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Modern Optical Engineering

The Design of Optical Systems

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Chapter 9

Basic Optical Devices

This chapter will be devoted to the first-order optics of several typical optical systems. The number of systems covered here is, of necessity, limited, and the emphasis is placed on those fundamental principles which are applicable to a broad range of optical systems. The rather straightforward algebraic manipulations and the considerations of image size and position which follow are quite typical of those encountered in the preliminary stages of optical system design. Constructional details of the optical components have been deliberately omitted and are discussed at considerable length in later chapters. Note that the system diagrams in this chapter show the components as simple lenses. These could equally well be mirrors instead of lenses, and typically are fairly complex assemblies of lens elements.

9.1 Telescopes, Afocal Systems

The primary function of a telescope is to enlarge the apparent size of a distant object. This is accomplished by presenting to the eye an image which subtends a larger angle (from the eye) than does the object. The magnification, or power, of a telescope is simply the ratio of the angle subtended by the image to the angle subtended by the object.* Nominally, a telescope works with both its object and image located at infinity; it is referred to as an afocal instrument, since it has no focal length. In the following material, a number of basic relationships for telescopes and afocals will be presented, all based on systems with both object and image located at infinity. In practice, small

*For large angles, the magnification is the ratio of the tangents of the half-angles.

departures from these infinite conjugates are the rule, but for the most part they may be neglected. However, the reader should be aware that the fact that the object and/or the image are not at infinity will occasionally have a noticeable effect and must then be taken into account. This is usually important only with low-power devices. See also the comments on instrument myopia in Sec. 5.4.

There are three major types of telescopes: astronomical (or inverting), terrestrial (or erecting), and Galilean. An astronomical or Keplerian telescope is composed of two positive (i.e., converging) components spaced so that the second focal point of the first component coincides with the first focal point of the second, as shown in Fig. 9.1a.The objective lens (the component nearer the object) forms an inverted image at its focal point; the eyelens then reimages the object at infinity where it may be comfortably viewed by a relaxed eye. Since the internal image is inverted, and the eyelens does not reinvert the image, the view presented to the eye is inverted top to bottom and reversed left to right.

In a Galilean, or "Dutch," telescope, 9.1b, the positive eyelens is replaced by a negative (diverging) eyelens; the spacing is the same, in that the focal points of objective and eyelens coincide. In the Galilean scope, however, the internal image is never actually formed; the object

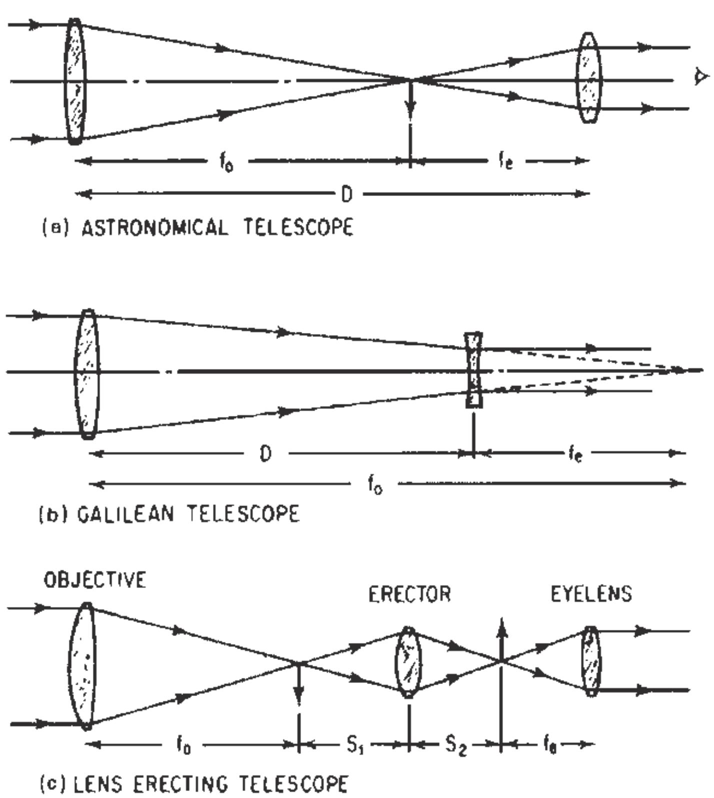

Figure 9.1 The three basic types of telescope.

for the eyelens is a "virtual" object, no inversion occurs, and the final image presented to the eye is erect and unreversed. Since there is no real image formed in a Galilean telescope, there is no location where cross hairs or a reticle may be inserted.

Assuming the components of the telescope to be thin lenses, we can derive several important relationships which apply to *all* telescopes and afocal systems and which are of great utility. First, it is readily apparent that the length (D) of a simple telescope is equal to the sum of the focal lengths of the objective and eyelens.

$$D = f_o + f_e \tag{9.1}$$

Note that in the Galilean telescope, the spacing is the difference between the absolute values of the focal lengths since f_e is negative.

The magnification, or magnifying power, of the telescope is the ratio between u_e, the angle subtended by the image, and u_o, the angle subtended by the object. The size (h) of the internal image formed by the objective will be

$$h = u_o f_o \tag{9.2}$$

and the angle subtended by this image from the first principal point of the eyelens will be

$$u_e = \frac{-h}{f_e} \tag{9.3}$$

Combining Eqs. 9.2 and 9.3, we get the magnification

$$\text{MP} = \frac{u_e}{u_o} = \frac{-f_o}{f_e} \tag{9.4}$$

and

$$f_e = D/(1 - \text{MP})$$

$$f_o = \text{MP}D/(1 - \text{MP})$$

The sign convention here is that a positive magnification indicates an erect image. Thus, if objective and eyelens both have positive focal lengths, MP is negative and the telescope is inverting. The Galilean scope with objective and eyelens of opposite sign produces a positive MP and an erect image.

Note that u_o can represent the *real* angular field of view of the telescope and u_e the *apparent* angular field of view, and that Eq. 9.4 defines the relationship between the real and apparent fields for small angles. For large angles, the tangents of the half-field angles should be substituted in this expression.

From Chap. 6 we recall that the exit pupil of a system is the image (formed by the system) of the entrance pupil. In most telescopes the objective clear aperture is the entrance pupil and the exit pupil is the image of the objective as formed by the eyelens. Using the newtonian expression relating object and image sizes ($h'=hf/x$), and substituting CA_e (the exit pupil diameter) and CA_o (the entrance pupil diameter) for h' and h, f_e for f, and $-f_o$ for x, we get

$$\frac{CA_o}{CA_e} = \frac{-f_o}{f_e} = \text{MP} \tag{9.5}$$

While the above derivation has assumed the entrance pupil to be at the objective, Eq. 9.5 is valid regardless of the pupil location, as is obvious from the rays sketched in Fig. 9.1.

We also can get a simple expression for the eye relief of the Kepler telescope as follows:

$$R = (\text{MP} - 1) f_e/\text{MP}$$

The amount of motion of the eyepiece needed to focus the telescope for someone who is nearsighted or farsighted is given by

$$\delta = Df_e^2/1000$$

where δ is in millimeters and D is in diopters.

Equations 9.4 and 9.5 can be combined to relate the external characteristics (magnifications, fields of view, and pupils) of *any* afocal system, regardless of its internal construction

$$\text{MP} = \frac{u_e}{u_o} = \frac{CA_o}{CA_e} \tag{9.6}$$

The erecting telescope, Fig. 9.1c, consists of positive objective and eyelenses with an erecting lens between the two. The erector reimages the image formed by the objective into the focal plane of the eyelens. Since it inverts the image in the process, the final image presented to the eye is erect. This is the form of telescope ordinarily used for observing terrestrial objects, where considerable confusion can result from an inverted image. (An erect image may also be obtained by the use of an erecting prism as discussed in Chap. 4.) The magnification of a terrestrial telescope is simply the magnification that the telescope would have without the erector, multiplied by the linear magnification of the erector system

$$\text{MP} = -\frac{f_o}{f_e} \cdot \frac{s_2}{s_1} \tag{9.7}$$

where s_2 and s_1 are the erector conjugates as indicated in Fig. 9.1c. For a scope as shown, f_o, f_e, and s_2 are positive signed quantities and s_1 is negative. The resulting MP is thus positive, indicating an erect image.

An afocal system is the basis of the *laser beam expander.* The beam diameter of a laser is enlarged by a factor equal to the MP when the laser beam is sent into the eyepiece end of the telescope. Expansion of the beam reduces the beam divergence. The Galilean form (Fig. 9.1b) is usually preferred because there is no focus (which can cause a breakdown of the air if the laser is powerful) and the optical design characteristics are more favorable. However, the Keplerian form (Fig. 9.1a) is used when a spatial filter (a pinhole at the focus) is necessary.

An afocal system can also be used to change the power, focal length, and/or the field of view of another system by inserting it in a space in the system where the light is collimated (i.e., where the object or image is at infinity.) (See Sec. 13.3 and Fig. 13.32.)

Note that an afocal system can be used to image objects which are not at an infinite distance. For example, the exit pupil of a telescope is the image of the aperture stop, which is usually at the objective lens. Again, a consideration of the rays diagramed in Fig. 9.1 will indicate that the linear magnification m is the same, regardless of where the object and image are located. The magnification $m=h'/h$ is equal to the reciprocal of the angular magnification, MP. Thus, $m=h'/h=1/\mathrm{MP}$. Note that if the aperture stop is placed at the internal focus, then the afocal system becomes telecentric in both object and image space.

9.2 Field Lenses and Relay Systems

In a simple two-element telescope as shown in Fig 9.2a, the field of view is limited by the diameter of the eyelens (as was discussed at greater length in Chap. 6). In the sketch, the solid rays indicate the largest field angle that a bundle may have and still pass through the telescope without vignetting; for the bundle represented by the dashed rays, only the ray through the upper rim of the objective gets through, and vignetting is effectively complete.

The function of a *field lens* is indicated in Fig. 9.2b. If the field lens is placed exactly at the internal image, it has no effect on the power of the telescope, but it bends the ray bundles (which would otherwise miss the eyelens) back toward the axis so that they pass through the eyelens. In this way the field of view may be increased without increasing the diameter of the eyelens. Note that the exit pupil is shifted to the left, closer to the eyelens, by the introduction of a positive field lens. The distance from the vertex of the eyelens to the exit pupil is called the "eye relief" (since the eye must be placed at the pupil to see the full field of view). The necessity for a positive eye relief obviously limits the

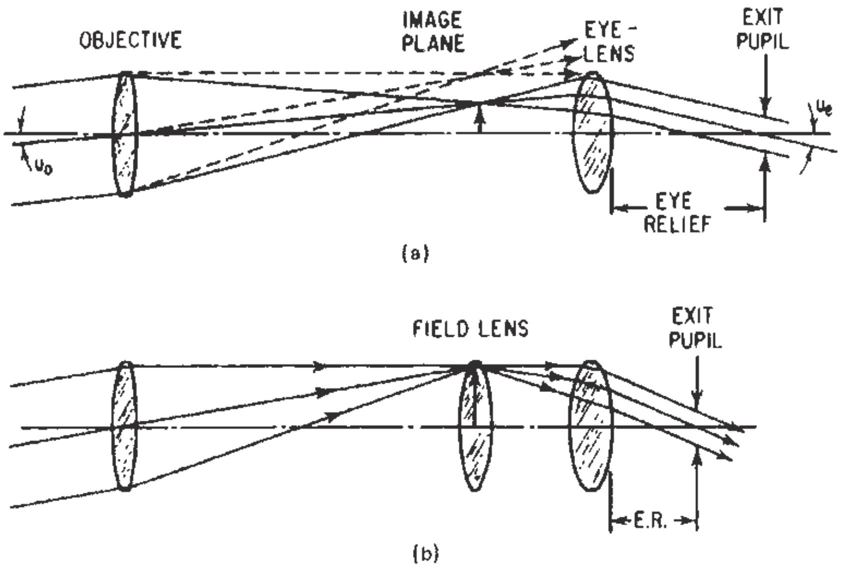

Figure 9.2 The action of the field lens in increasing the field of view.

strength of the field lens that can be used. In practice, field lenses are rarely located exactly at the image plane, but either ahead of or behind the image, so that imperfections in the field lens are out of focus and are not visible.

Periscopes and endoscopes

When it is desired to carry an image through a relatively long distance and the available space limits the diameter of the lenses which can be used, a system of *relay lenses* can be effective. In Fig. 9.3, the objective lens forms its image in field lens *A*. The image is then relayed to field lens *C* by lens *B* which functions like an erector lens. The image is then relayed again by lens *D*. The power of field lens *A* is chosen so that it forms an image of the objective at lens *B*; similarly, field lens *C* forms an image of lens *B* in lens *D*. In this way, the entrance pupil (which, in this example, is at the objective) is imaged at each of the relay lenses in turn and the image of the object is passed through the system without vignetting. The dashed rays emerging from lens *A* will indicate the large diameters which would otherwise be necessary to cover the same field of view. Tbis type of system is used in periscopes and endoscopes.

An optimum arrangement for most optical systems is often the layout with the least total amount of lens power. In a periscope system the minimum power system is simple to design. Given the maximum lens diameter (which is determined by the available space) the image at the field lenses is arranged to fill this diameter, and the clear aperture of the relay lens is filled with the beam. Thus, witb reference to Fig. 9.3, the focal length of the objective is set equal to the field lens *CA* divided

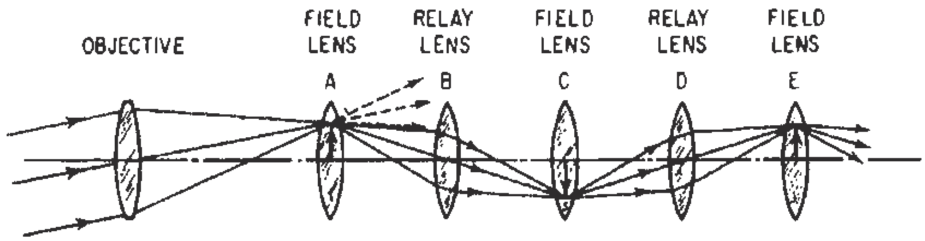

Figure 9.3 A system of relay lenses.

by the total field of view, and the distance from A to B is the product of the relay lens CA times the f-number of the objective lens. Lenses B, C, D, etc., all have the same focal length, which is half the distance from A to B, and lenses B, C, D, etc., are all working at unit magnification ($m = -1$). This arrangement yields the minimum lens power for the system; this is the best layout for a periscope system.

