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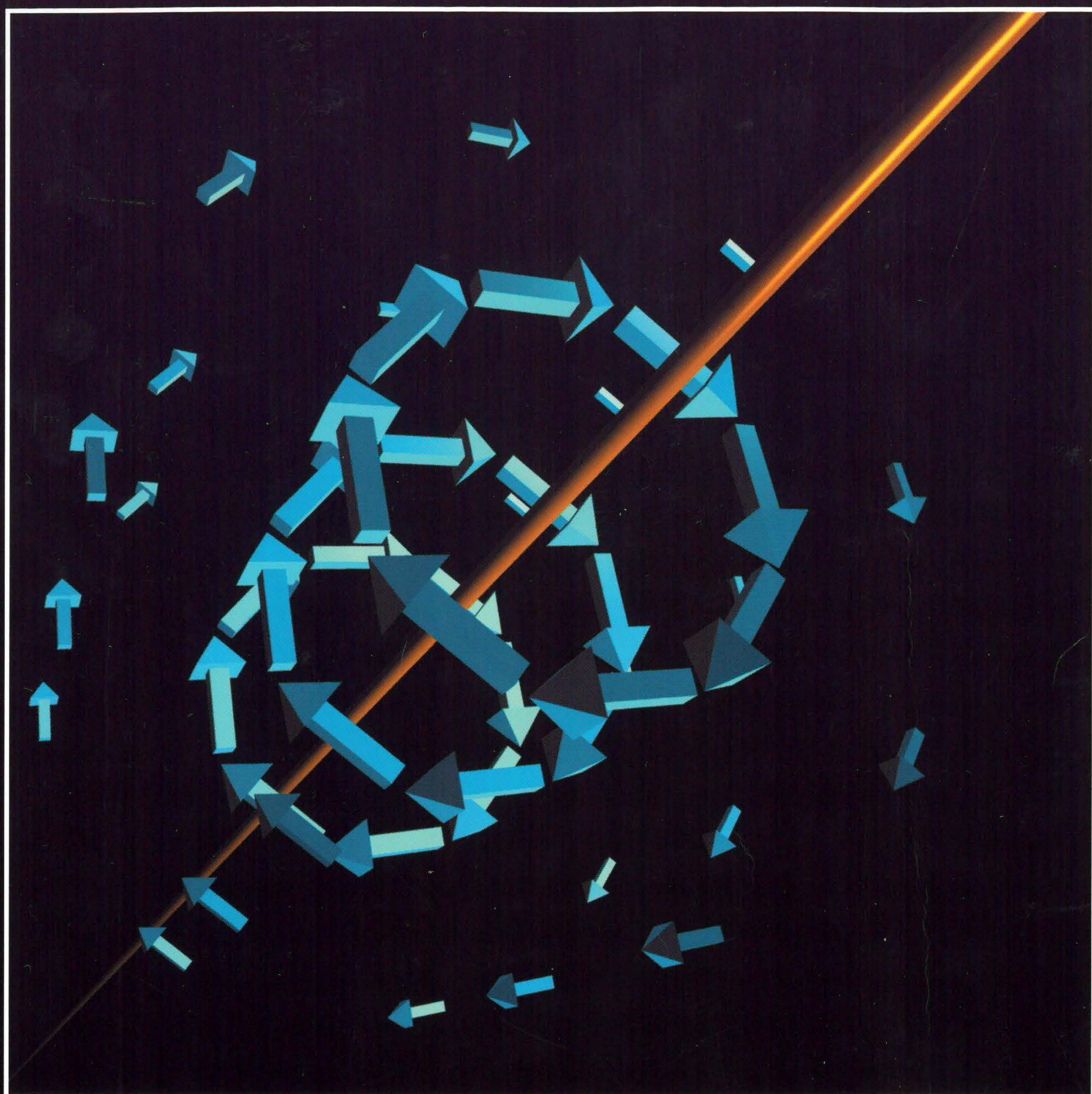

MATTER & INTERACTIONS II

ELECTRIC AND MAGNETIC INTERACTIONS

Second Edition

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ELECTRIC & MAGNETIC INTERACTIONS

SECOND EDITION

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CHAPTER 17

MAGNETIC FIELD

Key concepts

- Moving charged particles make a magnetic field, which is different from an electric field.
- The needle of a magnetic compass aligns with the direction of the net magnetic field at its location.
- A current is a continuous flow of charge.
    - Electron current is number of electrons per second entering a section of a conductor.
    - Conventional current is measured in coulombs/second (amperes), and is assumed to consist of moving positive charges (real or imagined). The direction of conventional current is opposite to the direction of electron current.
- The superposition principle can be applied to calculate the expected magnetic field from current-carrying wires in various configurations.
    - A current-carrying loop is a magnetic dipole.
    - A bar magnet is also a magnetic dipole.
    - A single atom can be a magnetic dipole.

Electric fields are not the only kind of field associated with charged particles. When a compass needle turns and points in a particular direction, we say that there is a "magnetic field" pointing in that direction, which forces the needle to line up with it (Figure 17.1). Initially we'll simply define magnetic field as "whatever it is that is detected by a compass." The twist of a compass needle is an indicator of magnetic fields, just as the twist of a suspended electric dipole is an indicator of electric fields, as you saw in Chapter 14.

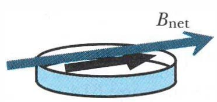


Figure 17.1 A compass needle points in the direction of the net magnetic field at its location.

Magnetic fields are made by moving charges, so we will need to have some moving charges at hand. We will assemble some simple electric circuits in which currents can run steadily, providing a convenient source of moving charges. In this chapter we will study magnetic field, and we will also develop an atomic-level description of magnets. Later, in Chapter 20, we will study the forces that magnetic fields exert on moving charges.

> Simple experiments
>
> There is a set of simple experiments related to the topics discussed in this chapter, which allow you to explore these phenomena yourself, using equipment such as batteries, wires, flashlight bulbs, and a compass. See Section 17.15 on page 612 for further information.

17.1 ELECTRON CURRENT

Magnetic fields are made by moving charges. A current in a wire provides a convenient source of moving charges, and allows us to experiment with producing and detecting magnetic fields.

In equilibrium, there is no net motion of the sea of mobile electrons inside a metal. In the electric circuits we will construct in this chapter, we can arrange things so the electron sea does keep moving continuously. This continuous flow of electrons is called an "electric current," and is an indication that the system is not in equilibrium. In order to be able to talk about what things affect electric current, we need a precise definition:

DEFINITION OF ELECTRON CURRENT

The electron current i is the number of electrons per second that enter a section of a conductor.

In an electric circuit with a steady current flowing, the electron current i is the same in every section of a wire of uniform thickness and composition (in Chapter 18 we'll see why). If we could count the number of electrons per second passing a particular point in a circuit (Figure 17.2), we could measure electron current directly. This is difficult to do, so we use indirect measurements to determine the magnitude of electron current in a wire. One such indirect measurement involves measuring the magnetic field created by the moving electrons.

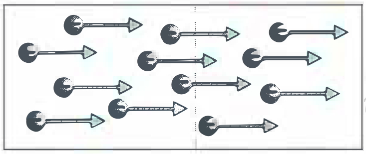


Figure 17.2 The electron current in this conductor is the number of electrons per second passing the dashed line.

As electrons drift through a wire, they collide with the atomic cores, and this "friction" heats the wire (and prevents the electrons from going faster and faster). Both the heating and the magnetic effects are proportional to the "electron current"—the number of electrons that enter the wire every second.

17.X.1 If 1.8×10^{16} electrons enter a light bulb in 3 milliseconds, what is the magnitude of the electron current at that point in the circuit?

17.X.2 If the electron current at a particular location in a circuit is 9×10^{18} electrons/s, how many electrons pass that point in 10 minutes?

