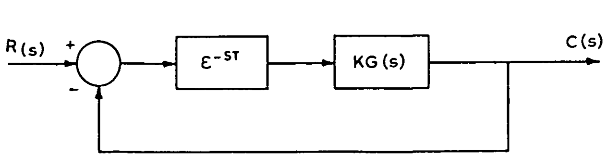

Fig. 1 *Block diagram of control system with time delay*

For a given value of the time delay T, the maximum value of K for stability can be obtained[1] by considering the intersection of the Nyquist locus of $G(j\omega)$ with the locus (circles of radius K^{-1}) of $(-K)^{-1}\exp(j\omega T)$ for different values of K and finding the value of K which gives identical frequencies on both the loci at the intersection point (Fig. 2).

The above procedure for finding the maximum value of the gain K for stability is rather time-consuming. When one is interested in finding the approximate maximum value of K for stability, this note suggests approximating the characteristic equation (eqn. 1) by

$$1 + K\,G(s) \sum_{k=0}^{n-m} \frac{(-sT)^k}{k!} = 0$$

when the time delay T is small, and using the Routh–Hurwitz criterion to find the maximum value of K for stability. This approximation avoids all the graphical work and gives a somewhat pessimistic maximum bound on K, as the following two examples show:

Example 1: Let $T = 0{\cdot}25$s, and

$$G(s) = \frac{1}{s(s+1)(s+2)(s+3)}$$

The characteristic equation is given by

$$1 + \frac{K\exp(-0{\cdot}25\,s)}{s(s+1)(s+2)(s+3)} = 0$$

Using the above approximation and considering only the first three terms of the series for the exponential, the characteristic equation may be written as

$$s^4 + 6s^3 + (11 + 0{\cdot}03125K)s^2 + (6 - 0{\cdot}25K)s + K = 0$$

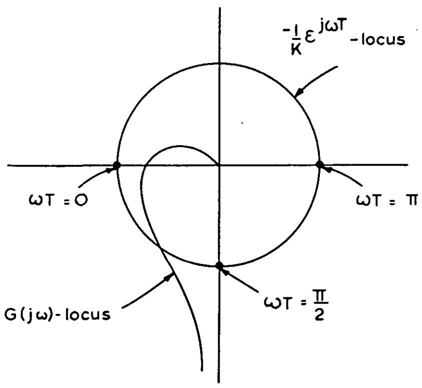

Fig. 2 *Nyquist diagram to find maximum permissible value of gain*

Application of the Routh–Hurwitz criterion to the above equation yields the maximum value of K for stability as 7·2, which compares well with the value of 7·33 obtained using the Nyquist criterion.

Example 2: Let $T = 0{\cdot}2$s, and

$$G(s) = \frac{(s + 0{\cdot}5)}{s(s+1)(s+2)}$$

The characteristic equation of the system is given by

$$1 + \frac{K(s + 0{\cdot}5)\exp(-0{\cdot}2s)}{s(s+1)(s+2)} = 0$$

Approximating the exponential as described earlier, the characteristic equation may be written as

$$(1 + 0{\cdot}02K)s^3 + (3 - 0{\cdot}19K)s^2 + (2 + 0{\cdot}9K)s + 0{\cdot}5K = 0$$

Applying the Routh–Hurwitz criterion to the above equation gives the maximum value of K for stability as 12·68, whereas the value obtained by using the Nyquist criterion is 13·66.

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Reference

1 KUO, B. C.: 'Automatic control systems' (Prentice-Hall, 1962)

TOUCH DISPLAY—A NOVEL INPUT/OUTPUT DEVICE FOR COMPUTERS

A novel input/output device for computer systems has wires, sensitive to the touch of a finger, on the face of a cathode-ray tube on which information can be written by the computer. This device, the 'touch display', provides a very efficient coupling between man and machine.

Touch wires

The key to the possibilities of the touch display is the provision of the 'touch wires', which are sensitive to the touch of a finger.

The touch wires themselves are completely passive, being simply pieces of tinned-copper wire (¾in long 22s.w.g.) attached to the front surface of a mask moulded to fit the front surface of a cathode-ray tube. Any convenient number, say 16, are attached to the mask and are disposed so that the computer can write an item of information against each one.