An *endoscope* is a miniature periscope used to examine the inside of a cavity through a small orifice; they are widely used in medical applications. The size of the optics in a medical endoscope is on the order of 2 or 3 mm in diameter. The *equivalent air path* is the actual physical path divided by the index of refraction. In an endoscope or periscope, the number of relay stages is determined by the length of the instrument. If the airspaces are filled with glass, the equivalent air path is shortened by a factor equal to the index of the glass, and the number of relay stages is thereby reduced. Rather than simply fill the spaces with rods of glass, the relay lenses are typically made as cemented doublets, with the flint (negative) element made thick enough to fill the space. The outer surface of the flint is made convex so that it functions as the field lens. This is often referred to as a *rod-lens endoscope*. The reduction in the number of relay components both reduces the cost of the endoscope and improves the image quality (especially by reducing the secondary spectrum and the Petzval field curvature).

9.3 Exit Pupils, the Eye, and Resolution

Since almost all telescopes are visual instruments, they must be designed to be compatible with the characteristics of the human eye. In Chap. 5, we saw that the pupil of the eye varied in diameter from 2 mm to about 8 mm, depending on the age of the viewer and the brightness of the scene being viewed. Since the pupil of the eye is, in effect, a stop of a telescopic system, its effect must be considered. For ordinary use, an exit pupil of 3 mm diameter will fill the pupil of the eye and no increase in retinal illumination will be obtained by providing a larger exit pupil. From Eq. 9.5, it is apparent that the maximum *effective* clear aperture for an ordinary telescope objective is thus limited to a diameter of about 3 mm times the magnification. In practice, this is, however, a fairly flexible situation. In surveying instruments exit pupils of 1.0 to 1.5 mm are common, since size and weight are at a premium and resolution is the most desired characteristic. In ordinary binoculars, a 5-mm pupil is usually provided; the added pupil diameter makes it much easier to align the binocular with the eyes. For the same reason, rifle scopes usually have exit pupils ranging in size from 5 to 10 mm. Telescopes and binoculars designed for use at low light levels (such as night glasses) usually have 7- or 8-mm exit pupils in order to obtain the maximum retinal illumination possible when the pupil of the eye is large.

In Chap. 5, it was indicated that the resolution of the eye was at best about one minute of arc; Chap. 6 indicated that the angular resolution of a perfect optical system was $(5.5/D)$ seconds of arc when the clear aperture of the system (D) was expressed in inches. One or both of these limitations will govern the effective performance of any telescope, and for the most efficient design of a telescope, both should be taken into account. If two objects which are to be resolved are separated by an angle α, after magnification by a telescope their images will be separated by $(\mathrm{MP})\alpha$. If $(\mathrm{MP})\alpha$ exceeds one minute of arc, the eye will be able to separate the two images; if $(\mathrm{MP})\alpha$ is less than one minute, the two objects will not be seen as separate and distinct. Thus, the magnification of a telescope should be chosen so that

$$\begin{aligned} \mathrm{MP} &> \frac{1}{\alpha} \qquad (\alpha \text{ in minutes}) \\ &> \frac{0.0003}{\alpha} \qquad (\alpha \text{ in radians}) \end{aligned} \tag{9.8}$$

where α is the angle to be resolved. For critical work, a magnification value considerably larger than indicated in Eq. 9.8 is often selected in order to minimize the visual fatigue of the viewer.

From the opposite point of view, since the resolution of a telescope (in object space) is limited to $(5.5/D)$ seconds, it is apparent that the smallest resolved detail in the image presented to the eye will subtend an angle of $(\mathrm{MP})\,(5.5/D)$ seconds, and if this angle equals or exceeds one minute, the eye can discern all of the resolved details. Equating this angle to one minute (60 seconds), we find that the maximum "useful" power for a telescope is

$$\mathrm{MP} = 11D \tag{9.9}$$

(when D is in inches). Magnification in excess of this power is termed *empty magnification,* since it produces no increase in resolution. *However, it is not unusual to utilize magnifications two or three times* *this amount to minimize visual effort.* The upper limit on effective magnification usually occurs at the point when the diffraction blurring of the image becomes a distraction sufficient to offset the gain in visual facility.

Example A

As numerical examples to illustrate the preceding sections, we will determine the necessary powers and spacings to produce a telescope with the following characteristics: a magnification of 4× and a length of 10 in. We will do this in turn for an inverting telescope, a Galilean telescope, and an erecting telescope, and will discuss the effects of arbitrarily limiting the element diameters to 1 in.

For a telescope with only two components, it is apparent that Eqs. 9.1 and 9.4 together determine the powers of the objective and eyelens. Thus, we have

$$D = f_o + f_e = 10 \text{ in}$$

and

$$\text{MP} = \frac{-f_o}{f_e} = \pm 4\times$$

where the sign of the magnification will determine whether the final image is erect (+) or inverted (−). Combining the two expressions and solving for the focal lengths, we get

$$f_o = \frac{(\text{MP})\, D}{(\text{MP}) - 1}$$

$$f_e = \frac{D}{1 - (\text{MP})}$$

For the inverting telescope, we simply substitute MP = −4 and D = 10 in, to find that the required focal length for the objective is 8 in; for the eyelens, it is 2 in. Since the lens diameters are to be 1 in, the exit pupil diameter is 0.25 in (from Eq. 9.5). The position of the exit pupil can be determined by tracing a ray from the center of the objective through the edge of the eyelens or by use of the thin-lens equation (Eq. 2.4), as follows:

$$\frac{1}{s'} = \frac{1}{f} + \frac{1}{s} = \frac{1}{f_e} + \frac{1}{(-D)} = \frac{1}{2} - \frac{1}{10} = 0.4$$

$$s' = 2.5 \text{ in}$$

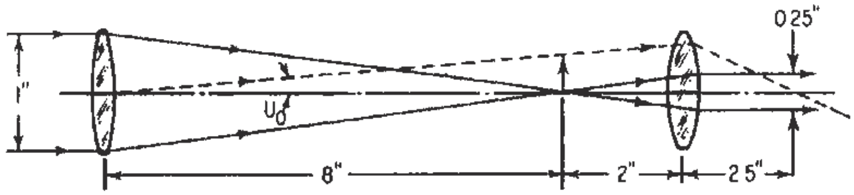

Figure 9.4 The inverting telescope of Example A.

Thus, the eye relief of our simple telescope is $2^1/_2$ in.

The field of view of this telescope is not clearly defined, since it is determined by vignetting at the eyelens, as consideration of Fig. 9.4 will indicate. The aperture will be 50 percent vignetted at a field angle such that the principal (or chief) ray passes through the rim of the eyelens. Under these conditions

$$u_o = \frac{\text{dia. eyelens}}{2D} = \frac{1}{2 \times 10} = \pm 0.05 \text{ radians}$$

and the real* field of view totals 0.1 radians, or about 5.7°.

This is a poor representation of what the eye will see, however, since the vignetted exit pupil at this angle closely approximates a semicircle 0.25 in in diameter and can thus completely fill a 3-mm eye pupil. The field angle at which no rays get through the telescope is a somewhat more representative value for the field of view. If we visualize the size of u_o in Fig. 9.4 as being slowly increased, it is apparent that the ray from the bottom of the objective will be the first to miss the eyelens and the ray from the top of the objective will be the last to be vignetted out. For the example we have chosen, with both lenses 1 in in diameter, it is apparent that the limiting diameter of the internal image will also be 1 in. (For differing lens diameters, it is a simple exercise in proportion to determine the height at which this ray strikes the internal focal plane.) The half field of view for 100 percent vignetting is then the quotient of the semidiameter of the image divided by the objective focal length, or ±0.0625 radians; the total real field is 0.125 radians, or about 7.1°.

Thus, for an exit pupil of 0.25 in, the field of view is totally vignetted at 0.125 rad, 50 percent vignetted at 0.1 rad, and unvignetted at 0.075 rad. These three conditions are illustrated in Fig. 9.5, and it is apparent that the "effective" position of the exit pupil shifts inward as the amount of vignetting increases.

*The *real* field of a telescope is the (angular) field in the object space. The *apparent* field is the (angular) field in the image (i.e., eye) space.

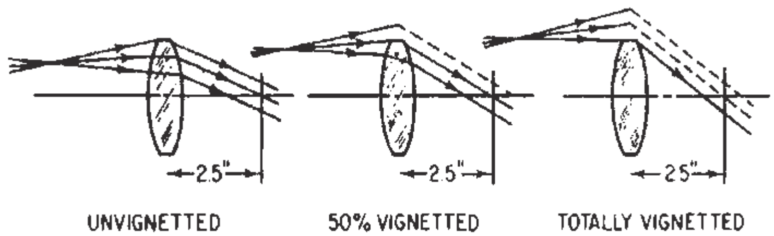

Figure 9.5 The vignetting action of the eyelens determines the field of view in an astronomical telescope.

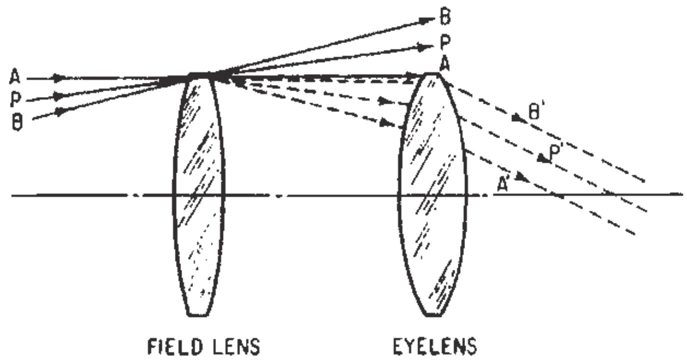

Figure 9.6 Ray diagram used to determine field lens power in Example A.

Let us now determine the minimum power for a field lens which will completely eliminate the vignetting at a field angle of ±0.0625 rad. From Fig. 9.6, it can be seen that the field lens must bend the rays from the objective so that ray B strikes no higher than the upper rim of the eyelens. The slope of ray B is equal to 1 in (the difference in the heights at which it strikes the objective and the field lens) divided by 8 in (the distance from field lens to objective), or +0.125. After passing through the field lens, we desire the slope to be zero (in this case) as indicated by the dashed ray B'. Using Eq. 2.41, we can solve for the power of the field lens as follows:

$$u' = u - y\phi_f$$

$$0.0 = +0.125 - (0.5)\,\phi_f$$

$$\phi_f = +0.25$$

$$f_f = \frac{1}{\phi} = 4 \text{ in}$$

We can now determine the new eye relief by tracing a principal ray from the center of the objective through the field and eye lenses.

$$u'_o = \frac{y_f}{f_o} = +0.0625 = u_f$$

$$u'_f = u_f - y_f\phi_f = +0.0625 - 0.5\,(0.25) = -0.0625$$

$$y_e = y_f + u'_f f_e = 0.5 - 0.0625\,(2) = 0.375$$

$$u'_e = u'_f - y_e\phi_e = -0.0625 - 0.375\,(0.5) = -0.25$$

$$l'_e = \text{eye relief} = \frac{-y_e}{u'_e} = \frac{-0.375}{-0.25} = 1.5 \text{ in}$$

Note that u'_e and u_o are still related by the magnification, as in Eq. 9.4, where

$$\text{MP} = \frac{u'_e}{u_o} = \frac{-0.25}{+0.0625} = -4\times$$

since the power of the system has not been changed by the introduction of the field lens located exactly at the focal plane. If we desire to locate the field lens slightly out of the focal plane, the general approach would be the same; the distances, ray heights, etc., in the computations would, of course, be modified accordingly. The power of the telescope would be increased if the field lens were placed to the right of the focus, and vice versa. In either case the scope is slightly shortened.

For the Galilean version of our telescope, we solve for the component focal lengths by substituting $+4\times$ for the magnification in the equations in the second paragraph of Example A and get

$$f_o = \frac{(\text{MP})\,D}{(\text{MP}) - 1} = \frac{(+4)\,10}{+4 - 1} = +13.33 \text{ in}$$

$$f_e = \frac{D}{1 - (\text{MP})} = \frac{10}{1 - (+4)} = -3.33 \text{ in}$$

If we assume the aperture stop to be at the objective lens of a Galilean telescope, the exit pupil will be found to be inside the telescope, and we obviously cannot put the viewer's eye there. Thus in a Galilean scope the aperture stop is not the objective lens but is the pupil of the user's eye, and the exit pupil is wherever the eye is located. This is usually about 5 mm behind the eyelens. To determine the field of view, we must trace a principal ray through the center of the pupil and passing through the edge of the objective, as indicated in Fig. 9.7. This can be done by assuming some arbitrary value for u_e and tracing the ray through, then scaling the ray data by an appropriate constant (as indicated in Chap. 6) to make the ray height at the objective equal to one-half its clear aperture. To simplify matters, we will assume here that the pupil is coincident with the eyelens; thus, u_e is equal to half the objec-

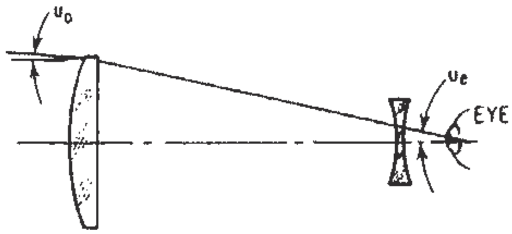

Figure 9.7 In a Galilean telescope, the field of view is determined by the diameter of the objective lens and the location of the exit pupil, which is usually the pupil of the observer's eye.

tive diameter divided by the spacing between the lenses, or 0.05 radians in this instance. Since MP$=u_e/u_o$ per Eq. 9.4, we can solve for u_o=0.05/4=0.0125 radians. The total real field is 0.025 radians (about 1.5°), considerably less than that of the inverting telescope discussed above. Note that the same type of field vignetting considerations as discussed related to the eyelens of the astronomical telescope may be applied to the objective of the Galilean telescope. One must also bear in mind that the *direction* of the Galilean field of view can be changed by a lateral shift of the viewer's eye; this is not true for a telescope with a real internal image when the field stop is located at the image.