Simple circuits

Related experiment:
17.EXP.17 on page 613

A simple electric circuit, involving a battery, wire, and a flashlight bulb, is a convenient means of producing a supply of moving electrons. We will refer to such circuits throughout this chapter. Despite the simplicity of the materials, the physics questions that can be investigated with such circuits are centrally important ones. If you have the appropriate equipment, you may wish to do these experiments yourself.

17.2 DETECTING MAGNETIC FIELDS

Related experiment:
17.EXP.18 on page 614

We can use a magnetic compass as a detector of magnetic fields. Just as the deflection of a charged piece of invisible tape or the twisting of a permanent electric dipole indicates the presence of an electric field, the twisting of a compass needle indicates the presence of a magnetic field.

? How can you be sure that a compass needle is not simply responding to electric fields?

In observing the behavior of a compass, we find that:

- The compass needle is affected by the proximity of objects made of iron or steel, even if these objects are electrically neutral (and therefore attract both positively and negatively charged tapes). It is also affected by the presence of nickel or cobalt, though these are less readily available.
- The compass needle is unaffected by objects made of most other elements, including aluminum, copper, zinc, and carbon, whereas charged tapes interact with these objects.
- If it is not near objects made of iron, the compass needle points toward the Earth's magnetic north pole, while neither electrically charged objects nor electric dipoles do this.

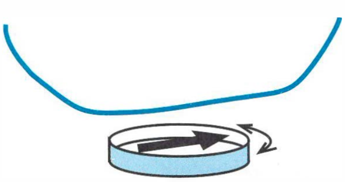


Figure 17.3 If a compass needle is originally pointing North, and a current-carrying wire aligned north-south is placed on the compass, the needle deflects. The deflection direction depends on whether the current runs northward or southward.

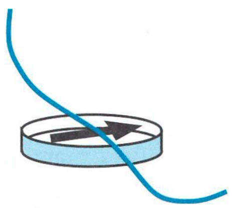


Figure 17.4 If a current-carrying wire aligned east-west is placed on a compass, the needle does not deflect.

If you bring a current-carrying wire near the compass, something very interesting happens: the compass needle is deflected while current is running in the wire, but not while the wire is disconnected from the battery. This effect was discovered by accident by the Danish scientist Oersted in 1820 while doing a lecture demonstration in a physics class. (He must have been surprised!) The phenomenon is often called "the Oersted effect." The biggest effect occurs if the wire points north-south (Figure 17.3); the deflection direction depends on whether the current runs northward or southward. The compass needle doesn't deflect if the wire points east-west (Figure 17.4).

From such experiments, one can draw the following conclusions:

- the magnitude of the magnetic field produced by a current of moving electrons depends on the amount of current
- a wire with no current running in it produces no magnetic field
- the magnetic field due to the current appears to be perpendicular to the direction of the current
- the direction of the magnetic field due to the current under the wire is opposite to the direction of the magnetic field due to the current above the wire

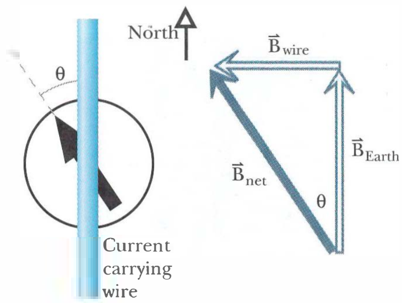


Figure 17.5 A compass needle points in the direction of the net magnetic field, which is the superposition of the magnetic field of the Earth and the magnetic field of the current-carrying wire which passes over the compass, aligned north-south.

A model for the observations

The moving electrons in a wire create a magnetic field at various locations in space, including at the location of the compass. The vector sum of the Earth's magnetic field $\vec{B}_{Earth}$ plus the magnetic field $\vec{B}_{wire}$ of a current-carrying wire makes a net magnetic field $\vec{B}_{net}$ (Figure 17.5). Since a compass needle points in the direction of the net magnetic field at its location, the needle turns away from its original north-south direction to align with the net field. Because the magnetic field made by the wire is perpendicular to the wire and is in opposite directions above and below the wire, the pattern of magnetic field made by the wire must look like Figure 17.6.

It happens (conveniently) to be the case that the magnitudes of $\vec{B}_{Earth}$ and $\vec{B}_{wire}$ are similar. If B_{Earth} were much larger than B_{wire}, then we would see almost no response from the compass.

? What would we observe if B_{wire} were much larger than B_{Earth}?

If B_{wire} were much larger than B_{Earth}, then the compass deflection would be nearly 90°, since the net magnetic field would be primarily due to the magnetic field made by the current in the wire.

There is hardly any observable electric interaction of the current-carrying wire with nearby materials, because the wires have no net charge (actually, we'll see later that there are tiny amounts of excess charge on the surface, but too little to observe easily).

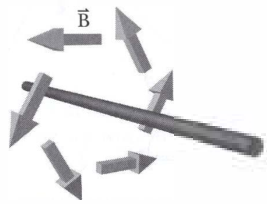


Figure 17.6 The magnetic field made by a current-carrying wire. At a location directly under the wire, the magnetic field is perpendicular to the wire.

Example: Compass deflection

A current-carrying wire is oriented north-south and laid on top of a compass. At the location of the compass needle the magnetic field due to the wire points west and has a magnitude of 3×10^{-6} T. The horizontal component of Earth's magnetic field has a magnitude of about 2×10^{-5} tesla. What compass deflection will you observe?

The compass needle will point in the direction of the net magnetic field, so from the diagram in Figure 17.7:

$$\theta = \tan^{-1}\left(\frac{B_{wire}}{B_{Earth}}\right) = \tan^{-1}\left(\frac{3\times10^{-6}\,\mathrm{T}}{2\times10^{-5}\,\mathrm{T}}\right) = 8.5°\ \text{west}$$

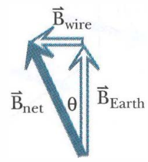


Figure 17.7 The compass points in the direction of the net magnetic field.

17.X.3 A current-carrying wire oriented north-south and laid over a compass deflects the compass 15° east. What are the magnitude and direction of the magnetic field made by the current? The horizontal component of Earth's magnetic field is about 2×10^{-5} tesla.

17.3 Biot-Savart Law: Single Moving Charge

Careful experimentation has shown that a stationary point charge makes an electric field given by this equation, called Coulomb's law:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

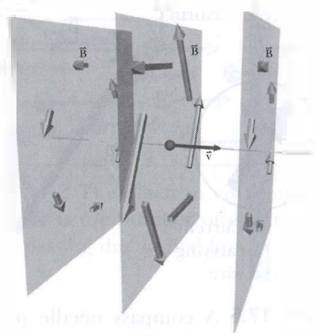


Figure 17.8 The magnetic field made by a moving positive charge, shown in three planes normal to $\vec{v}$.

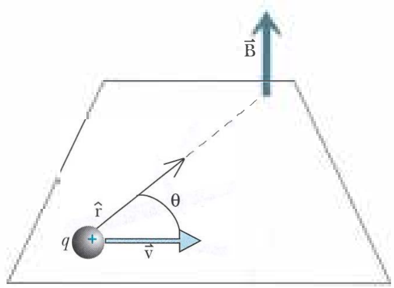


Figure 17.9 The magnetic field made by a moving charge is perpendicular to the plane defined by $\vec{v}$ and $\hat{r}$.

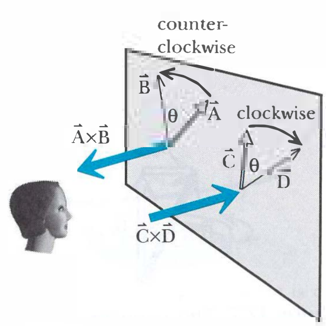


Figure 17.10 The cross product is out of or into the plane defined by the two vectors, depending on whether rotating the first vector toward the second vector (through an angle less than 180°) is counterclockwise or clockwise.