These touch wires are connected by fine insulated wires, running in grooves in the rear surface of the mask, to screened cables from the edge of the mask to the electronic equipment.

Each touch wire is made sensitive by connecting it via the screened lead to become one arm of an a.c. bridge circuit. The bridge is initially balanced but becomes unbalanced by the electrostatic capacitance of the operator.

A good clean signal is obtained (signal/noise ratio $\simeq$ 100 : 1) on touching, but the proximity effect is negligible. Each bridge can accept two touch wires, and a number of bridges can use one amplifier and a phase-sensitive detector on a time-sharing basis.

Use of the touch display

The touch display coupled to a computer can be considered as a keyboard, the engravings of which can be varied to suit the needs of the moment; more fundamentally, it can be considered as a means by which a computer presents a list of those 'choices' which are available to a controller at any given instant and as the means by which the controller indicates his choice back to the computer.

In the way that in mathematics it is profitable to consider a complex function to be built up of simpler functions (e.g. sine waves, as in Fourier analysis), it is profitable to consider the 'control function', i.e. whatever a human controller does in a real-time computing system, as a series of 'choices', each choice being made from a limited list and the content of each list being determined by the previous choice.

It is in the light of this hypothesis, i.e. that the control function can be equated to a series of choices, that the concept of the touch display is seen to be important, because it combines the computer's ability to set up lists of choices with a facility for the controller to indicate his choice in a natural manner.

In one possible use of the touch display, at an air-traffic-control position, the computer can present to the controller the following lists on successive pictures, in each picture one item of the list being against one touch wire:

- list of call signs of the aircraft under control
- list of items in the flight plan of the aircraft selected
- list of values for the first element of the item
- list of values for the second element of the item
- list of values for the third element of the item
- list of executive actions.

A list of values for an element of an item may be the numerals 0 to 9 or symbols such as those for 'climbing', 'holding' etc., and can be displayed exactly according to the possibilities of that element; e.g. the list for the first digit of a time need not show a value greater than five (tens of minutes).

During the operation the display may show, in addition to the current list and away from the touch wire, reference data, such as the complete flight plan and the amendments so far put in.

One of the touch displays has been made up and programmed for the air-traffic-control task, and an evaluation of it in this role is being arranged.

Acknowledment

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ONLINE DIGITAL PREDICTION AND CONTROL OF THE OUTPUT OF A LINEAR TIME-VARYING SYSTEM

The system time-varying weighting function is approximated by a grid of sample estimates. Criteria are given for specifying the sample spacing in each dimension and for predicting the sample estimates at the next sampling instant. The criteria are based on maximum sample smoothness, subject to a prescribed signal/noise ratio at the system output.

Consider the identification scheme of Fig. 1. Let us assume that the physical system has a lowpass characteristic, which for practical purposes may be represented by a weighting function bandlimited to $|\omega| < \pi/T_1$ rad/s, with a time variation of the weighting function bandlimited to $|\omega| < \pi/T_2$ rad/s. From the Nyquist sampling theorem[1] the weighting function is uniquely described by an infinite set of time-varying weights spaced at time interval T_1, where each time-varying weight is sampled over all time at instants spaced T_2 apart. The digital model of the physical system must be described by a finite number of weights; specifically, let the model output be

$$y_m = \sum_{n=0}^{N-1} w_{nm} x_{m+n} \qquad m = 0, \ldots M-1 \quad . \quad . \quad (1)$$

where w_{nm} are time-varying weights to be determined over an output measurement interval $0 \leqslant m \leqslant M-1$ from bandlimited sampled values x_m, z_m of normal operating records $x(t)$, $z(t)$; m counts sampling instants backwards in time from the present instant $m = 0$. Let us assume that x_m, z_m are bandlimited to $|\omega| < \pi/T_3$ rad/s, and the system is able to respond to the highest significant frequency components of input; i.e. $T_3 > T_1$. The assumption that the higher-frequency components of the weighting function are not excited by the input means that a smoothed weighting function is acceptable. The choice of N in eqn. 1 and the estimation of sampling intervals T_2 and T_3 (T_3 is normalised to 1 s) are considered later. The choice of M in eqn. 1 can be shown* to be expressible in terms of N and certain statistical properties of the input signal and output noise.