For the erecting telescope example, we will lay out a telescopic rifle sight, with a magnification of +4×, a length of 10 in, and a maximum lens diameter of 1 in, as before. For small-caliber (.22) rifles, a 2-in eye relief is acceptable; for heavier guns, eye reliefs of 3 to 5 in are common. Let us assume that we desire an eye relief of 4 in and design the telescope accordingly. The entrance pupil (at the objective) has a diameter of 1 in; by Eq. 9.6, the exit pupil diameter is thus 0.25 in. Again by Eq. 9.6, the apparent field at the eyepiece (u_e) is equal to $4u_o$, where u_o is the real field. With reference to Fig. 9.8, it is apparent that u_e is limited by the diameter of the eyelens and that for an *unvignetted* pupil and a 1-in-diameter eyelens, the 4-in eye relief R limits us to an apparent field as follows:

$$u_e = 4u_o = \pm\frac{1}{2R}\ \text{(eyelens dia. − pupil dia.)}$$

$$= \pm\frac{1}{2 \times 4}\ (1 - 0.25) = \pm 0.09375$$

$$u_o = \pm 0.0234 = (\pm 1.3°)$$

To determine the spacing and powers of the components, we note that the length will be

$$L = f_o - s_1 + s_2 + f_e$$

and the magnification will be

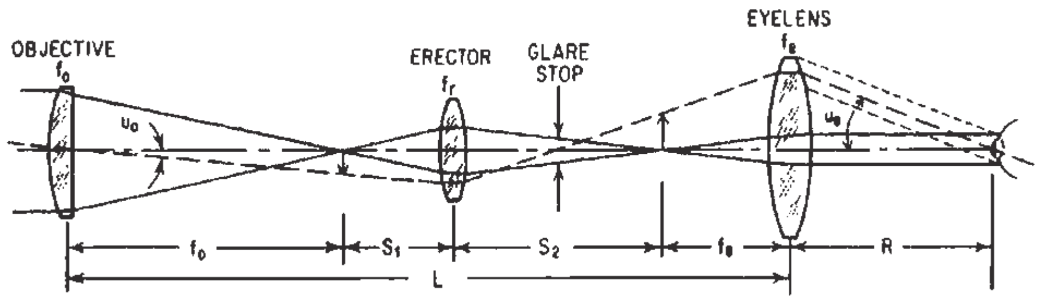

Figure 9.8 Optics of a simple erecting telescope.

$$M = \frac{-f_o s_2}{f_e s_1}$$

We can combine these expressions and derive equations for s_1, s_2, and f_r in terms of M, L, f_o, and f_e as follows:

$$s_1 = \frac{-f_o (L - f_o - f_e)}{(Mf_e + f_o)}$$

$$s_2 = \frac{-s_1 Mf_e}{f_o} = \frac{Mf_e (L - f_o - f_e)}{(Mf_e + f_o)}$$

$$f_r = \frac{s_1 s_2}{s_1 - s_2} = \frac{Mf_e f_o (L - f_o - f_e)}{(Mf_e + f_o)^2}$$

At this point, we are faced with a situation which is very common in the layout stages of optical design. We can elect to proceed algebraically to find an expression for f_o and f_e which will yield a scope with the desired eye relief R, or we can proceed numerically. In general, for a one-time solution, the numerical approach is usually the better choice, especially if the system under consideration is well understood. If one is likely to design a number of systems of the same type with various parameters, or if one is "exploring" and wishes to locate *all* possible solutions, the often tedious labor of an algebraic solution may be well repaid.

The preceding equations indicate that we have two choices (or degrees of freedom) which we can make, namely f_o and f_e, and arrive at a 4× scope of 10-in length; we have not, however, included the eye relief in these equations. To resolve this situation numerically, we would now assume some reasonable value for f_o, then proceed to test various values of f_e, selecting the value of f_e which yields the desired value for the eye relief R. Since R is not a critical dimension, a graphic solution (after a few values of f_e have been tried), plotting R versus f_e would be quite adequate for our purpose. Repeating the process for several additional values of f_o would then indicate the range of solutions available.

To arrive at a solution analytically, we would proceed as follows: a principal ray, starting at the center of the objective lens with some arbitrary slope angle would be ray-traced by thin-lens equations (2.41, 2.42, and 2.43), using the symbolic values for the spacings and lens powers derived from the three equations immediately preceding. The symbolic values for the powers and spacings involved would thus be:

$$\text{First airspace} = f_o - s_1 = f_o + \frac{f_o(L - f_o - f_e)}{(Mf_e + f_o)}$$

$$\text{Erector power } \phi_r = \frac{1}{f_r} = \frac{(Mf_e + f_o)^2}{Mf_e f_o(L - f_o - f_e)}$$

$$\text{Second airspace} = s_2 + f_e = f_e + \frac{Mf_e(L - f_o - f_e)}{(Mf_e + f_o)}$$

$$\text{Eyelens power } \phi_e = \frac{1}{f_e}$$

The expression for the final intercept length of this ray, $l'_e = -y_e/u'_e$ is then equated to the eye relief R, and a solution for f_e expressed in terms of f_o, M, L, and R is extracted. As can be imagined, the procedure is lengthy and the probability of making an error in the derivation is approximately unity for the first few attempts. Careful work and frequent checking are not only advisable, they are mandatory. When the smoke has cleared away, one finds that

$$f_e = \frac{M^2RL - f_o(M^2R + L)}{M^2(R + L) - f_o(M-1)^2}$$

and that for any chosen value for f_o (which is less than L and more than zero), a set of powers and spacings can be obtained which will satisfy our original conditions for power M, length L, and eye relief R.

We are now faced, regardless of whether we have arrived via numbers or symbols, with the problem of determining what is a suitable value for f_o upon which to base our solution. There are a number of criteria by which to judge the value of a given solution. In general, one desires to minimize the power of the components in any given system; in subsequent chapters, it will become apparent that it is often advisable to minimize one or all of the following: $\Sigma|\phi|$, $\Sigma|y\phi|$, $\Sigma|y^2\phi|$ (where the symbol $|x|$ indicates the absolute value of x), ϕ is the component power, and y represents the height of either the axial or principal ray on the component, or the element semiclear aperture.

Avoiding, for a few chapters at least, the rationale behind these desiderata, we shall proceed to indicate the technique. For a number of arbitrarily chosen values of f_o, we determine the required values for f_r and f_e (as well as s_1 and s_2). Then the values of the component powers ϕ_o, ϕ_r, and ϕ_e (where $\phi = 1/f$) as well as $\Sigma|\phi| = |\phi_o| + |\phi_r| + |\phi_e|$ are plotted against f_o, resulting in a graph as shown in Fig. 9.9. Note that the minimum $\Sigma|\phi|$ occurs in the region of f_o=3.5; for want of a better criterion, this is a reasonable choice.

To carry the matter a bit further, we can trace an axial ray and a principal ray through each solution. The axial ray has starting data (at the objective) of y=0.5 and u=0; the principal ray starting data is y_p=0 and u_p=0.0234375, chosen on the basis of eye relief and eyelens diameter considerations as discussed several paragraphs above. From these ray traces, we can determine the axial ray height y at each lens, y^2, and the necessary minimum clear diameter at each lens $D=2(|y|+|y_p|)$ to pass the full bundle of rays at the edge of the field. It turns out that *under the conditions we have established,* the diameter for the objective and eyelens must be 1 in, and the diameter of the erector lens is 0.3125 in for all values of f_o. From this information, a graph as shown in Fig. 9.10 can be plotted. The choice of which of the four minima to select must be made on the basis of material which is contained in subsequent chapters. In general, however, a minimum $\Sigma|\phi|$ in this example would reduce the Petzval curvature of field, a minimum $\Sigma|D\phi|$ would reduce the cost of making the optics, and minimum $\Sigma|D\phi|$, $\Sigma|y\phi|$, or $\Sigma|y^2\phi|$ would tend to reduce other aberrations, the choice being dependent upon which aberration one most desired to reduce.

Assuming that we have chosen f_o=+4, the values of the lens powers and spacings would be determined as follows:

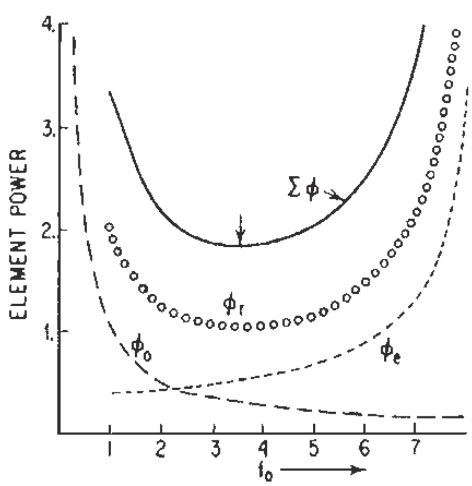

Figure 9.9 Plot of the element powers for a 10-in-long erecting telescope with 4-in eye relief versus the arbitrarily chosen objective focal length. ϕ_0, ϕ_r, and ϕ_e are tbe powers of the objective, erector, and eyelens, respectively.

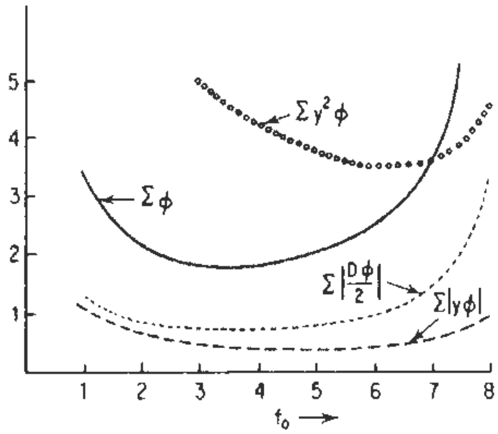

Figure 9.10

$$f_o = +4$$

$$f_e = \frac{4 \times 4 \times 4 \times 10 - 4\,(4 \times 4 \times 4 + 10)}{4 \times 4\,(4 + 10) - 4\,(4 - 1)\,(4 - 1)} = +\,1.8298$$

$$s_1 = \frac{-4\,(10 - 4 - 1.8298)}{(4 \times 1.8298 + 4)} = -1.4737$$

$$s_2 = \frac{-\,(-1.4737) \times 4 \times 1.8298}{4} = +2.6965$$

$$f_r = \frac{-\,(-1.4737) \times 4 \times 1.8298}{(4 \times 1.8298 + 4)} = +0.9529$$

9.4 The Simple Microscope or Magnifier

A microscope is an optical system which presents to the eye an enlarged image of a near object. The image is enlarged in the sense that it subtends (from the eye) a greater angle than the object does when viewed at normal viewing distance. The "normal viewing distance" is conventionally considered to be about 10 in; this represents an average value for the distance at which most people see detail most clearly. (Obviously, very young people can see detail in objects a few inches from the eye and mature persons whose visual accommodation is failing may have difficulty focusing on objects several feet away.) The magnification or magnifying power of a microscope is defined as the ratio of the visual angle subtended by the image to the angle subtended by the object at a distance of 10 in from the eye.

The simple microscope or magnifying glass consists of a lens with the object located at or within its first focal point. In Fig. 9.11, the object h, a distance s from the magnifier, is imaged at a distance s' with a

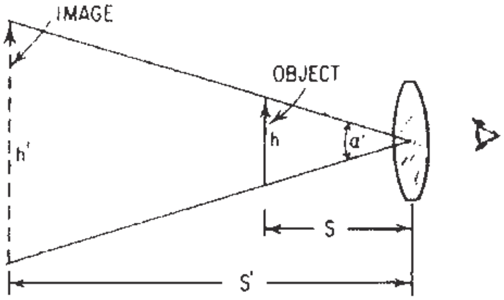

Figure 9.11 The simple microscope, or magnifier, forms an erect, virtual image of the object.

height h'. As shown, the image is virtual and both s and s' are negative quantities according to our sign convention. We can readily determine the magnification by using the first-order equations (2.4 and 2.7) as follows. The object and image distance equation

$$\frac{1}{s'} = \frac{1}{f} + \frac{1}{s}$$

is solved for s

$$s = \frac{fs'}{f - s'}$$

and substituted into the equation for the image height

$$h' = \frac{hs'}{s} = \frac{h\,(f - s')}{f}$$

Now if the eye is located at the lens, the angle subtended by the image is given by

$$\alpha' = \frac{h'}{s'} = \frac{h\,(f - s')}{fs'}$$

If the unaided eye were to view the object at a distance of −10 in, the angle subtended would be

$$\alpha = \frac{-h}{10 \text{ in}}$$

The magnifying power is the ratio between these two angles

$$\text{MP} = \frac{\alpha'}{\alpha} = \frac{h\,(f - s')}{fs'} \times \frac{(-10 \text{ in})}{h}$$

$$= \frac{10 \text{ in}}{f} - \frac{10 \text{ in}}{s'} \qquad (9.10)$$

Thus we find that the magnification produced by a simple microscope depends not only on its focal length but on the focus position chosen. If one adjusts the object distance so that the image is at infinity (i.e., $s = -f$ and $s' = \infty$) and can be viewed with a relaxed eye, then the magnification becomes simply

$$MP = \frac{10 \text{ in}}{f} \tag{9.10a}$$

If the focus is set so that the image appears to be 10 in away (i.e., $s' = -10$ in) then

$$MP = \frac{10 \text{ in}}{f} + 1 \tag{9.10b}$$

The value of MP given by Eq. 9.10a is conventionally used to express the power of magnifiers, eyepieces, and even compound microscopes.

The preceding assumed that the eye was located at the lens. If the image is not located at infinity, the magnifying power will be reduced as the eye is moved away from the lens. If R is the lens-to-eye distance, the magnification becomes

$$MP = \frac{10\,(f - s')}{f\,(s' - R)} \tag{9.10c}$$

Note that if the dimensions are in millimeters, the constant 10 becomes 254.

9.5 The Compound Microscope

As illustrated in Fig. 9.12, a compound microscope consists of an objective lens and an eyelens. The objective lens produces a real inverted image (usually enlarged) of the object. The eyelens reimages the object at a comfortable viewing distance and magnifies the image still further. The magnifying power of the system can be determined by substituting the value of the combined focal length of the two components (as given by Eq. 2.45) into Eq. 9.10a

$$f_{eo} = \frac{f_e f_o}{f_e + f_o - d} \tag{9.11}$$

$$MP = \frac{10 \text{ in}}{f_{eo}} = \frac{(f_e + f_o - d)\,10 \text{ in}}{f_e f_o}$$

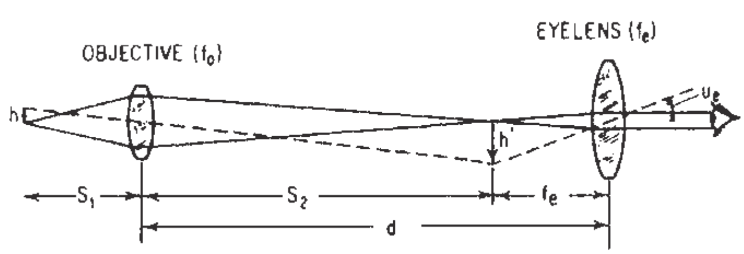

Figure 9.12 The compound microscope.

The more conventional way to determine the magnification is to view it as the product of the objective magnification times the eyepiece magnification. With reference to Fig. 9.12, this approach gives

$$\mathrm{MP} = M_o \times M_e = \frac{s_2}{s_1} \cdot \frac{10 \text{ in}}{f_e} \tag{9.12}$$

Equations 9.11 and 9.12 yield exactly the same value of magnification, as can be shown by substituting $(d - f_e)$ for s_2; determining s_1 in terms of d, f_e, and f_o (from Eq. 2.4); and substituting in Eq. 9.12 to get Eq. 9.11.