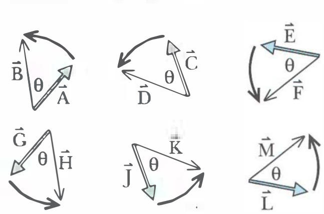


Figure 17.11 All these cross products have the same magnitude and the same direction (out of the plane).

Similarly, careful experimentation shows that a moving point charge not only makes an electric field but also makes a magnetic field (Figure 17.8), which curls around the moving charge. This curly pattern is characteristic of magnetic fields. (In contrast, we saw in Chapter 16 that it is impossible to produce a curly electric field by arranging stationary point charges.) Magnetic field is measured in "tesla" and its magnitude and direction are given by the "Biot-Savart law" (pronounced bee-oh sah-VAR):

THE BIOT-SAVART LAW FOR A SINGLE CHARGE

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}, \text{ where}$$

$$\frac{\mu_0}{4\pi} = 10^{-7}\,\frac{\text{tesla}\cdot\text{m}^2}{\text{coulomb}\cdot\text{m/s}} \quad \textit{exactly}$$

$\vec{v}$ is the velocity of the point charge q, and $\hat{r}$ is a unit vector that points from the source charge toward the observation location.

The vector cross product

The Biot-Savart law involves $\vec{v}\times\hat{r}$, a "vector cross product," which you encountered in the study of angular momentum. The magnetic field made by a moving charge is perpendicular to the plane defined by $\vec{v}$ and $\hat{r}$ (Figure 17.9).

There are two ways of calculating the cross product of two vectors $\vec{A}$ and $\vec{B}$. The first is an algebraic vector calculation; the second involves finding the magnitude and direction separately, using geometry. Both are useful.

CROSS PRODUCT $\vec{A}\times\vec{B}$

Vector: $\vec{A}\times\vec{B} = \langle (A_yB_z - A_zB_y), (A_zB_x - A_xB_z), (A_xB_y - A_yB_x) \rangle$

Magnitude of vector: $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

Direction of vector: see below

To determine the direction and magnitude of a vector cross product $\vec{A}\times\vec{B}$ geometrically, do the following:

- Place the tails of the vectors $\vec{A}$ and $\vec{B}$ together. These vectors define a plane. Look at this plane (Figure 17.10).
- Imagine rotating $\vec{A}$ toward $\vec{B}$, through the smaller of the two possible angles (never more than 180°). If the rotation is counterclockwise, $\vec{A}\times\vec{B}$ is out of the plane (toward the person in Figure 17.10). If the rotation is clockwise the result is into the plane ($\vec{C}\times\vec{D}$ in Figure 17.10 is into the plane, pointing away from the person)
- Find the magnitude of $\vec{A}\times\vec{B}$ by $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$, where the angle θ is the angle between $\vec{A}$ and $\vec{B}$. All the cross products in Figure 17.11 have the same magnitude (and the same direction).

The right-hand rule

There is another way to determine the direction of the cross product, using a "right-hand rule":

- Point fingers of right hand in direction of first vector $\vec{A}$ (Figure 17.12)
- Rotate wrist, if necessary, to make it possible to
  - Bend fingers of right hand toward second vector $\vec{B}$ through an angle θ less than 180° (Figure 17.13)
  - Stick out thumb, which points in direction of cross product $\vec{A}\times\vec{B}$

Wrist rotation

The rotation of the wrist is an important part of the right-hand rule. Consider the situation in Figure 17.14. With your right hand in this position, you can't bend your fingers backwards from the first vector $\vec{A}$ toward the second vector $\vec{B}$—it is a physical impossibility. You need to rotate your wrist into a position from which it is possible to bend the fingers, as shown in Figure 17.15. Pay attention to the size of the angle through which you bend your fingers. This is the angle whose sine is part of the definition of the magnitude of the cross product. This angle should never be more than 180°. If it is, you have made a mistake in orienting your hand and are in danger of piercing your palm with your nails! Probably you need to rotate your wrist.

Try this right-hand rule on the examples in Figure 17.10. When looking at $\vec{A} \times \vec{B}$ in the plane in Figure 17.10 you will find that your fingers bend counterclockwise and your thumb points out of the plane. When looking at $\vec{C} \times \vec{D}$ in the plane in Figure 17.10 you will find that you have to rotate your wrist before you can bend your fingers, and your thumb points into the plane. So this right-hand rule and the counterclockwise/clockwise rule used in Figure 17.10 are equivalent. In Figure 17.11 you'll find that in all cases the cross product is out of the plane.

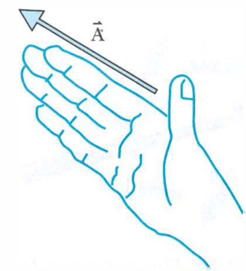


Figure 17.12 Open right hand, fingers point in direction of $\vec{A}$.

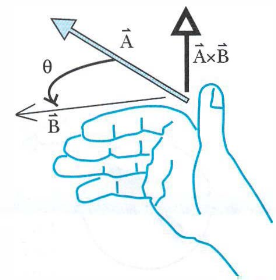


Figure 17.13 Bend fingers less than 180° toward lining up with $\vec{B}$. Thumb points in direction of $\vec{A} \times \vec{B}$.

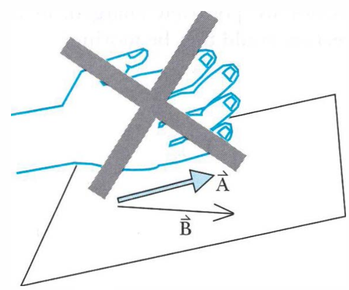


Figure 17.14 You can't bend your fingers backward. You must rotate the wrist into a position that lets you bend the fingers.

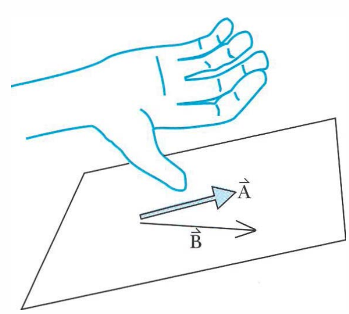


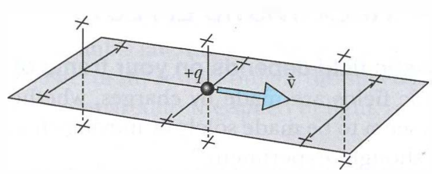
Figure 17.15 After rotating the wrist, it is possible to bend the fingers.

Two-dimensional projections

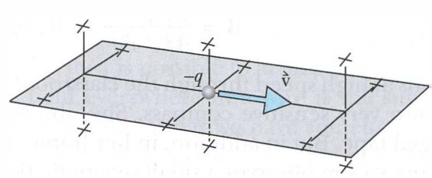
Because it is more difficult to sketch a situation in three dimensions, whenever possible we will work with two-dimensional projections onto the x-y plane. If $\vec{A}$ and $\vec{B}$ lie in the x-y plane, the cross product vector $\vec{A} \times \vec{B}$ points in the $+z$ direction (out of the page, ⊙) or in the $-z$ direction (into the page, ⊗).

17.X.4 Given that $\vec{v} = \langle v_x, v_y, v_z \rangle$ and $\hat{r} = \dfrac{\langle x, y, z \rangle}{\sqrt{x^2 + y^2 + z^2}}$, write out $\vec{v} \times \hat{r}$ as a vector.

17.X.5 The following is an important exercise. At the locations marked with × in the accompanying figure, determine the direction of the magnetic-field vectors due to a positive charge $+q$ moving with a velocity $\vec{v}$. For each observation location, draw the unit vector $\hat{r}$ *from* the charge *to* that location, then consider the cross product $\vec{v} \times \hat{r}$. Pay attention not only to the directions of the magnetic field but also to the relative magnitudes of the vectors.