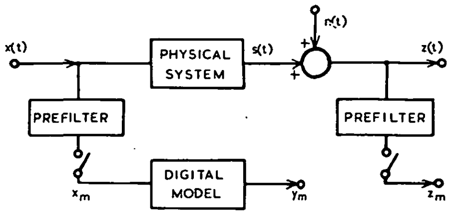

Fig. 1 *Notation used to describe the identification problem*

Smoothness of the weights

Let the sample estimates of the smoothed weighting function ($T_3 > T_1$) at any instant be called a set of weights, and let the sets of weights at instants spaced T_2 apart be called sample sets. For simplicity, let the estimate of T_2 be $(M-1)/(K-1)$, an integer, and let k $(0 \leqslant k \leqslant K-1)$ be the count of sample sets in the measurement interval $M-1$.

The kth sample set may be regarded as the strengths of a sequence of impulses spaced at 1 s intervals. Considering the sequence of impulses to be passed through an ideal lowpass filter of bandwidth $\sigma\pi$ rad/s $(0 < \sigma \leqslant 1)$, the filter output is

$$w_\sigma'(k, \tau) = \sum_{n=0}^{N-1} w_{nk} \operatorname{sinc} \sigma\pi(\tau - n) \qquad k = 0, \ldots K-1 \quad . \quad . \quad . \quad . \quad (2)$$

where $\operatorname{sinc} \theta = (\sin \theta)/\theta$. $w_\sigma'(k, \tau)$ will be referred to as the σ factor Nyquist envelope of the kth sample set; in the case of the unit-factor Nyquist envelope, the weights are sample points of the envelope. We define the coefficient

$$F_\sigma' = \frac{\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} [w_\sigma'(k, n) - w_{nk}]^2}{\sum_{k=0}^{K-1} \sum_{n=0}^{N-1} w_{nk}^2} \quad . \quad . \quad . \quad (3)$$

as a normalised measure, averaged over the K sample sets, of the relative power content of the weighting-function spectral density in the frequency band $\sigma\pi \leqslant \omega \leqslant \pi$ (π rad/s is the Nyquist frequency). The frequency band $(1-\sigma)\pi$ may be thought of as a guardband, and F_σ' may be regarded as an indicator of the rate of roll-off of the spectral density adjacent to the Nyquist frequency. For a preset value of $\sigma(\sigma = \frac{1}{2}$ gives a reasonable guardband), we note that the smaller the value of F_σ', the smoother is the sample set. Likewise, the σ factor Nyquist envelope of the time variation of the nth weight $(n = 0, \ldots N-1)$ is

$$w_\sigma''(t, n) = \sum_{k=-\infty}^{\infty} w_{nk} \operatorname{sinc} \sigma\pi \left(\frac{K-1}{M-1}\right)(t-k) \qquad n = 0, \ldots N-1 \quad . \quad . \quad . \quad (4)$$

where, mathematically, the contribution from all sample sets, past and future, must be included. Analogous to the coefficient F_σ', we define a coefficient

$$F_\sigma'' = \frac{\sum_{n=0}^{N-1} \sum_{k=0}^{K-1} [w_\sigma''(k, n) - w_{nk}]^2}{\sum_{n=0}^{N-1} \sum_{k=0}^{K-1} w_{nk}^2} \quad . \quad . \quad . \quad (5)$$

Note that $w_\sigma'(k, n) \neq w_\sigma''(k, n)$, because ideal-filter smoothing is applied independently in the two time dimensions t and τ. Since eqn. 4 is not realisable, let us arbitrarily set future values of sample sets $(k = -1, -2 \ldots)$ equal to the present sample set $(k = 0)$. Also, let us disregard the effect of sample sets further in the past than, say, $k = 2K-1$; i.e. let

* BROWN, R. F.: 'Optimum measurement interval in real-time synthesis of linear time-varying systems'. Submitted for publication in *Proc. IEE*