An ordinary laboratory microscope has a tube length of 160 mm. The tube length is the distance from the second (i.e., internal) focal point of the objective to the first focal point of the eyepiece. Thus, by Eq. 2.6, the objective magnification is $160/f_o$, and rewriting Eq. 9.12 for millimeter measure, we get

$$\mathrm{MP} = \frac{-160}{f_o} \cdot \frac{254}{f_e} \tag{9.13}$$

Standard microscope optics are usually referred to by their power. Thus, a 16-mm focal length objective has a power of 10× and an 0.5-in focal length eyepiece has a power of 20×. The combination of the two would have a magnifying power of 200×, or 200 diameters.

The resolution of a microscope is limited by both diffraction and the resolution of the eye in the same manner as in a telescope. In the case of the microscope, however, we are interested in the linear resolution rather than angular resolution. By Rayleigh's criterion, the smallest separation between two object points that will allow them to be resolved is given by Eq. 6.20

$$Z = \frac{0.61\lambda}{\mathrm{NA}}$$

where λ is wavelength and $\mathrm{NA} = n \sin U$, the numerical aperture of the system. Note that the index n and the slope of the marginal ray U are those at the object. Because of the importance of the numerical aperture in this regard, microscope objectives are usually specified by power and numerical aperture; for example, a 16-mm objective is usually listed as a 10×NA 0.25.

At a distance of 10 in, the visual resolution of one minute of arc (0.0003 radians) corresponds to a linear resolution of about 0.003 in, or 0.076 mm. When the object is magnified by an optical system, the *visual* resolution at the object is thus

$$R = \frac{0.003 \text{ in}}{\mathrm{MP}} = \frac{0.076 \text{ mm}}{\mathrm{MP}} \tag{9.14}$$

If we now equate the visual resolution R with the diffraction limit Z and solve for the magnification, we find that

$$\mathrm{MP} = \frac{0.12\ \mathrm{NA}}{\lambda} \tag{9.15}$$

with λ in millimeters, is the magnifcation at which the diffraction limit and visual limit match. At this power the eye can resolve all the detail present in the image, and setting $\lambda = 0.55\ \mu\mathrm{m}$, any magnification beyond 225 NA is "empty magnification." However, as with telescopes, magnifications several times this amount are regularly used, as discussed in Sec. 9.3.

9.6 Rangefinders

Figure 9.13 is a schematic diagram of a simplified triangulation rangefinder. The eye views the object by two paths; directly through semitransparent mirror M_1 and by an offset path via M_1 and fully reflecting mirror M_2. The angular position of one of the mirrors is adjusted until both images coincide. In the rudimentary instrument shown here, a pointer attached to mirror M_2 can be used to read the value of $\theta/2$; the distance to the object is found from

$$D = \frac{B}{\tan \theta} \tag{9.16}$$

where B is the base length of the instrument. In actual rangefinders, a telescope is often combined with the mirror system to increase the

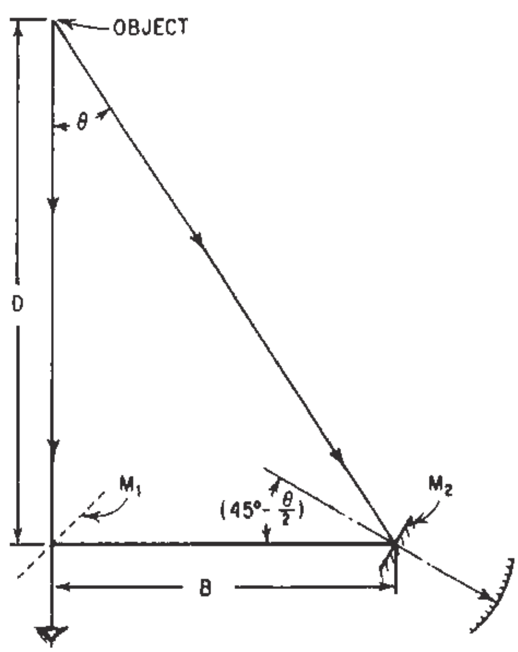

Figure 9.13 Basic rangefinder optical system. The eye views the object directly through semireflector M_1 and also through movable mirror M_2. The angular setting of M_2 which brings both views into coincidence determines the range.

accuracy of the reading, and any one of a number of devices may be used to determine θ; the distance is usually read directly from a suitable range scale so that no calculation is necessary.

The accuracy of the value of D depends on how accurately θ can be measured. For large ratios of D/B, we can write

$$D = \frac{B}{\theta} \tag{9.17}$$

and differentiating with respect to θ, we get

$$dD = -B\theta^{-2}\,d\theta \tag{9.18a}$$

Substituting $\theta = B/D$ into Eq. 9.18a, we find that the error in D due to a setting error of $d\theta$ is

$$dD = \frac{-D^2}{B}\,d\theta \tag{9.18b}$$

Now $d\theta$ is primarily limited by how well the eye can determine when the two images are in coincidence. This is essentially the vernier acuity of the eye and is about 10 seconds of arc (0.00005 radians). If the magnification of the rangefinder optical system is M, then $d\theta$ is $0.00005/M$ radians, and the ranging error is

$$dD = \pm\frac{5 \times 10^{-5} D^2}{MB} \tag{9.18c}$$

Thus, the greater the base B and the greater the magnification M, the more accurate the value of the range D.

A few of the devices encountered in rangefinders are illustrated in Fig. 9.14. In Fig. 9.14a the end mirrors are replaced by penta-prisms (or "penta"-reflectors), which are constant-deviation devices, bending the line of sight 90° regardless of their orientation. The reason for their use is to remove a source of error, since no change in the relative angular position of the two images is produced by misalignment of the penta-prisms as would be the case with simple 45° mirrors. A double telescope is built into the system to provide magnification; the power of each branch of the telescope must be carefully matched to avoid errors. The coincidence prism is provided to split the field of view into two halves, with a sharply focused dividing line between. In the system as shown, the final image is inverted; an erecting system, either prism or lens, is frequently included. Actual coincidence prisms are usually much more complex than that shown here.

A great variety of devices may be utilized to bring the two images into coincidence. Those shown in Fig. 9.14b to d are located between

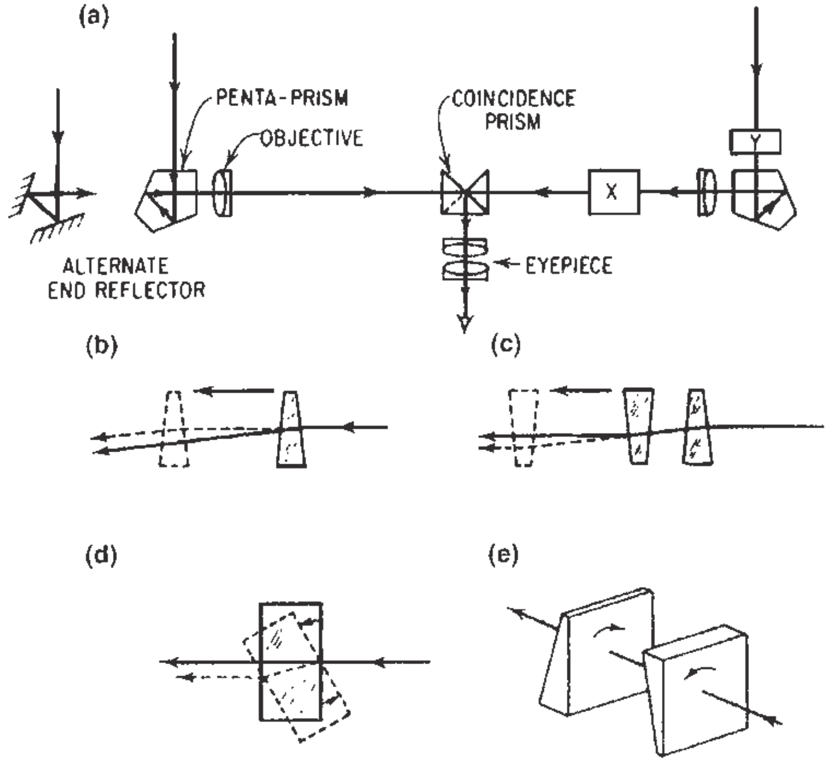

Figure 9.14 Typical rangefinder optical devices. (a) A telescopic rangefinder with coincidence prism and penta-prism end reflectors. (b) Sliding prism used at X to establish coincidence. (c) Pair of sliding prisms used at X. (d) Rotating parallel plate used at X. (e) Counter-rotating prisms used at Y to establish coincidence.

the objective and eyelens, usually in the region marked X in Fig. 9.14a. The sliding prism of Fig. 9.14b produces a displacement at the image plane which increases with its distance from the image; it is usually an achromatic prism. Figure 9.14c shows two identical prisms with variable spacing, which displace but do not deviate the rays. The rotating block in Fig. 9.14d operates on the same principle. All of the above tend to introduce astigmatism (that is, a difference of focal position in vertically and horizontally aligned images) since they are tilted surfaces in a convergent beam. The counterrotating wedges of Fig. 9.14e can be located in parallel light (region Y in Fig. 9.14a) and thus avoid this difficulty. Note that as one wedge turns clockwise, the other must rotate counterclockwise through exactly the same angle; in this way the vertical deviation is maintained at zero while the horizontal deviation can be varied plus or minus twice the deviation of an individual wedge. These are sometimes called Risley prisms.

Another device to produce a variable angle of deviation consists of a fixed plano concave lens and a movable plano convex lens of the same radius with their curved surfaces nested together. When the convex lens is located so that its plane surface is parallel to that of the concave lens, the pair produces no angular deviation. However, if the convex lens is rotated (about its center of curvature), the pair effectively becomes a prism and will produce an angular deviation. This device can be executed with spherical surfaces or with cylindrical surfaces.

Single-lens reflex (SLR) cameras often incorporate a split-image rangefinder which is based on an entirely different principle than the coincidence rangefinder described above. The viewfinder of an SLR camera consists of the camera objective lens, a field lens, and an eyelens. The field lens is divided into three zones as indicated in Fig. 9.15b. The outer zone functions as a straightforward field lens, redirecting the light at the edge of the field so that it passes through the eyelens. It is made in the form of a plastic *Fresnel lens,* in which the curved surface of a lens is collapsed in annular zones to a thin plate, as shown in Fig. 9.15a. This has the refracting effect of the lens without its thickness or weight. Such Fresnel lenses are also used as condensers in overhead projectors, as well as in spotlights and signal lamps. The center zone of the SLR field lens is split into two halves. Each half is a wedge prism; the two prisms are oriented in opposite directions. If the image formed by the objective lens is in focus, it is located in the plane of the wedges and the two halves of the image line up with each other. If the image is out of focus, the image through one-half of the split wedge is deviated in one direction; through the other half the deviation is in the other direction and the image is split. The intermediate zone of the field lens has a surface comprised of tiny pyramidal prisms which deviate and break up an out-of-focus image so as to exaggerate the out-of-focus blurring.

For many applications the optical rangefinder has been superceded by the laser rangefinder. This is essentially optical radar, where the distance to the target is obtained by measuring the travel time for a pulse of light to reflect from the target and return. In military applications a high-power laser is used; in surveying applications a cooperative target such as a retrodirector (corner-cube prism) is used and a much lower power source is adequate.

9.7 Radiometer and Detector Optics

A radiometer is a device for measuring the radiation from a source. In a simple form, it may consist of an objective lens (or mirror) which collects the radiation from the source and images it on the sensitive surface of a detector capable of converting the incident radiation into an electrical signal. A "chopper," which may be as simple as a miniature fan blade, is usually interposed in front of the detector to provide an alternating signal for the benefit of the electronic circuitry which must amplify and process the detector output.

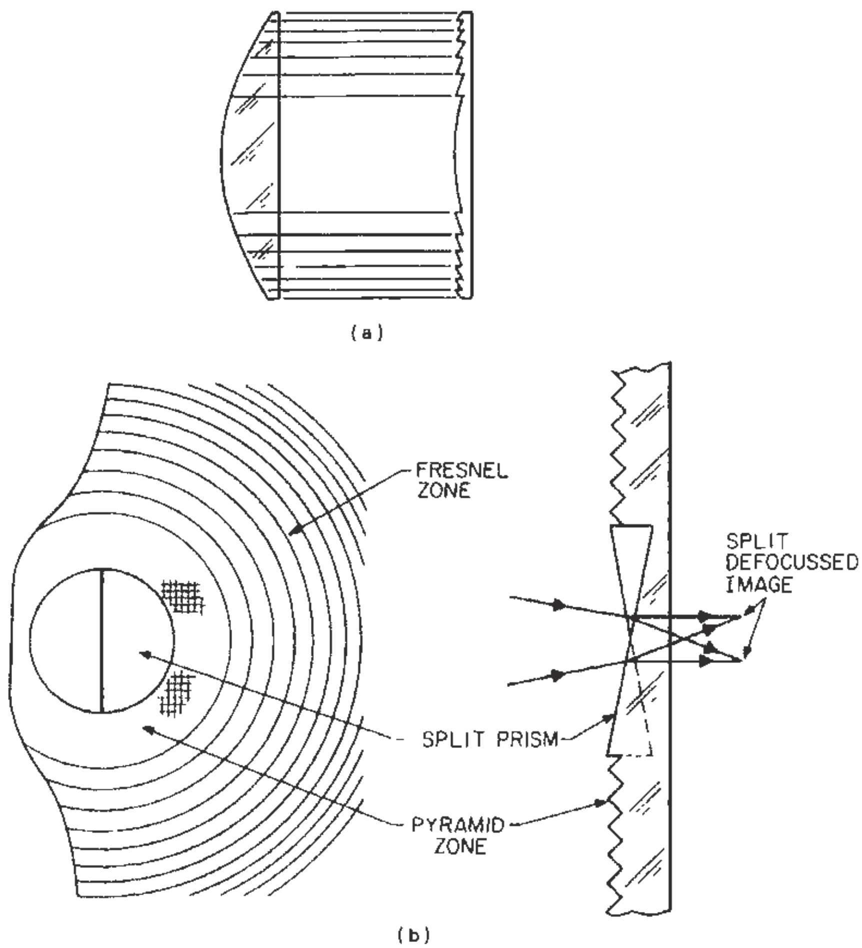

Figure 9.15 (a) A Fresnel lens is shown with the equivalent lens from which it is derived. Each annular zone of the Fresnel lens has the same surface slope as the corresponding zone of the lens. (b) The split-prism rangefinder of a 35-mm SLR camera splits an out-of-focus image in two by means of oppositely oriented wedge prisms in its central zone. If the image is focused on the wedge surface, it is not deviated or split. The area surrounding the split prism is comprised of tiny pyramidal prisms which break up an out-of-focus image and exaggerate its blur. The outer zone is a Fresnel lens acting as a field lens for the camera viewfinder.