17.X.6 If the charge is negative ($-q$) as in the adjacent figure, how does this change the pattern of magnetic field?



17.X.7 Explain why the magnetic field is zero straight ahead of and straight behind a moving charge.

17.X.8 How does the magnetic field of a moving point charge fall off with distance at a given angle: like $1/r$, $1/r^2$, or $1/r^3$?

17.X.9 Describe the magnetic field made by a charge that is not moving.

17.X.10 To get an idea of the size of magnetic fields at the atomic level, consider the magnitude of the magnetic field due to the electron in the simple Bohr model of the hydrogen atom. In the ground state the Bohr model predicts that the electron speed would be 2.2×10^6 m/s, and the distance from the proton would be 0.5×10^{-10} m. What is B at the location of the proton?

In the Appendix on vectors you can see how to calculate cross products in terms of unit vectors along the axes, $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

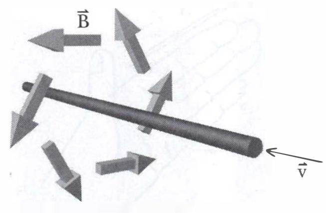


Figure 17.16 The magnetic field due to moving charges in the wire curls around the wire. Here the moving charges are assumed to be negative.

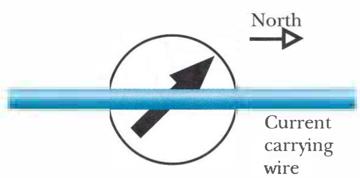


Figure 17.17 If the moving particles in the wire are electrons, in what direction are they moving? If we assume that the particles are positively charged, in which direction would they be moving?

Sign of the moving charge

Assume that the moving charged particles in a current-carrying wire are negatively charged, and move in the direction shown in Figure 17.16.

> **?** Use the Biot-Savart law and the right-hand rule to predict the direction of the magnetic field at several locations around the wire (Figure 17.16).

Convince yourself that the cross product $\vec{v} \times \hat{r}$ does correctly give the direction and pattern of magnetic field near a wire. From this curly pattern you can see that the direction of compass deflection will be different if the wire is above or below the compass, which is observed.

> **?** Try the analysis again, but assume that the moving charges are positively charged particles moving in a direction opposite to that shown. Do you still predict the curly magnetic field as shown in Figure 17.16?

You should have concluded that for the purpose of predicting the direction of magnetic field, it does not matter whether we assume that the moving charges are negative and moving in a given direction, or positive and moving in the opposite direction. It is common to describe current flow in terms of "conventional current," which means assuming that the moving charges are positive, even if this is known not to be the case in a particular situation.

17.X.11 In Figure 17.17 a current-carrying wire lies on top of a compass. Judging from the deflection of the compass away from north, what is the direction of the electron current in the wire? If the current were due to the motion of positive charges, which way would they be moving?

17.4 RELATIVISTIC EFFECTS

Magnetic field depends on your frame of reference

Electric fields are made by charges, whether at rest or moving. Magnetic fields seem to be made solely by moving charges. But consider the following odd "thought experiment."

Suppose Jack sits in the classroom with a charged piece of invisible tape stuck to the edge of his desk. He can of course observe an electric field due to the charged tape, but he doesn't observe any magnetic field. His compass is unaffected by the charged tape, since those charges aren't moving:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} = 0, \text{ since } v = 0$$

Jill runs at high speed through the classroom past the charged tape, carrying her own very sensitive compass. She observes an electric field due to the charged tape. But in addition, in her frame of reference the charged tape is moving, so she observes a small magnetic field due to the moving charges, which affects her compass! As far as Jack is concerned, the charged tape just makes an electric field. But apparently Jill sees a mixture of electric and magnetic fields made by what is for her a moving charged tape.

Up until now we have implied that electric fields and magnetic fields are fundamentally different, but this "thought experiment" shows that they are in fact closely related. Moreover, this connection raises questions about the Biot-Savart law:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

Just what velocity are we supposed to use in this formula? The velocity of the tape relative to Jack (which is zero) or the velocity of the tape relative to Jill (opposite her own velocity)?

The correct answer is that you use the velocities of the charges as you observe them in your frame of reference. Using these velocities in the Biot-Savart law you calculate the magnetic field; in your frame of reference you observe a magnetic field that agrees with your prediction. Observers in a different reference frame use the velocities observed in their frame to calculate the magnetic field using the Biot-Savart law, and in their frame of reference they observe a magnetic field that agrees with their prediction. You and they make different predictions, and observe different magnetic fields, but both you and they find agreement between theory and experiment.

Evidently there is a deep connection between electric fields and magnetic fields, and this connection is made explicit in Einstein's special theory of relativity. Later in the text we will see further aspects of this connection.

Retardation

Remember that when you move a charge, the electric field of that charge at a distance from the charge doesn't change instantaneously (Section 13.6). The electric field doesn't change until a time sufficient for light to reach the observation location, and you measure a change in the electric field at the same instant that you see the charge move.

The same retardation effect is observed with magnetic fields. If you suddenly change the current in a wire, the magnetic field at some distance from the source of the magnetic field stays the same until enough time has elapsed for light to reach the observation location. So magnetic field has some reality in its own right, independent of the moving charges that originally produced it.

The Biot-Savart law does not contain any reference to time, so it cannot be relativistically correct. Like Coulomb's law, the Biot-Savart law is only approximately correct and will give accurate results only if the speeds of the moving source charges are small compared to the speed of light.

17.5 ELECTRON CURRENT & CONVENTIONAL CURRENT

An easy way to observe the magnetic fields made by moving charged particles is to initiate and sustain a current—a continual flow of charged particles in one direction. In order to do this, we need to find a way to produce and sustain an electric field inside a wire, since we know that if the electric field inside a metal becomes zero, the metal object will be in equilibrium, and no current will flow. How exactly it is possible to arrange charges in order to create such an electric field everywhere in a wire is the subject of Chapter 18. For the moment, we'll assume that we have somehow accomplished this by assembling a circuit from batteries, wires, and perhaps light bulbs, since evidently currents do flow in such circuits.

In order to apply the Biot-Savart law to predict the magnitude and direction of the magnetic field associated with this current, we need to know the number of moving charges making the field, and how fast they are moving.

A formula for electron current

Consider a section of a metal wire through which the mobile-electron sea is continuously shifting, under the influence of a a nonzero electric field, as indicated in Figure 17.18. There can be no excess charge anywhere inside the wire. To every mobile electron there corresponds a singly charged positive atomic core—an atom minus one electron which has been released to roam freely in the mobile-electron sea. Averaged over a few atomic diame-

Figure 17.18 The mobile electron sea drifts to the left with a speed $\bar{v}$.

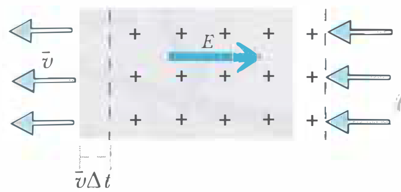


Figure 17.19 During a time Δt the electron sea shifts a distance $\bar{v}\Delta t$.

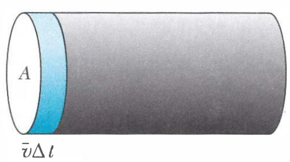


Figure 17.20 The volume of the electron sea that has flowed past a point in the wire during a time Δt is $A\bar{v}\Delta t$.

Related experiment:
17.EXP.19 on page 614

ters, the interior of the metal is neutral, and the repulsion between the mobile electrons is balanced out on the average by the attractions of the positive cores.

Suppose that in a metal wire the mobile-electron sea as a whole has an average "drift speed" $\bar{v}$. Assume that the current is evenly distributed across the entire cross section of the wire (in a later chapter we will be able to show that this is true.) After a short time Δt, this section of the electron sea will drift to the left a distance $\bar{v}\Delta t$, as indicated in Figure 17.19.

If the cross-sectional area of this wire is A, the volume of the disk-shaped portion of the sea that has flowed out of the left end of this section of the wire (and into the adjoining section of wire) in the time Δt is $A\bar{v}\Delta t$ (Figure 17.20).

? If the density of mobile electrons (that is, the number of mobile electrons per unit volume) everywhere in the metal is n, how many electrons are in this disk?

$$\left(n\ \frac{\text{electrons}}{\text{m}^3}\right)(A\ \text{m}^2)\left(\bar{v}\ \frac{\text{m}}{\text{s}}\right)(\Delta t\ \text{s}) = nA\bar{v}\Delta t\ \text{electrons}$$

The number of electrons in this disk is the number of electrons that have flowed out of this section of the wire in the short time Δt.

? What is the number of mobile electrons per second that flow past the left end of this piece of the wire?

We divide by the time Δt to calculate the number of mobile electrons per second that flow past the left end of this piece of the wire:

$$\frac{\#\text{electrons}}{\text{s}} = \frac{nA\bar{v}\Delta t}{\Delta t} = nA\bar{v}$$

This result, that the number of electrons passing some section of the wire per second is $nA\bar{v}$, is sufficiently important to be worth remembering (though it is even better to remember the simple derivation that gives this result).

ELECTRON CURRENT

The rate i at which electrons pass a section of a wire:

$$i = nA\bar{v} \quad (\text{\# of electrons per second})$$

n is the mobile electron density (the number of mobile electrons per unit volume), A is the cross-sectional area of wire, and $\bar{v}$ is the average drift speed of electrons.

In a circuit, electrons leave the negative end of the battery (the end marked "–") and flow through the wire to the positive end of the battery.

17.X.12 You have used copper wires in your circuits. Let's calculate the mobile electron density n for copper. A mole of copper has a mass of 64 g (0.064 kg), and one mobile electron is released by each atom in metallic copper. The density of copper is about 9 grams per cubic centimeter, or $9\times10^3\ \text{kg/m}^3$. Show that the number of mobile electrons per cubic meter in copper is $8.4\times10^{28}\ \text{m}^{-3}$.

17.X.13 Suppose that $i = 3.4\times10^{18}$ electrons/s are drifting through a copper wire. (This is a typical value for a simple circuit.) The cross-sectional area of the wire is $8\times10^{-7}\ \text{m}^2$ (corresponding to a diameter of 1 mm), and the density of mobile electrons in copper is $8.4\times10^{28}\ \text{m}^{-3}$. What is the drift speed of the electrons?

17.X.14 At the drift speed found in the previous exercise, about how many minutes would it take for a single electron in the electron sea to drift from one end to the other end of a wire 30 cm long, about one foot? (A puzzle: if the drift speed is so slow, how can a lamp light up as soon as you turn it on? We'll come back to this in the next chapter.)

Conventional current

In most metals, current consists of drifting electrons. However there are a few materials in which the moving charges are positive "holes" in the sea of mobile electrons. The positive holes act in every way like real positive particles. For example, holes drift in the direction of the electric field, and they have a charge of $+e$.

Electron current moves from the negative end of a battery through a circuit to the positive end of the battery; hole current would go in the opposite direction. Given this information, consider the following question:

? From observations of the direction of magnetic field around a copper wire, can one tell whether the current in copper consists of electrons, or holes?

Observations of the magnetic field due to a current-carrying wire are not sufficient to tell the difference, because if the sign of the moving charge is changed, the direction of the drift velocity is also changed: $(+e)(+\bar{v}) \rightarrow (-e)(-\bar{v})$ (Figure 17.21 and Figure 17.22). Therefore the prediction of the Biot-Savart law is exactly the same in either case.

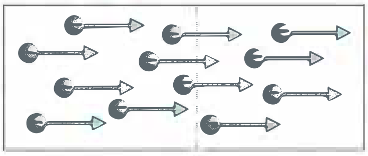


Figure 17.21 An electron current to the right moves negative charge from left to right.

In copper, as in most metals, the current is due to moving electrons. The few metals that have hole current include aluminum and zinc. In the doped semiconductors important in electronics, "p-type" material (positive type) involves hole current, and "n-type" (negative type) involves electron current. In Chapter 20 we will study the Hall effect, which can be used to determine whether the current in a wire is due to electrons or to holes.

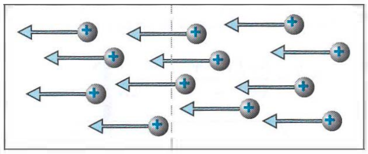


Figure 17.22 A conventional current to the left would move positive charge from right to left, which would have the same effect as moving negative charge from left to right, as in the previous figure.

Because most effects (other than the Hall effect) are the same for electron current and hole current, it is traditional to define "conventional current" to run in the direction of hole current, even if the actual current consists of moving electrons, in which case the "conventional current" runs in the opposite direction to the electrons (Figure 17.22). This simplifies calculations by eliminating the minus sign associated with electrons. (This sign ultimately goes back to a choice made by Benjamin Franklin when he arbitrarily assigned "positive" charge to be the charge we now know to be carried by protons.)

In addition, conventional current I is defined not as the number of holes passing some point per second but rather as the amount of charge (in coulombs) passing that point per second. This is the number of holes per second multiplied by the (positive) charge $|q|$ associated with one hole:

CONVENTIONAL CURRENT

$$I = |q| n A \bar{v} \text{ (coulombs per second, or amperes)}$$

The direction of conventional current is opposite to the direction of electron current. The moving charged particles are assumed to be positive.

In a metal, $|q| = e$, but in an ionic solution the moving ions might have charges that are a multiple of e.

17.X.15 In Exercise 17.X.12 you calculated an electron current of $i = 3.4\times10^{18}$ electrons/s. What was the conventional current, including units?

17.6 THE BIOT-SAVART LAW FOR CURRENTS

We don't often observe the magnetic field of a single moving charge. Usually we are interested in the magnetic field produced by a large number of charges moving through a wire in a circuit. The superposition principle is valid for magnetic fields, so we need to add up the magnetic-field contributions of the individual charges.

Let's calculate the magnetic field due to a bunch of moving positive charges contained in a small thin wire of length Δl and cross-sectional area A (Figure 17.23). If there are n moving charges per unit volume, there are $nA\Delta l$ moving charges in this small volume. We will measure the magnetic field at a location far enough away from this small volume that each moving charge produces approximately the same magnetic field at that location.

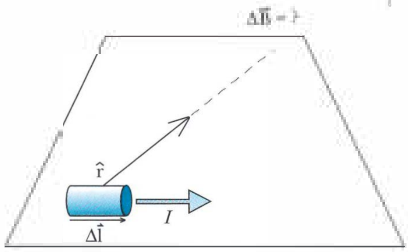


Figure 17.23 Magnetic field contributed by a short thin length Δl of current-carrying wire.

? Show that the net magnetic field of all the moving charges in this small volume, far from the volume, is

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2},$$

where $\Delta\vec{l}$ is a vector with magnitude Δl, (the length of the segment of wire) pointing in the direction of the conventional current I.

This results from the fact that the volume $A\Delta l$ contains $n(A\Delta l)$ moving charges, so the magnetic field is $n(A\Delta l)$ times as large as the magnetic field of one of the charges. Each moving charge has a charge q, and the conventional current is $I = |q|(nA\bar{v})$. Collecting terms, we find the result given above.

So we have an alternative form of the Biot-Savart law for the magnetic field contributed by a short thin length of current-carrying wire (Figure 17.24):

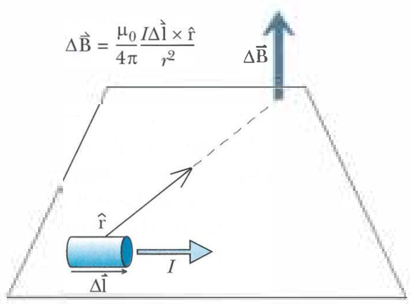


Figure 17.24 The Biot-Savart law for the magnetic field contributed by a short thin length of wire carrying conventional current I. The magnetic field due to this current segment is perpendicular to the plane defined by $\Delta\vec{l}$ and $\hat{r}$.