The radiometer is widely used for the purpose its name would seem to imply, to measure radiation. However, it is also the basis of many other applications. The receiver in a communications system by which one talks over a beam of light is a sort of radiometer whose output is converted into audible form. The seeker head of an infrared homing air-to-air missile (e.g., the Sidewinder) is basically a radiometer whose output is arranged to indicate whether the hot exhaust of an enemy jet is on or off the line of sight.

A simple radiometer is sketched in Fig. 9.16. The detector, with a diameter D, is located at the focus of an objective with a focal distance F and a diameter A. The half-field of view of the system is α, and since

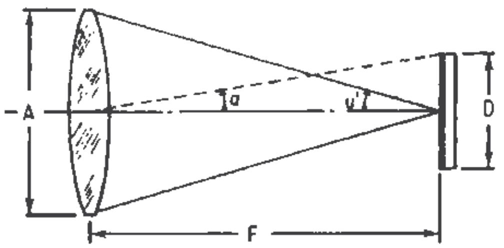

Figure 9.16 A simple radiometer with an objective lens which forms an image of the radiation source directly on the detector cell.

the detector is at the focus of the system, it is apparent that the half-field of view is given by

$$\alpha = \frac{D}{2F} \tag{9.19}$$

In the various applications of radiometers, the following characteristics are frequently desirable in the optical system

1. In order to collect a large quantity of power from the source, the diameter A of the system should be as large as possible.
2. In order to increase the signal-to-noise ratio, the size D of the detector should be as small as possible.
3. In order to cover a practical field of view, the field angle α should be of reasonable size (and often, should be as large as possible).

The relationship between A and F is, as we have previously noted, a limited one. If the optical system is to be aplanatic* (that is, free of spherical aberration and coma), the second principal surface (or principal "plane") must be spherical; for this reason, tbe effective diameter A cannot exceed twice the focal distance F, and the slope of the marginal ray at the image cannot exceed 90°. This limits the numerical aperture of the system to NA=n' sin 90°=n'; for systems in air with distant sources the limiting relative aperture becomes f/0.5. There are other limits imposed on the speed of the objective lens; the design of the system may be incapable of whatever resolution is required at large aperture ratios, or physical limitations (or predetermined relationships) may limit the acceptable speed of the objective.

We can introduce the effective f/# of the objective by multiplying both sides of Eq. 9.19 by A; setting (f/#) = F/A and rearranging, to get, for systems in air,

$$(f/\#) = \frac{D}{2A\alpha} \tag{9.20}$$

*The frequent assumption of aplanatic systems in the analysis of radiometric systems is based (1) on the usual need for good image quality and (2) on the fact that the image illumination (irradiance) produced by an aplanatic system cannot be exceeded, so that the assumption provides a limiting case.

or for systems with the final image in a medium of index n'

$$\mathrm{NA} = n' \sin u' = \frac{A\alpha}{D} \tag{9.21}$$

Equation 9.21 can also be demonstrated by setting the optical invariant (Eq. 2.54) at the objective ($I = A\alpha/2$) equal to the invariant at the image ($I = \frac{1}{2}Dn'u'$) and substituting $\sin u'$ for u' (in accordance with our requirement for aplanatism).

Since the (f/#) cannot be less than 0.5 and $\sin u'$ cannot exceed 1.0, it is apparent that the objective aperture A, half-field angle α, and detector size D, are related by

$$\left| \frac{A\alpha}{n'D} \right| \leq 1.0 \tag{9.22}$$

It should be noted that Eq. 9.22, since it can be derived by way of the optical invariant with no assumptions as to the system between object and detector, is valid for all types of optical systems, including reflecting and refracting objectives with or without field lenses, immersion lenses, light pipes, etc. *It is thus quite futile to attempt a design with the left member of Eq. 9.22 larger than unity; in fact, it is sometimes difficult to exceed (efficiently) a value of 0.5 when good imagery is required.* This limit is applicable to *any* optical system, no matter how simple or complex. Equation 9.22 is exactly analogous to Eq. 8.24 for projection or illumination systems.

As an example of the application of Eq. 9.22, let us determine the largest field of view possible for a radiometer with a 5-in aperture and a 1-mm (0.04-in) detector. If the detector is in air ($n' = 1.0$) we then have, from Eq. 9.22,

$$\frac{5\alpha}{0.04} \leq 1.0 \text{ or } \alpha \leq 0.008 \text{ radians}$$

and the absolute maximum total field (0.016 radians) is a little less than one degree (0.01745 radians). An immersion lens at the detector (described below) with an index n' would increase the maximum field angle to $0.016n'$.

An *immersion lens* is a means of increasing the numerical aperture of an optical system by a factor of the index n of the immersion lens, usually without modifying the characteristics of the system. Another way of considering the immersion lens is to think of it as a magnifier which enlarges the apparent size of the detector. The most frequently utilized form of immersion lens is a hemispherical element in optical contact with the detector. In Fig. 9.17, a concentric immersion lens of index n' has reduced the size of the image to h'/n'. Since the first surface of the immersion lens is concentric with the axial image point, rays directed toward this point are normal to this surface and are not

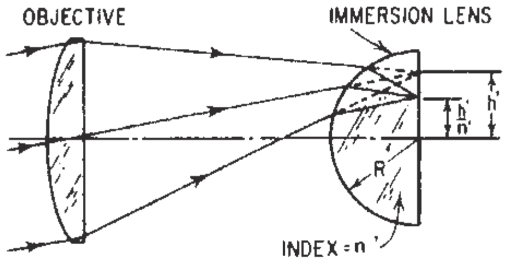

Figure 9.17 A hemispherical immersion lens concentric with the focus of an optical system reduces the linear size of the image by a factor of its index.

refracted. For this reason, neither spherical aberration nor axial coma nor axial chromatic is introduced. The optical invariant at the image is $h'n'u'$, and since u' is not changed by the immersion lens, it is apparent that as n' increases, h' must decrease.

In the use of immersion lenses, one must beware of reflection (especially total internal reflection) at the plane surface. Ideally, the detector layer should be deposited directly on the immersion lens. Since immersion lenses are usually resorted to in cases where the angles of incidence are large, total internal reflection can occur if the immersion lens index is high and a low-index layer (air or cement, for example) separates it from the detector.

In the application of radiometer-type systems, it is not unusual that one wishes to use an objective of relatively low speed with a small detector and still cover a large field of view. This is readily accomplished by means of a field lens. The field lens is located at (or more frequently, near) the image plane of the objective system and redirects the rays at the edge of the field toward the detector, as indicated in Fig. 9.18. As can be seen from a brief consideration of the figure, the field lens actually images the clear aperture of the objective on the surface of the detector. The optimum arrangement is when the image of the objective aperture is the same size as the detector and

$$\frac{s_1}{s_2} = (-)\frac{A}{D}$$

This arrangement not only makes a larger field angle possible, but has the advantage of providing an even illumination over a large portion of the detector surface. Most detectors vary in sensitivity from point to point over their surface; with a field lens of focal length given by

$$f = \frac{s_1 s_2}{s_1 - s_2}$$

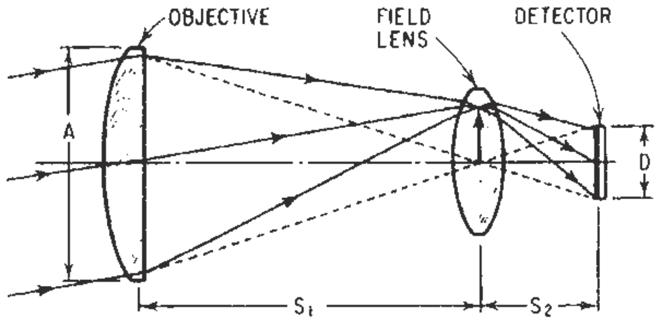

Figure 9.18 Radiometer with field lens to increase the field of view with a small detector.

the same area of the detector is illuminated regardless of where the source is imaged in the field of view. Field lenses and immersion lenses are frequently combined. Note that the insertion of a field lens in a radiometer does not change the limitations of Eqs. 9.21 and 9.22; it simply permits the use of an objective system with a low numerical aperture by raising the numerical aperture at the detector.

Another device to enlarge the field of view of a radiometer with a small detector is the light pipe, or cone channel condenser. In Fig. 9.19, a principal ray from the objective is shown being reflected from the walls of a tapered light pipe. Note that without the light pipe, the ray would completely miss the detector.

It is instructive to consider the "unfolded" path of a ray through such a system, as indicated in Fig. 9.20. The actual reflective walls of the light pipe are shown as solid lines; the dashed lines are the images of the walls formed by reflection from each other. This layout is analogous to the prism unfolding technique explained in Chap. 4 as a "tunnel diagram" and allows us to draw the path of a ray through the system as a straight line. Note that ray *A* in the figure undergoes three reflections before it reaches the detector end of the pipe. Ray *B*, entering at a greater angle, never does reach the detector, but is turned around and comes back out the large end of the pipe. This is a limit on the effectiveness of the pipe and is analogous to the *f*/# or numerical aperture limit on ordinary optical systems discussed above in the derivation of Eqs. 9.20 et seq.

A light pipe may be constructed as a hollow cone or pyramid with reflective walls in the manner indicated in Figs. 9.19 and 9.20. It is also common to construct them out of a solid piece of transparent optical material. The walls may then be reflective coated or one may rely on total internal reflection if the angles are properly chosen. Note that with a solid light pipe, total internal reflection may occur at the exit face; this can be avoided by "immersing" the detector at the exit end of the pipe. The use of a solid pipe effectively increases its acceptance angle by a factor of the index n of the pipe material; the effect on the

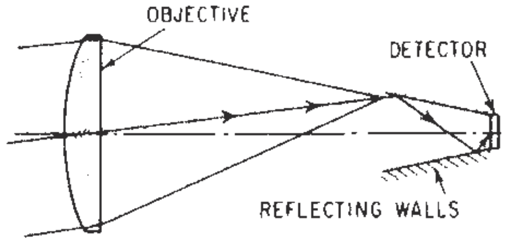

Figure 9.19 The action of a reflecting light pipe in increasing the field of view of a radiometer.

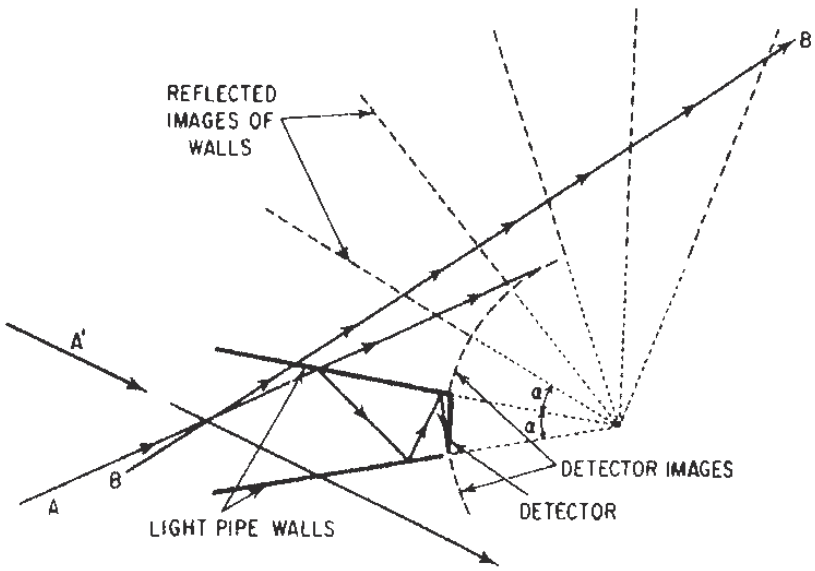

Figure 9.20 Ray tracing through a light pipe by means of an "unfolded" diagram.

system is exactly analogous to the use of an immersion lens, and the total radiometer system is still governed by Eq. 9.22 as before. Light pipes may be used with field lenses; the most common arrangement is to put a convex spherical surface on the entrance face of a solid pipe.

If one were to look into the large end of a pyramidal light pipe, one would see a sort of checkerboard multiple image of the exit face (or detector), as indicated in Fig. 9.20 for a two-dimensional case. The checkerboard is wrapped around a sphere centered on the apex of the pyramidal pipe. This image is, of course, the effective size of the ("magnified") detector, and the cone of light from the objective, as indicated by rays A and A' is spread out over this array. This effect is occasionally useful in decorrelating the point-for-point relationship between the detector surface and the objective aperture which is established when a field lens is used. The effect is even more pronounced in a conical pipe.

The discussion in this section has been devoted to condensing radiation onto a small detector. The tables can be turned. If we replace the detector with a small source of radiation, devices such as field lenses and light pipes can be used to increase the apparent size of the source and to reduce the angle through which it radiates (or vice versa).

A common application of the light pipe is in *illumination systems,* especially where extremely uniform illumination is required and the source is very nonuniform, such as a high-pressure mercury or xenon or metal halide arc lamp. If the light pipe is made with parallel sides (either as a cylinder or with a square or rectangular cross section) as shown in Fig. 9.21, the image of the light source can be focused on one end of the pipe; the other end is then quite uniformly illuminated. As can be seen from the figure, the multiple reflections of the source form a checkerboard array of images which is effectively a new light source, and the illumination across the exit end of the pipe is quite uniform. Of course there is no reason that a tapered pipe cannot be used in this way, and this is occasionally done. Note that the proportions of the light pipe (length, diameter) and the convergence of the imaging beam will determine the number of reflections and the number of reflected source images.

9.8 Fiber Optics

A long, polished cylinder of glass can transmit light from one end to the other without leakage, provided that the light strikes the walls of the cylinder with an angle of incidence greater than the critical angle for total internal reflection. The path of a meridional ray through such a cylinder is shown in Fig. 9.22. The geometric optics of meridional

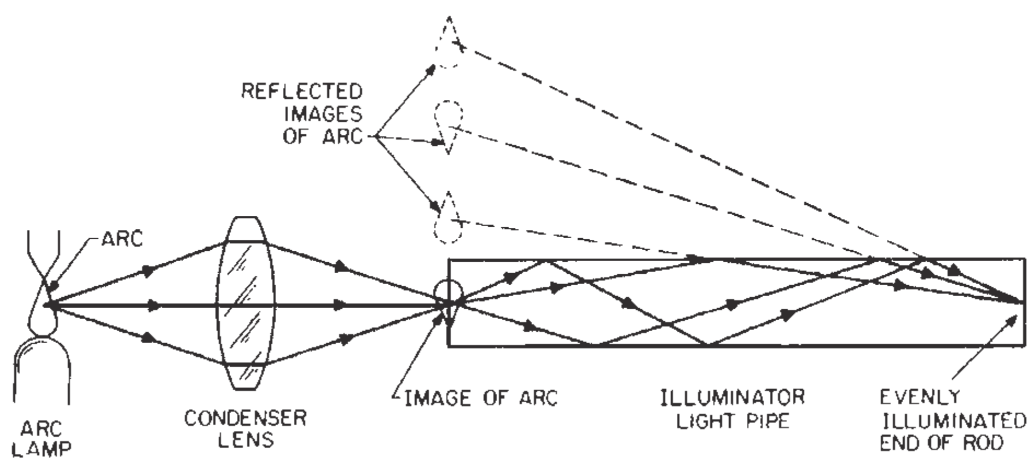

Figure 9.21 A light pipe can be used to produce very uniform illumination at its exit face when a light source is focused on the other end. The multiple images produced by reflections from the pipe walls become the illuminating source for the exit face.

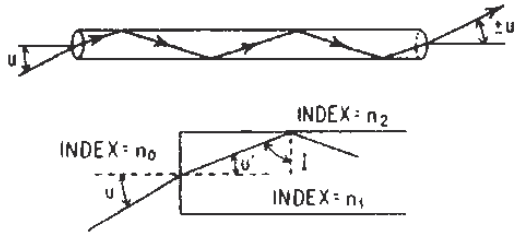

Figure 9.22 Light is transmitted through a long polished cylinder by means of total internal reflection.

rays through such a device are relatively simple. For a cylinder of length L, the path traveled by the meridional ray has a length given by

$$\text{Path length} = \frac{L}{\cos U'} \tag{9.23}$$

and the number of reflections undergone by the ray is

$$\text{No. reflections} = \frac{\text{path length}}{(d/\sin U')} = \frac{L}{d}\tan U' \pm 1 \tag{9.24}$$

where U' is slope of the ray inside the cylinder, d is the cylinder diameter, and L its length. For the light to be transmitted without reflection loss, it is necessary that the angle I exceed the critical angle

$$\sin I_c = \frac{n_2}{n_1}$$

where n_1 is the index of the cylinder and n_2 the index of the medium surrounding the cylinder. From this one can determine that the maximum external slope of a meridional ray which is to be totally reflected is

$$\sin U = \frac{1}{n_0}\sqrt{n_1^2 - n_2^2} \tag{9.25}$$

This "acceptance cone" of a cylinder is often specified as a numerical aperture; by rearranging Eq. 9.25, we get

$$\text{NA} = n_0 \sin U = \sqrt{n_1^2 - n_2^2} \tag{9.26}$$

This is the minimum value for the numerical aperture; as indicated below and in Fig. 9.23, skew rays have a larger NA than do meridional rays.

Again, with reference to Fig. 9.22, it is apparent that if the meridional ray had entered the cylinder well above or well below the axis, it would have emerged with a slope angle of $-U$. The path of a pair of skew rays is indicated (in an end-on view) in Fig. 9.23. Note that a skew ray is rotated with each reflection and that the amount of

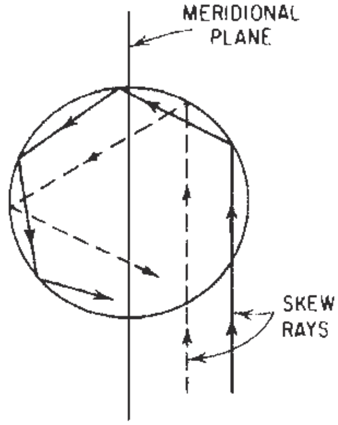

Figure 9.23 The path of skew (nonmeridional) rays through a reflecting cylinder is a sort of helix. The amount of rotation a ray undergoes in traversing a given length depends on its entrance position.

rotation depends on the distance of the ray from the meridional plane. Thus, a bundle of parallel rays incident on one end of a cylinder will emerge from the other end as a hollow cone of rays with an apex angle of $2U$. If the diameter of the cylinder is small, diffraction effects may diffuse the hollow cone to a great extent. It is also worth noting that since the skew rays strike the surface of the cylinder at a greater angle of incidence than the meridional rays, the numerical aperture for skew rays is larger than that for meridional rays.

If the light-transmitting cylinder is bent into a moderate curve, a certain amount of light will leak out the sides of the cylinder. However, the major portion of the light is still trapped inside the cylinder, and a simple curved rod is occasionally a convenient device to pipe light from one location to another.

Optical fibers are extremely thin filaments of glass or plastic. Typical diameters for the fibers range from 1 to 2 μm to 25 μm or more. At these small diameters, glass is quite flexible, and a bundle of optical fibers constitutes a flexible light pipe. Figure 9.24 shows a few of the applications of fiber optics. Figure 9.24a indicates the basic property of an oriented, or "coherent," bundle of fibers in transmitting an image from one end of the fiber to the other. If the bundle is constrained at both ends so that each fiber occupies the same relative position at each end, then the fiber rope may literally be tied in knots without affecting its image-transmitting properties. Fiber bundles with lengths of many feet are obtainable with surprisingly high transmissions. The limiting resolution (in line pairs per unit length) of a coherent fiber bundle is approximately equal to half the reciprocal of the fiber diameter; by synchronously oscillating or scanning both ends of the fiber, this resolution can be doubled. When the fibers are tightly packed, their surfaces contact each other and leakage of light from one fiber to the next will occur. Moisture, oil, or dirt on the fiber surface can also interfere with total internal reflection. This is prevented by coating or "cladding" each fiber with a thin layer of lower-index

(a)

(b)

(c)

(d)

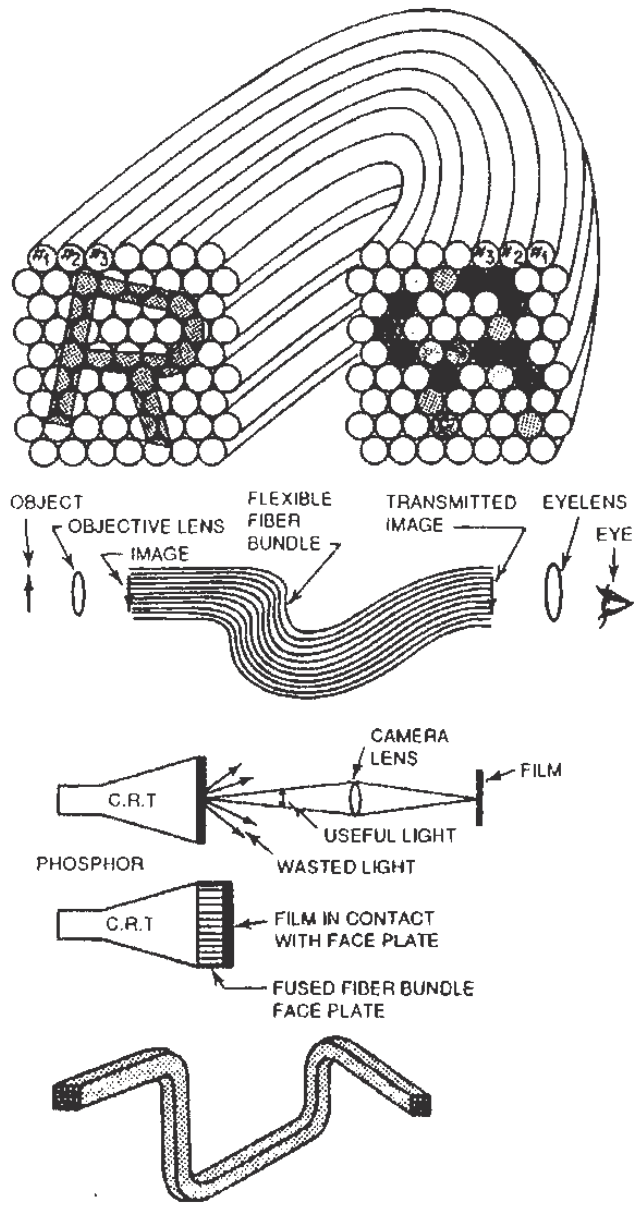

Figure 9.24 Fiber optics.

glass or plastic. For example, the core glass may have $n_1=1.72$ and the cladding $n_2=1.52$, yielding a numerical aperture according to Eq. 9.26 of the order of 0.8. Since the total internal reflection (TIR) occurs at the core-cladding interface, moisture or contact between the outer surfaces does not frustrate the TIR if the cladding is thick enough.

Figure 9.24b shows a flexible gastroscope or sigmoidoscope. An objective lens forms an image of the object on one end of a coherent fiber bundle; at the other end the transmitted image is viewed with the aid of an eyepiece or video camera.

Ordinary photography of a cathode ray tube face is an inefficient process. The phosphor radiates in all directions and a camera lens intercepts only a small portion of the radiated light. A tube face composed of a hermetically fused fiber array (Fig. 9.24c) can transmit all the energy radiated into a cone defined by its NA to a contacted photographic film with negligible loss. Fused fibers are always clad with low-index glass to separate the fibers; frequently an absorbing layer or absorbing fibers are added to prevent contrast reduction by stray light which is emitted at angles larger than the numerical aperture of the fibers. Fiber optics are also available as optical conduit, that is, rigid fused bundles, for efficient transmission of light through labyrinthian paths, as shown in Fig. 9.24d.

Flexible plastic fibers with diameters on the order of 0.5 in are used as single fibers in illumination systems.

A tapered, coherent, fused-fiber bundle can be used as either a magnifier or minifier (depending on whether the original object is placed at the small or large end of the taper). By twisting a coherent bundle of fibers, either fused or not, an image erector can be made which will carry out the function of the erector prisms described in Chap. 4. These are often found in image-intensifier systems such as those used in night vision goggles.

Hollow glass fibers in diameters from 0.5 to 1.0 mm, internally coated, are moderately flexible and have been used to transmit radiation in the 10-μm wavelength region. These fibers do a reasonable job of maintaining the gaussian distribution of the laser light.

Gradient index fibers

The preceding descriptions have dealt with fibers whose principal function was to transmit power from one end to the other, with little or no concern for any coherence; energy incident on one end of the fiber is effectively homogenized or scrambled and transmitted to the other end. But if the index of the fiber is made high in the center, gradually changing to low at the outside, then the ray paths through the fiber will be curved rather than straight lines. If the index gradient is properly chosen (i.e., approximately a function of the reciprocal of the square of the radial distance from the center of the fiber), the ray paths are sinusoidal as shown in Fig. 9.25. This has two significant effects. Rays originating from a point are brought to a focus periodically along the fiber; thus the fiber is capable of forming an image just as a lens is. This is the basis of the GRIN or SELFOC rod. For example, if the index is given as a function of the radial distance r as

$$n(r) = n_0 (1 - kr^2/2)$$

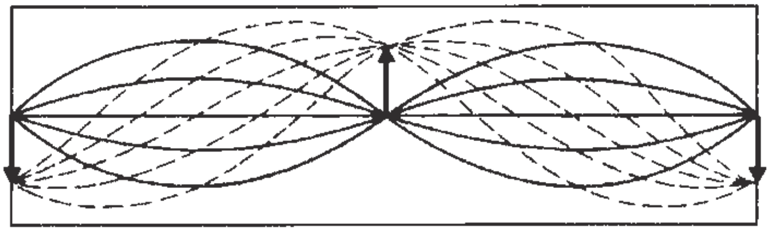

Figure 9.25 In a gradient index rod or fiber (GRIN or SELFOC rod), light rays travel in sinusoidal paths because the index is high at the center of the rod and lower at the edge. Such a rod can form an image just as a lens does. The rod length shown is the equivalent of two relay lenses and an intermediate-field lens. A short length of rod will act like a single lens element, and a longer length can act like a periscope.

then the focal length of a rod with an axial length of t is

$$\text{efl} = \frac{1}{n_0 \sqrt{k} \sin (t \sqrt{k})}$$

and the back focus is

$$\text{bfl} = \frac{1}{n_0 \sqrt{k} \tan (t \sqrt{k})}$$

The "pitch" of the sinusoidal ray path is $2\pi/\sqrt{k}$.

Since the focusing effect is continuous along the length of the rod, such a device is the equivalent of the periscope system of relay and field lenses described in Sec. 9.2. A length of rod corresponding to two relay lenses and one intermediate field lens as shown in Fig. 9.25 will thus produce an erect image of an area approximately equal to the rod diameter. A row, or a double row, of such rods is the basis of compact table top (scanning) copy machines. Obviously, a long GRIN rod can function as an endoscope and a short rod (less than a quarter of the length shown in Fig. 9.25) will function like an ordinary lens. This latter is called a *Wood lens.*

The other significant aspect of such an index gradient is that because the light rays travel in sinusoidal paths, they never reach the walls of the fiber and do not depend on reflection at a low-index cladding layer to confine them to the fiber. Also, the optical path (index times distance) is the same for all paths; obviously the axial path is the shortest, but it is at the highest index. This constancy of optical path means that the travel time is the same for all paths over the full numerical aperture; contrast this with the path length given by Eq. 9.23, which varies with the cosine of the ray slope angle.

Fibers for communications

Another application for optical fibers is in communication. Using light as an extremely high frequency carrier wave, the data transmission rate can be very, very high. Fibers can be made with extremely low absorption (less than 0.1-dB loss per kilometer) so that transmission of information over distances of several miles becomes practical. However, if the lengths of the possible ray paths differ from each other, the elapsed time for light to travel from one end of the fiber to the other will vary from ray to ray. At high data rates, only a small amount of travel time difference is enough to introduce a phase shift sufficient to reduce the signal modulation to a useless level. Again, Eq. 9.23 indicates the path length variation involved. The fibers used for telephone and data transmission are typically single-mode fibers (with core diameters on the order of 10 μm) which will not support propagation of a light wave except directly down the length of the fiber. In addition to the variation of path length, another source of trouble results from the fact that in most materials the index varies with wavelength, and thus, even with a constant path length, the optical path would vary with wavelength. Communication fiber materials, in addition to low absorption, are characterized by a very low dispersion in the (narrow) region of the spectrum in which they are used. Silica (SiO_2) fibers are made with near zero dispersion at 1.3 μm wavelength and very low absorption at 1.55 μm. Multilayer cladding can shift the zero dispersion to 1.55 μm and flatten it, to make 1.3 to 1.6 μm useful.