THE BIOT-SAVART LAW FOR A SHORT THIN LENGTH OF WIRE

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2}$$

$$\text{where } \frac{\mu_0}{4\pi} = 10^{-7}\ \frac{\text{tesla}\cdot\text{m}^2}{\text{coulomb}\cdot\text{m/s}}\ \textit{exactly},$$

$\Delta\vec{l}$ is a vector in the direction of the conventional current I, and whose magnitude Δl is the length of the segment of wire. $\hat{r}$ is a unit vector that points *from* the source charge *toward* the observation location.

? Explain why this formula also gives the right results if the moving charges are (negative) electrons, as long as we interpret $\Delta\vec{l}$ as a vector in the direction of the conventional current.

The law works for moving electrons because while they have the opposite charge, they also move in the direction opposite to the direction of the conventional current. These two changes in sign cancel each other.

It is important to keep in mind that there is really only one Biot-Savart law, not two. Always remember that

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2}$$

is simply the result of adding up the effects of many moving charges in a short thin length of wire, each of which contributes a magnetic field

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$

The key point is that there are $nA\Delta l$ electrons in a short length of wire, each moving with average speed $\bar{v}$, so that the sum of all the $q\bar{v}$ contributions is $(nA\Delta l)|q|\bar{v} = (|q|nA\bar{v})\Delta l = I\Delta l$.

17.7 THE MAGNETIC FIELD OF CURRENT DISTRIBUTIONS

In the next sections we will apply the Biot-Savart law to find the magnetic field of various distributions of currents in wires. We will use the same four-step approach we used to find the electric field of distributed charges in Chapter 15:

1) Cut up the current distribution into pieces; draw $\Delta\vec{B}$ for a representative piece.
2) Write an expression for the magnetic field due to one piece.
3) Add up the contributions of all the pieces.
4) Check the result.

Applying the Biot-Savart law: A long straight wire

Using the procedure outlined above, we can calculate the magnetic field near a long straight wire. This will make it possible to predict the compass needle deflection that you observe when you bring a wire near your compass, and you can compare your prediction with your experimental observation.

The long straight wire is one of the few cases that we can calculate completely by hand. Except for a few special cases, calculating the magnetic field due to distributed currents is often best done by computer. The basic concepts you would use in a computer program are the same as we will use here, but the computer would carry out the tedious arithmetic involved in the summation step of the procedure.

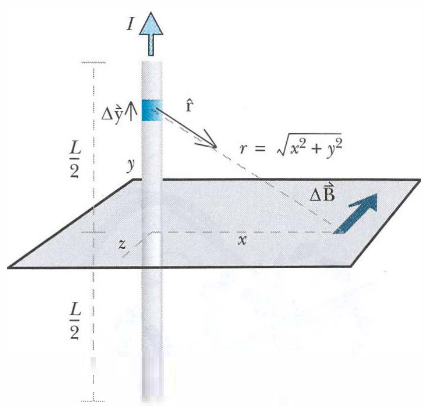


Figure 17.25 Cut the wire into short pieces and draw the magnetic field contributed by one of the pieces.

Step 1: Cut up the distribution into pieces and draw $\Delta\vec{B}$

We will consider a wire of length L (Figure 17.25). We cut up the wire into very short sections each of length Δy, where Δy is a small portion of the total length. We will calculate the magnetic field a perpendicular distance x from the center of the wire. Using the right-hand rule, we can draw the direction of $\Delta\vec{B}$ for this "piece" of current.

Step 2: Write an expression for the magnetic field due to one piece

origin: Center of wire

vector $\vec{r}$: $\vec{r} = \langle x, 0, 0\rangle - \langle 0, y, 0\rangle = \langle x, -y, 0\rangle$

magnitude of $\vec{r}$: $r = [x^2 + (-y)^2]^{1/2}$

unit vector $\hat{r} = \frac{\vec{r}}{r} = \frac{\langle x, -y, 0\rangle}{[x^2+(-y)^2]^{1/2}}$

As usual, $\vec{r}$ extends from the source location to the observation location.

location of piece: depends on y (so y will be the integration variable)

$\Delta\vec{l}$ *in terms of* y: $\Delta\vec{l} = \Delta y\langle 0, 1, 0\rangle$

magnetic field due to one piece:

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta y\hat{j}\times\hat{r}}{(x^2+y^2)} = \frac{\mu_0}{4\pi}\frac{I\Delta y\langle 0,1,0\rangle}{(x^2+y^2)}\times\frac{\langle x,-y,0\rangle}{[x^2+(-y)^2]^{1/2}}$$

evaluate cross product:

$$\langle 0, 1, 0\rangle \times \langle x, -y, 0\rangle = \langle 0, 0, -x\rangle$$

expression for $\Delta\vec{B}$:

$$\Delta\vec{B} = -\frac{\mu_0}{4\pi}\frac{Ix\Delta y}{(x^2+y^2)^{3/2}}\langle 0, 0, 1\rangle$$

components: Every piece of the straight wire contributes some magnetic field in the $-z$ direction, so we need to calculate only the z component.

Step 3: Add up the contributions of all the pieces

By letting $\Delta y \to 0$, we can write the sum as an integral:

$$B = \frac{\mu_0}{4\pi}Ix\int_{y=-L/2}^{y=+L/2}\frac{dy}{(x^2+y^2)^{3/2}}$$

Fortunately, this integral can be found in standard tables of integrals:

$$B = \frac{\mu_0}{4\pi}Ix\left[\frac{y}{x^2\sqrt{x^2+y^2}}\right]_{y=-L/2}^{y=+L/2}$$

$$B = \frac{\mu_0}{4\pi}Ix\left[\frac{L/2}{x^2\sqrt{x^2+(L/2)^2}} - \frac{-L/2}{x^2\sqrt{x^2+(L/2)^2}}\right]$$

$$B = \frac{\mu_0}{4\pi}\frac{LI}{x\sqrt{x^2+(L/2)^2}}$$

An extremely important case is that of a very long wire ($L >> x$) or, equivalently, the magnetic field very near a short wire ($x << L$).

? Show that you get

$$B \approx \frac{\mu_0}{4\pi}\frac{2I}{x} \text{ if } L >> x$$

If $L >> x$, $\sqrt{x^2+(L/2)^2} \approx L/2$, which leads to the stated result.

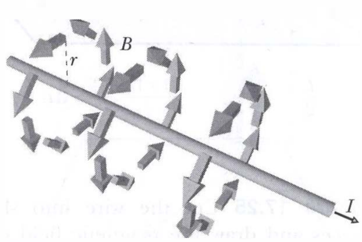


Figure 17.26 Magnetic field of a straight current-carrying wire, at selected locations a distance r radially outward from the wire.

Related experiment:
17.EXP.20 on page 614

In summary, Figure 17.26 shows the pattern of magnetic field a distance x from the center of a straight wire of length L. Because of the symmetry of this field, it does not matter where we draw our x axis—the field will be the same all around the rod. To indicate this, we replace x in our formula with r, the radial distance from the rod.

Although the magnitude of the field is constant at constant r, the direction of the field is different at each angle, since the field curls around the wire. To express this as a vector equation we would have to include a cross product in the expression; it's simpler to get the direction using the right-hand rule.

MAGNETIC FIELD OF A STRAIGHT WIRE

$$B_{wire} = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}}$$

(length L, conventional current I, a perpendicular distance r from center of wire)

$$B_{wire} \approx \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ if } L >> r$$

Historically, the result for a very long straight wire was first obtained by the French physicists Biot and Savart. Their names have come to be associated with the more fundamental principle (the "Biot-Savart law") that leads to this result.