9.9 Anamorphic Systems

An anamorphic optical system is one which has a different power or magnification in one principal meridian than in the other. Such devices usually make use of either cylinder lenses or prisms. With reference to Fig. 9.26c, consider the fan of rays shown in the figure. The left-hand cylindrically surfaced lens is the equivalent of a plane parallel plate for these rays. However, the right-hand lens refracts these rays just as a spherical lens would, because its cylinder axes are at 90° to the left lens. The magnification of this fan of rays is about $-0.5\times$ as drawn. If we consider a fan of rays in the other prime meridian, however, the situation is reversed; the lens effect occurs at the left lens and the magnification is about $-2.0\times$. Thus the square object figure is imaged as a rectangle four times as wide as it is high. Since the focusing effect of a cylinder varies as the square of the cosine of the angle that a ray fan makes to its power meridian, if both prime meridians are in focus, then all meridians are in focus.

Another typical anamorphic system consists of an ordinary spherical objective lens combined with a Galilean telescope composed of cylinder lenses, as indicated in Fig. 9.26. In the upper sketch (a), it is apparent that the cylindrical afocal combination serves to shorten the focal length of the prime lens and thus widen its field of view (for a given film size). In the other meridian (Fig. 9.26b), the cylinder lenses are

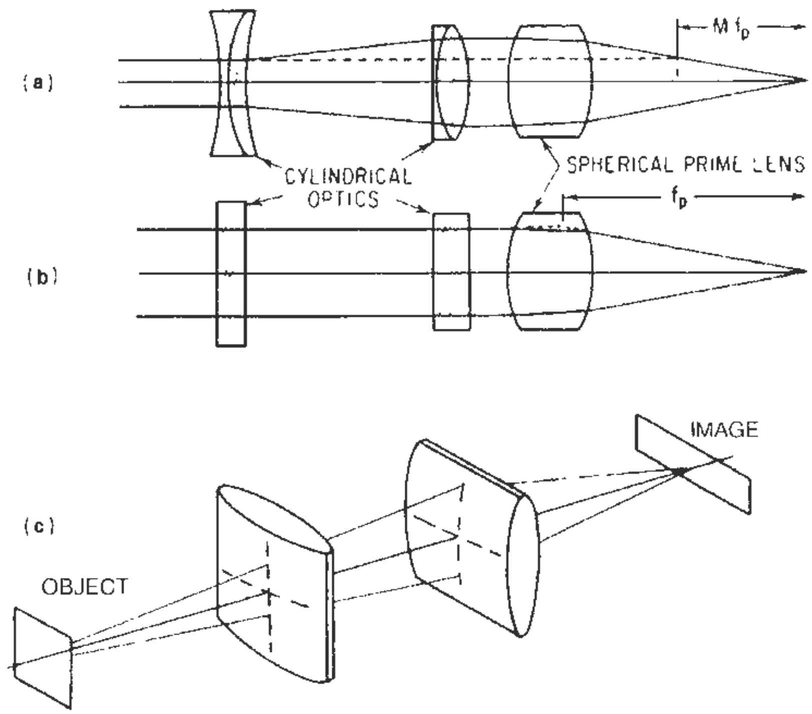

Figure 9.26 Cylindrical anamorphic systems.

equivalent to plane parallel plates of glass and do not affect the focal length or coverage of the prime lens. Thus, the system has a focal length equal to that of the prime lens f_p in one direction and a focal length equal to the magnification of the attachment times the prime lens focal length Mf_p in the other. In Fig. 9.26 the system is shown as a reversed Galilean telescope with a magnification of less than unity, and Mf_p is less than f_p. This is the type of system used in many wide-screen motion picture processes. The wide angular field is used to compress a large horizontal field of view into a normal film format. The distorted picture which results is expanded to normal proportions by projecting the film through a projection lens equipped with a similar attachment. Note that these attachments are used with ordinary camera and projector equipment.

Note that because an anamorphic system has a different equivalent focal length in each meridian, if it is to be focused at a finite distance, it will require a different shift of the lens to focus in each meridian. Thus the prime (spherical) lens must be focused separately from the cylindrical attachment (which is then focused by changing the space between the two components). This type of focusing has the unfortunate effect of changing the anamorphic ratio in a way which makes the face in a close-up appear fatter than it actually is. This is not a popular effect among the acting profession. There are two alternatives to this. One is to put a focusing component in front of the system. This is usually a pair of weak spherical elements, one positive and one negative, so that when closely spaced their power is zero; as the spacing between them is increased, their power becomes positive and the system is focused on a close distance. This is, in effect, a collimator for the object. The other alternative is called a Stokes lens, which consists of a pair of weak cylinders of equal but opposite powers, placed between the two components of the afocal cylindrical attachment, with their axes tilted at 45° to the axes of the attachment. When the two Stokes cylinders are counterrotated, both meridians of the system are focused at the tame time.

A *Bravais system* is the finite conjugate analog of an afocal power changer. Figure 9.27 shows the principle of a Bravais system inserted into the image space of an optical system for the purpose of increasing the size of the image without changing the image location. The component powers of this type of system can be determined from Eqs. 2.49 and 2.50 by setting the object to image distance T (the "track length") equal to zero. (Note that the arrangement shown here is usually much more satisfactory than that with the component powers reversed, which reduces the image size.) If a Bravais system is made with cylindrical optics, the image can be enlarged in one meridian and not in the other. This is of course an anamorphic system and has been successfully used for motion picture work. The value of such a "rear" anamorphic attachment is that its size is much less than that of the equivalent afocal attachment placed in front of the lens; this feature is especially important for use with long-focus zoom lenses, where the necessary size for a "front" anamorph can be overwhelming. In addition, there is no focus problem and no "fat" problem.

Cylinder lenses are also used to produce line images where a narrow slit of light is required. The image of a small light source formed by a cylinder lens is a line of light parallel to the axes of the cylindrical surfaces of the lens. The width of the line is equal to the image height

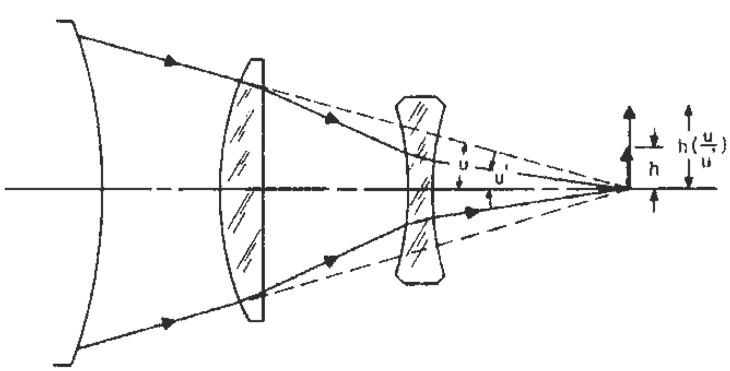

Figure 9.27 Bravais system.

given by the first-order optical equations; the length of the line is limited by the length of the lens, or as shown in Fig. 9.26c, it may be controlled by another cylindrical lens oriented at 90° to the first.

A prism may also be used to produce an anamorphic effect. In Sec. 9.1 (Eqs. 9.5 and 9.6), we saw that the magnification of an afocal optical system was given by the ratio of the diameters of its entrance and exit pupils. A refracting prism, used at other than minimum deviation, has different-sized exit and entrance beams and thus produces a magnification in the meridian in which it produces a deviation. Thus a single prism may be used as an anamorphic system. To eliminate the angular deviation, two prisms, arranged so that their deviations cancel and their magnifications combine, are usually used. Figure 9.28

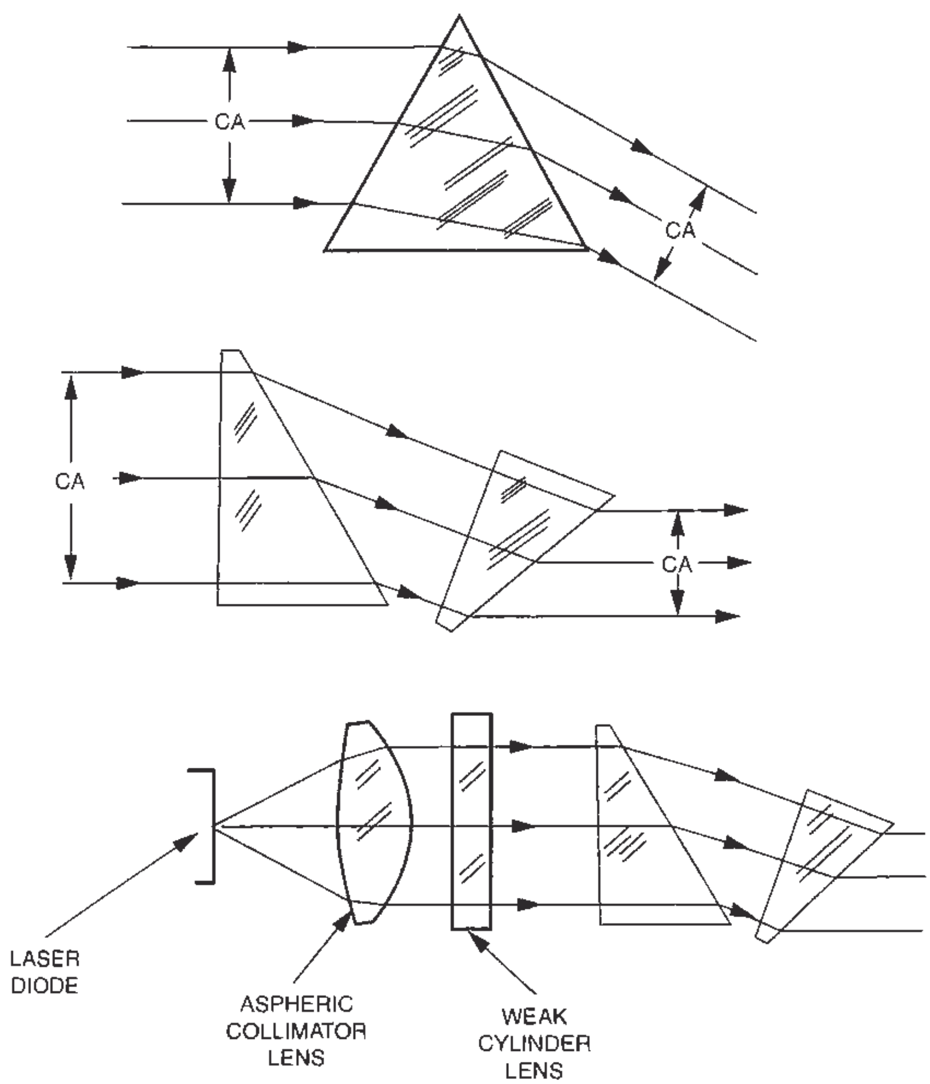

Figure 9.28 The anamorphic action of refracting prisms.

illustrates the action of a single prism and also shows a compound anamorphic attachment made up of two prisms. Since the anamorphic "magnification" of a prism is a function of the angle at which the beam enters the prism, a variable-power anamorphic can be made by simultaneously rotating both prisms in such a way that their deviations always cancel. Prism anamorphic systems are "in focus" and free of axial astigmatism only when used in parallel (collimated) light. Unlike cylindrical systems, they cannot be focused by changing the space between elements. For this reason, prism anamorphics are frequently preceded by a focusable pair of spherical elements which collimate the light from the object.

For use in systems which are not monochromatic, the prisms must be achromatized (as discussed in Sec. 4.5). Prism anamorphs have been used to project wide-screen (anamorphosed) movies; in this application, each achromatic prism component typically consisted of two or three prism elements. The useful field of such a device is rather small; being completely unsymmetric, it has all (both odd and even) orders of aberrations, including some unusual kinds of lateral color and distortion.

A laser diode is a useful light source, but it has two properties which ordinarily are a handicap: The output beam is not circular in cross section, but elliptical, and the source itself has a small but significant amount of astigmatism so that instead of appearing as a simple point, it appears as a point in different longitudinal locations for each meridian. The lower sketch in Fig. 9.28 shows a laser diode collimator, consisting of an aspheric surfaced collimator singlet, a weak cylindrical lens to cancel out the source astigmatism, and an anamorphic prism pair to convert the elliptical beam to one with a circular cross section. Note that the nearly monochromatic character of the output radiation makes achromatism unnecessary.

9.10 Variable-Power (Zoom) Systems

The simplest variable-power system is a lens working at unit power. If the lens is shifted toward the object, the image will become larger and will move further from the object. If the lens is moved away from the object, the image will become smaller and will again move away from the object. Thus one may find any number of conjugate pairs for which the object-to-image distance is the same but which have magnifications which are reciprocals of each other.

Figure 9.29 indicates the relationships involved in this arrangement. The algebraic expressions shown can be derived readily by manipulation of the thin-lens equation (Eq. 2.4).

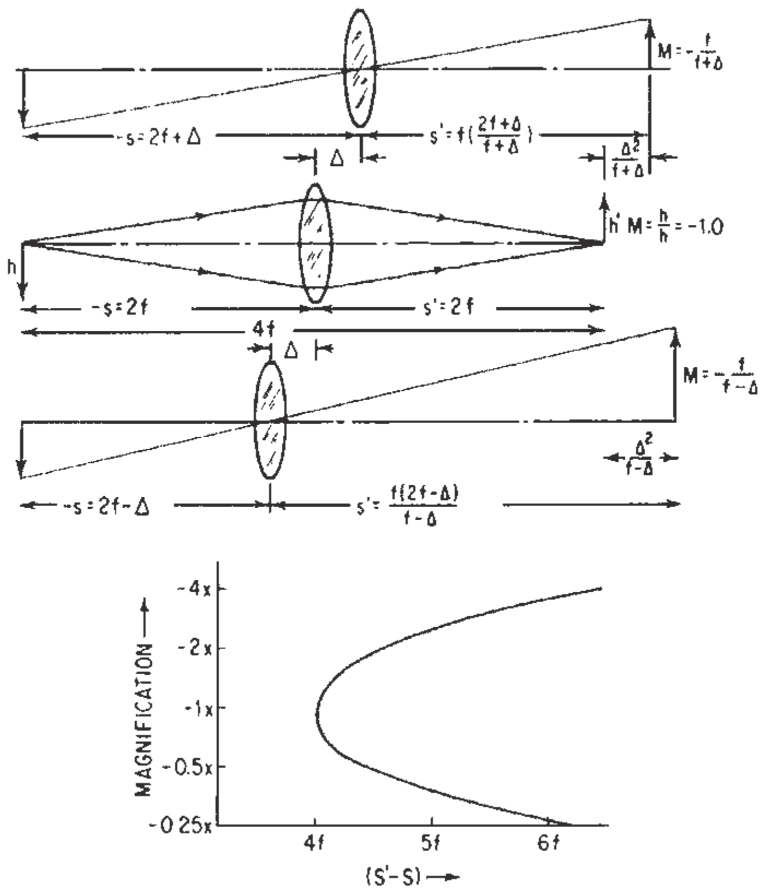

Figure 9.29 The basic unit power zoom lens. The graph indicates the shift of the image as the lens is moved to change the magnification.

The applicability of this particular zoom system is limited, since the commercial demand for variable-power systems at unit magnification is quite modest. However, by combining the moving element with one or two additional elements (usually of opposite sign), the zoom system can be made to operate at any desired set of conjugates. Several such arrangements are shown in Fig. 9.30. Note that in each system the moving lens passes through a point at which it works at unit magnification. By adding either a positive or negative eyelens or by simply adjusting the power of the last lens of the system, as indicated in the lower sketch, a telescope or afocal attachment may be made.

A system which is in focus only at two different magnifications is called a *bang-bang zoom.* It can be quite useful if what is wanted is a system with just two magnifications (and a continuous "zooming" action is not necessary). Since a bang-bang system is much easier and cheaper to design and build than a continuously in-focus zoom, it is often well worth considering whether a true zoom is really needed in a

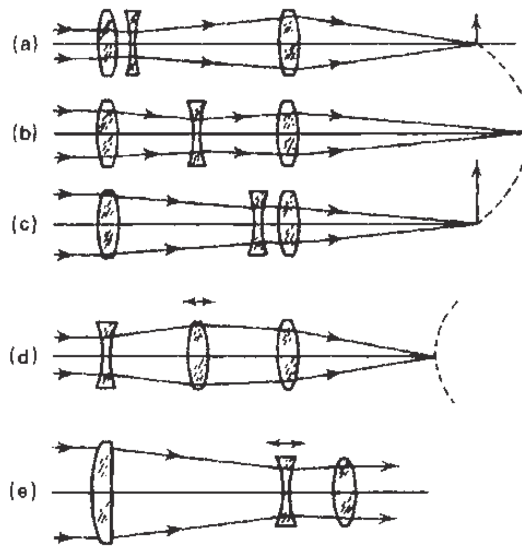

Figure 9.30 Zoom systems based on the unit power principle.

given application, or whether a simple choice of two magnifications, focal lengths, or powers would be sufficient.

All variable-power systems with a single moving component have the same characteristic relationship between image shift and magnification (or focal length). Thus for an uncompensated "single-lens" zoom system, there can be at most two magnifications at which the image is in exact focus. At all other powers, the image will be defocused. This situation can be alleviated in two ways. A "mechanically compensated" zoom system is one in which the defocusing is eliminated by introducing a compensating shift of one of the other elements of the system, as exemplified by Fig. 9.31. Since the motion of the compensating element is nonlinear, it is usually effected by a cam arrangement, hence the name "mechanically compensated."

In a zoom system, the motion of the elements will, of course, cause the ray heights, angles, etc. to change. It is apparent that the chromatic contributions of a single element (which are proportional to $y^2\phi/V$ and $yy_p\phi/V$ for axial and lateral chromatic, respectively) will vary accordingly. Thus, in order to achieve a *fully* achromatic system through the zoom, each component must be individually achromatized. However, since a small amount of chromatic often can be tolerated, singlet components are not uncommon.

The formulas for a thin-lens layout of this type of system are shown in Fig. 9.31 and can be derived by manipulation of the first-order expressions of Chap. 2. To use the formulas, one may arbitrarily select a value for ϕ_A, the power of the first element, then determine ϕ_B, ϕ_C, and the spacings for the "minimum shift" setting. To find the spacings for other positions of the moving lens, choose a value for one space and solve for the position of the compensating element to maintain the final focus at the same distance from the fixed element.

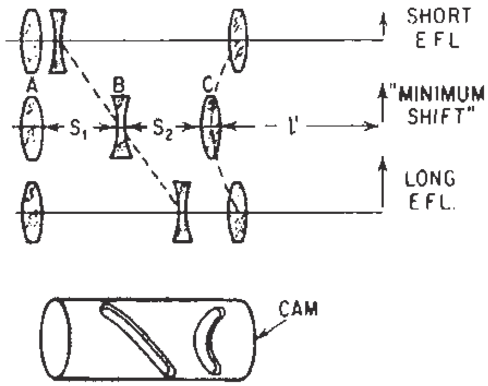

Figure 9.31 Mechanically compensated zoom system.

Given: Φ, power (1/efl) of a system at "minimum shift"

M, ratio of power at $S_1=0$ to power at $S_1=(R-1)/R\Phi_A$

$R=\sqrt{M}$

Choose: Φ_A, power of the first element. May be an arbitrary choice, or set

$\Phi_A=(R-1)/R(S_1+S_2)$ to control the length, (S_1+S_2), at "minimum shift"

Then: $\Phi_B=-\Phi_A(R+1)=(1-M)/R(S_1+S_2)$

$\Phi_C=(\Phi_A+\Phi)R(R+1)/(3R-1)$ to get Φ at the "minimum shift" position

"minimum shift" occurs at

$S_1=(R-1)/\Phi_A(R+1)=RS_2=R(S_1+S_2)/(1+R)$

$S_2=(R-1)/\Phi_A R(R+1)=S_1/R=(S_1+S_2)/(1+R)$

$l'=(3R-1)/\Phi R(R+1)$

$$S_1+S_2+l'=\frac{(R-1)}{\Phi_A R}+\frac{(3R-1)}{\Phi R(R+1)}$$

Motion of lens C is computed to hold the distance from lens A to the focal point at a constant value as lens B is moved.

It should be apparent that despite the use of three components in the preceding discussion, only two components are necessary to make a mechanically compensated zoom lens. Given any two components, if we change the space between them, Eqs. 2.44 and 2.45 indicate that the effective focal length will be changed. Of course, the back focal length will also change (according to Eq. 2.46), and the entire system will have to be shifted to maintain the focus. It usually turns out to be advantageous if one component is positive and the other negative. There are thus two possible arrangements, depending on which power comes first, and one's choice can be based on size and focal-length considerations. Many of the newer 35-mm camera zoom lenses are of this type.

Many of the newer zoom lens designs have more than two moving components. The extra motion may be used to improve the image quality through the zoom or to stabilize the image quality when the lens is focused at a near distance.

The other technique for reducing the focus shift in a variable-power system is called optical compensation. If two (or more) *alternate* lenses are linked and moved together with respect to the lenses between them, the powers and spaces can be so chosen that there are more than two magnifications at which the image is in exact focus. Two systems of this type are shown in Fig. 9.32. In the upper sketch, the first and third elements are linked and move to produce the varifocal effect. The second element, the other elements, and the film plane are all held in a fixed relationship with each other. The image motion produced by this type of system is a cubic curve, as shown in the upper graph. It is thus possible to arrange the powers and spaces so that the image is in exact focus for three positions in the zoom. The defocusing between these points is greatly reduced in comparison with the simpler systems described above, and if the range of powers is modest and the focal length of the system is short, a nonlinear compensating motion of one of the elements is not necessary. In the second system of Fig. 9.32, the motion of the image is described by a still-higher-order curve, and four points of exact compensation are possible; the residual image shift is about one-twentieth of the shift of the upper system. It turns out that the maximum number of points of exact compensation is equal to the number of variable airspaces. (Note that in Fig. 9.30 this number is 2, and the image motion is parabolic with two possible points of compensation.)

Originally it was thought that the fabrication of a mechanically compensated zoom lens would be almost impossibly difficult, requiring an unattainable level of precision, which could not be maintained as the cams, etc., wore with use. This turned out to be an incorrect assumption, and mechanically compensated zoom systems are widely used for almost all applications. Optical compensation is rare for several

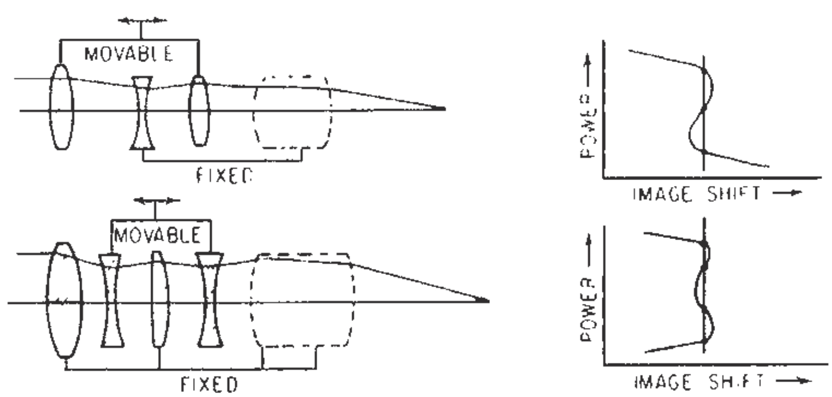

Figure 9.32 Optically compensated zoom systems. The upper system has three "active" components and three points of compensation as indicated in the upper graph. The lower system has four "active" components and four compensation points.

reasons. The requirements of the power and space layout to achieve optical compensation are extremely stringent and restrict the lens designer's ability to maintain the correction of the lens system throughout the zoom range. In addition, the size of the optically compensated system is significantly larger than the equivalent mechanically compensated system. Despite the fact that the optically compensated lens with its simple and undemanding mechanics is less expensive to fabricate, provided size is not a problem, the optically compensated zoom is effectively obsolete.

In zoom systems the focal lengths of a stationary first element and of the elements following the last moving lens may be changed at will, *provided the relationship between the focal points of the elements is maintained.* Such changes modify the focal length (or power) of the overall system and, in the case of the following elements, the amount of image shift as well. However, since a change in object position will shift the focus point of the first element with respect to the other elements, a zoom system is sensitive to object position. In order to maintain precise compensation, most zoom lenses are focused by moving an element of the first component with respect to the rest to offset this effect. As with the anamorphic systems discussed in Sec. 9.9, the leading component serves to collimate the light from the object.

9.11 The Diffractive Surface

The diffractive surface (or "kinoform" or "binary surface") as used in imaging optics is discussed at some length in connection with the design of telescope objectives in Chap. 12. In this section we are concerned not with the kinoform's Fresnel surface modulo 2π but with those surfaces which operate on the basis of diffraction in order to introduce a controlled diffusion or to produce a message or a pattern from a simple laser beam. Often these surfaces are simple two-, four-, or eight-level patterns with randomized surface elevations. These devices are made feasible by the recent advances in fabrication technology which make it possible to produce the microscopic wavelength-sized surface details required to produce these effects.

To those who think in terms of the phasefront or wavefront, the form of such a device is derived by describing the phasefront which will produce the desired effect and then determining the surface contour which will impose this phasefront on the input beam. However, for those who think geometrically, this is a less than satisfying explanation of how such a surface functions. The following are not elegant depictions, but they do serve the purpose of taking some of the mystery out of such devices.

The diffusing surface can be visualized as a surface randomly covered with microscopic lenses of a scale on the order of several wavelengths, either concave or convex, whose ratio of diameter to focal length equals the diffusion angle. Such diffusers are commercially available in diffusions of $^1/_2$°, 1°, etc. They can be useful in a number of applications, such as where one desires to destroy the spatial coherence in a laser system in order to eliminate interference patterns. The surface lens concept is not necessary; the same result can be produced by a stepped surface which locally alters the phase of the wavefront.

The pattern-generating surface is a little more difficult. Visualize a surface covered with weak prisms, each of which directs its portion of the incoming laser beam in a direction which will form a specific part of the desired pattern. When such a surface is produced on a microscopic wavelength scale, there are many, many tiny prisms in the area covered by the beam, and when the beam is translated across the surface, there are always enough prisms within the beam to produce the pattern. The bigger the beam diameter, the more prisms will be involved, and the better the definition of the pattern will be. There is an inherent "speckle" produced in this process which shows up as a random pattern of dots in the final image. Again, the effect can be produced by stepped surfaces which alter the wavefront diffractively to produce the desired patterns.

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Exercises

1 (a) What focal lenghts are required for the eyelens and objective of a 20 × astronomical telescope which is 10 in long? (b) What is the eye relief? (c) What is the minimum objective diameter if the diffraction limit of resolution is to match the resolution of the eye? (d) What is the maximum real field of the telescope if the diameter of the eye lens is 0.5 in?

ANSWER: (a) 10 in/21; 200 in/21 (b) $^1/_2$ in (c) 1.83 in (d) ±0.0296 radians

2 It is desired to add an afocal attachment in front of a 10-in *f*/10 camera lens to convert it to a 5-in focal length. (a) What element powers are necessary for a 3-in length reverse Galilean telescope to accomplish this? (b) What diameter must the outer element have if vignetting is not to exceed 50 percent for an object field of ±60°? Sketch the system. Is this a reasonable diameter?

ANSWER: (a) $f_0 = -3$ in; $f_e = +6$ in (b) $3^1/_2$ in

3 A microscope is required to work at a distance of 3 in from the object to the objective. If the objective and eyepiece both have 2-in focal lengths, what is the length of the microscope and what is its power?

ANSWER: Length=8 in; power=10×

4 What is the magnification produced by a telescope made up of a 5-in focal length objective and a 5-in focal length eyepiece (and thus nominally of unit power) when it is set at minus 2 diopters (i.e., the image of an infinitely distant object is −20 in from the eyelens)?

ANSWER: −1.25× (with eye at eyelens) or −0.8× (with eye at exit pupil)

5 What base length must a rangefinder have to measure a range of 2000 m to an accuracy of ±0.5 percent if it incorporates a 20-power telescope?

ANSWER: 1 m

6 Determine the focal length, diameter, and position (relative to the detector) for a radiometer field lens. The objective is a 5-in diameter *f*/4 paraboloid and the detector is 0.2-in square. The field to be covered is ±0.02 radians.

ANSWER: $f = 0.77$ in; diameter = 0.8 in minimum; $s_2 = 0.8$ in

7 The entrance opening of a tapered hollow light pipe is twice the exit opening. What is the largest angle a ray through the center of the entrance opening can make with the axis and still emerge from the small end of the pipe?

ANSWER: 30° (for a long pipe) and <90° (for a short pipe)

8 A hemicylindrical rod (plano-convex) with a cylindrical radius of 2.5 mm, which is 20 mm long, is located 50 mm from a 1-mm-square source of light. At the "focus," what is the size of the illuminated area? (Assume the rod index is 1.5)

ANSWER: 0.111 mm × 22.222 mm

9 Determine the element powers and spacings for a zoom lens of 10-in vertex length ($s_1+s_2=10$ in) with a zoom ratio of 4 which is to have a 10-in focal length at the "minimum shift" position. Plot the compensating motion of element C against the focal length of the lens as the element B is moved. Use Fig. 9.31.

ANSWER: $M=4$; powers: +0.05, −0.15, +0.18; spacings: 6.67 in, 3.33 in back focus: 8.33 in

$M={}^1/_4$; powers: −0.1, +0.15, 0; spacings: 3.33 in, 6.67 in; back focus: 6.67 in