Step 4: Check the result

direction: In Figure 17.26 the right-hand rule is consistent with the diagram.
units:

$$\left(\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{(\text{m}\cdot\text{A})}{(\text{m}\cdot\text{m})} = \text{T}$$

far away ($r >> L$):

$$B_{\text{wire}} \approx \frac{\mu_0}{4\pi}\frac{LI}{r^2} = \frac{\mu_0}{4\pi}\frac{|I\Delta\vec{l} \times \hat{r}|}{r^2}$$

which is indeed the magnetic field contributed by a short wire of length Δl.

Another right-hand rule

There is another right-hand rule which is often convenient to use with current-carrying wires. As you can see in Figure 17.27, if you grasp the wire in your right hand with your thumb pointing in the direction of conventional current, your fingers curl around in the direction of the magnetic field. Clearly this right-hand rule is consistent with the one we used for the Biot-Savart law.

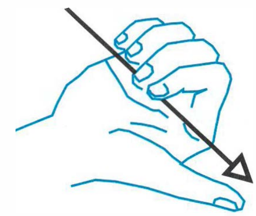


Figure 17.27 The thumb of the right hand points in the direction of conventional current flow I, and the fingers curl around in the direction of the magnetic field.

17.8 A CIRCULAR LOOP OF WIRE

Next we'll calculate the magnetic field of a circular loop of wire that carries a conventional current I. We'll do only the easiest case—the magnetic field at any location along the axis of the loop, which is a line going through the center and perpendicular to the loop.

This calculation is important for two reasons. First, many scientific and technological applications of magnetism involve circular loops of current-carrying wire. Second, the calculation will also lead into an analysis of atomic current loops in your bar magnet. After calculating the magnetic field we will compare our predictions with experiments.

Step 1: Cut up the distribution into pieces and draw $\Delta\vec{B}$

See Figure 17.28. We cut up the loop into very short sections each of length Δl (a small portion of the total circumference $2\pi R$) (Figure 17.28; side view in Figure 17.29). Determine direction of $\Delta\vec{B}$ with right-hand rule. Note that the angle between $\Delta\vec{l}$ and $\hat{r}$ is 90° for every location on the ring.

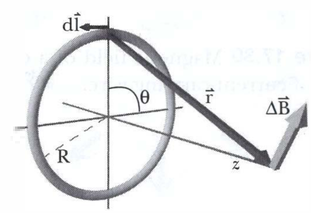


Figure 17.28 Cut the loop into short pieces and draw the magnetic field contributed by one of the pieces. Note that $d\vec{l} \perp \hat{r}$ for every piece of the loop. The loop is in the xy plane, with x to the right and y up. $\Delta\vec{B}$ is perpendicular to the plane defined by $\Delta\vec{l}$ and $\vec{r}$.

Step 2: Write an expression for the magnetic field due to one piece

To work out $\Delta\vec{B}$ for an arbitrary $\Delta\vec{l}$ will be algebraically messy, since

$$\Delta\vec{l} = \langle R\cos(\theta + d\theta), R\sin(\theta + d\theta), 0\rangle - \langle R\cos\theta, R\sin\theta, 0\rangle\text{, and}$$

$$\vec{r} = \langle 0, 0, z\rangle - \langle R\cos\theta, R\sin\theta, 0\rangle$$

However, we can simplify the problem considerably by noticing the symmetry of the situation. Because of the circular symmetry of the ring, the ΔB_x and ΔB_y contributed by one piece will be canceled by the contributions of a piece located on the other side of the loop.

Furthermore, the ΔB_z contributed by each piece of the ring will be exactly the same. This allows us to select one piece for which ΔB_z is easy to calculate, and use this value for every piece in the sum. We will select the piece shown in the diagram, located on the y axis.

origin: Center of loop

vector $\vec{r}$: $\vec{r} = \langle\text{obs. loc.}\rangle - \langle\text{source}\rangle = \langle 0, 0, z\rangle - \langle 0, R, 0\rangle = \langle 0, -R, z\rangle$

magnitude of $\vec{r}$: $r = [R^2 + z^2]^{1/2}$

unit vector $\hat{r} = \frac{\vec{r}}{r} = \frac{\langle 0, -R, z\rangle}{[R^2 + z^2]^{1/2}}$

location of piece: depends on θ (so θ will be the integration variable)

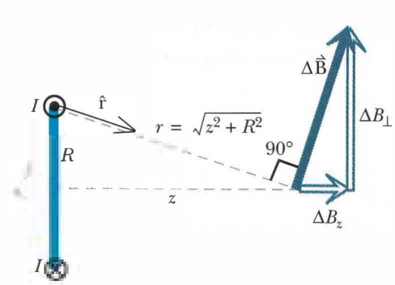


Figure 17.29 Side view of the current-carrying loop. Again note that $d\vec{l} \perp \hat{r}$ for every piece of the loop.

$\Delta\vec{l} : |\Delta\vec{l}| = \langle -R\Delta\theta, 0, 0\rangle$ the magnitude of $\Delta\vec{l}$ is $R\Delta\theta$.

magnetic field due to one piece:

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} I \frac{\langle -R\Delta\theta, 0, 0\rangle \times \langle 0, -R, z\rangle}{[R^2 + z^2]^{3/2}}$$

evaluate cross product:

$$\langle -R\Delta\theta, 0, 0\rangle \times \langle 0, -R, z\rangle = \langle 0, zR\Delta\theta, R^2\Delta\theta\rangle$$

We need only the z component, since the others will add up to zero:

expression for ΔB_z:

$$\Delta B_z = \frac{\mu_0}{4\pi}\frac{IR^2\Delta\theta}{[R^2 + z^2]^{3/2}}$$

In Figure 17.29 we show the component ΔB_z along the axis and the component $\Delta B_\perp$ perpendicular to the axis.

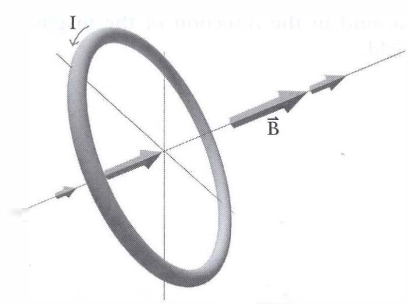


Figure 17.30 Magnetic field of a circular loop of current-carrying wire.

Step 3: Add up the contributions of all the pieces

We can express the sum as an integral, where θ, which specifies the location of a piece of the ring, runs all the way around the ring. Remember that ΔB_z contributed by each piece is the same.

$$B_z = \int_0^{2\pi} \frac{\mu_0}{4\pi}\frac{IR^2 d\theta}{(z^2 + R^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{IR^2}{(z^2 + R^2)^{3/2}}\int_0^{2\pi} d\theta$$

The integral $\int_0^{2\pi} d\theta$ is just 2π. Here is the result (Figure 17.30):

MAGNETIC FIELD OF A LOOP

$$B_{loop} = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}}$$

for a circular loop of radius R and conventional current I at a distance z from the center, along the axis

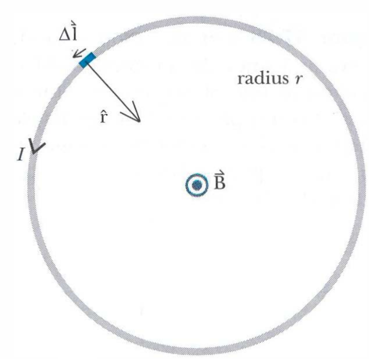


Figure 17.31 Magnetic field at the center of a loop of current-carrying wire.

Step 4: Check the result

units:

$$\left(\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{(\text{m}^2\cdot\text{A})}{(\text{m}^2)^{3/2}} = \text{T}$$

direction: Check several pieces with the right-hand rule.

special case: Center of the loop (Figure 17.31), where the magnetic field is especially easy to calculate from scratch. By the right-hand rule, $\Delta\vec{l} \times \hat{r}$ points out of the page for every piece of the loop, and its magnitude is simply Δl, since $\Delta\vec{l} \perp \hat{r}$ —the magnitude of $\hat{r}$ is 1, and $\sin(90°)=1$.

So we have this:

$$B = \sum \frac{\mu_0}{4\pi}\frac{I\Delta l}{R^2} = \frac{\mu_0}{4\pi}\frac{I(2\pi R)}{R^2} = \frac{\mu_0}{4\pi}\frac{2\pi I}{R}$$

because the sum of all the Δl's is just $2\pi R$, the circumference of the ring.

? Show that the general formula for the magnetic field of a loop reduces to this result if you let $z = 0$, which is another kind of check.

Related experiment:
17.EXP.21 on page 614

Qualitative features of the magnetic field of a loop of wire

It is interesting to see what the magnetic field is *far* from the loop, along the axis of the loop.

? Show that if z is very much larger than the radius R, the magnetic field is approximately equal to

$$\frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{z^3}$$

The key to this result is that if $z >> R$, $(z^2 + R^2)^{3/2} \approx (z^2)^{3/2} = z^3$. We see that the magnetic field of a circular loop falls off like $1/z^3$.

? Figure 17.32 shows the pattern of magnetic field along the axis of a coil containing many loops. What is the direction of the magnetic field at the points above and below the coil? (Hint: Apply the Biot-Savart law qualitatively to the near and far halves of the circular loop.)

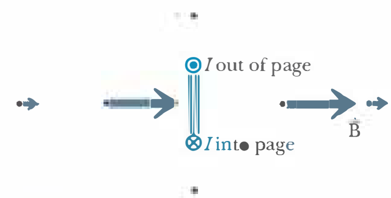


Figure 17.32 Predict the direction of the magnetic field of the coil, above and below the coil.

Above the coil in Figure 17.33 the upper part of the coil contributes a larger magnetic field to the left than does the lower part of the coil to the right, so the net field points to the left. Compare the pattern of magnetic field in Figure 17.33 with the pattern of *electric* field around an electric dipole.

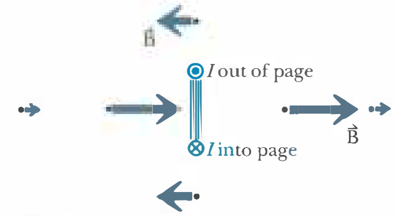


Figure 17.33 The magnetic field above and below the coil points in the opposite direction to the field along the axis.

Magnetic field at other locations outside the loop

The magnetic field at other locations outside the loop is more difficult to calculate analytically, but the magnetic field has a characteristic dipole pattern (Figure 17.34).

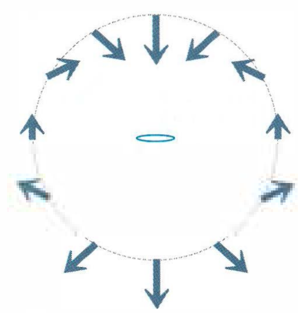


Figure 17.34 The magnetic field of a current loop (which lies in the xz plane, viewed edge-on), at locations outside the loop, in a plane containing the axis of the loop.

A special right-hand rule for current loops

There is another "right-hand rule" that is often used to get the direction of the magnetic field along the axis of a loop. Let the fingers of your right hand curl around in the direction of the conventional current, and your thumb will point in the direction of the magnetic field.

? Try using this right-hand rule to determine the direction of the magnetic field at the indicated observation location in Figure 17.35.

You should find that the magnetic field points down. This right-hand rule should of course give the same result as applying the more general right-hand rule to the cross product $\Delta\vec{l} \times \hat{r}$ and adding up the contributions of the various parts of the loop, as called for by the Biot-Savart law.

? On the diagram, consider $\Delta\vec{l} \times \hat{r}$ for two short pieces of the loop, on opposite sides of the loop. Show that the two pieces together contribute a magnetic field in the downward direction above the loop.

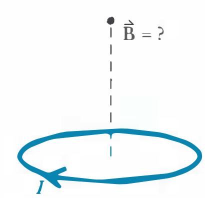


Figure 17.35 Curl the fingers of your right hand in the direction of the conventional current, and your thumb will point in the direction of the magnetic field.

17.9 MAGNETIC DIPOLE MOMENT

Recall the formula for the electric field along the axis of an electric dipole, at a distance r far from the dipole:

$$E_{\text{axis}} \approx \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}, \text{ where the "electric dipole moment" } p = qs$$

Similarly, in the formula for the magnetic field along the axis of a current-carrying coil, at a distance r far from the coil, we can write this:

$$B_{\text{axis}} \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}, \text{ where the "magnetic dipole moment" } \mu = IA$$

Here A is the area of the loop (πR^2 for circular loops). This formula for magnetic field is approximately valid even if the loop is not circular. The magnetic dipole moment $\vec{\mu}$ is considered to be a vector pointing in the direction of the magnetic field along the axis (Figure 17.36). This means that the direction of the magnetic dipole moment can be obtained by curling the fingers of your right hand in the direction of the conventional current, and your thumb points in the direction of the magnetic dipole moment.

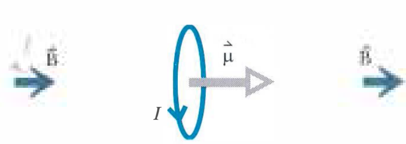


Figure 17.36 The magnetic dipole moment $\vec{\mu}$ is considered to be a vector pointing in the direction of the magnetic field along the axis.

We will see later that the concept of magnetic dipole moment also applies to magnets and provides a way of characterizing the strength of a magnet.

17.X.16 What is the magnetic dipole moment of a 3000-turn rectangular coil that measures 3 cm by 5 cm and carries a current of 2 amperes?

Twisting of a magnetic dipole

Related experiment:
17.EXP.22 on page 615

For reasons that we will discuss in a later chapter, the magnetic dipole moment vector $\vec{\mu}$ acts just like a compass needle. In an applied magnetic field, a current-carrying loop rotates so as to align the magnetic dipole moment $\vec{\mu}$ with the field.

17.10 THE MAGNETIC FIELD OF A BAR MAGNET

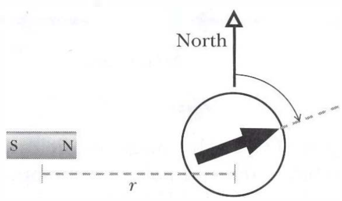


Figure 17.37 A bar magnet affects a compass.

From its name, you might guess that a magnet also makes a magnetic field. We say that a magnetic field is present if we see a compass needle twist. If you bring a bar magnet near a compass, you see that a bar magnet does make a compass needle deflect (Figure 17.37). Two magnets interact with each other, attracting or repelling depending on their relative orientations.

Related experiments:
17.EXP.23 and 17.EXP.24 on page 615

Magnetic interactions have some similarity to electric interactions: one can observe both attraction and repulsion, the superposition principle is valid, and the interactions pass through matter. However, there are also some major differences. A bar magnet interacts only with iron or steel objects, while a charged invisible tape interacts with *all* objects. A magnet is permanently magnetic, while a charged invisible tape or plastic pen loses its charge within a relatively short time. Two negatively charged objects always repel each other, while two magnets may repel or attract, depending on their orientations. The magnetic field around a current-carrying wire has a "curly" pattern that we don't see with electric fields.

The magnetic field of the Earth

The magnetic field of the Earth has a pattern that looks like that of a bar magnet (Figure 17.38). In the Northern Hemisphere, the Earth's magnetic field dips downward toward the Earth. A horizontally held compass is affected only by the horizontal component of the magnetic field, and this horizontal component gets smaller as you move closer to the magnetic poles.

For example, the horizontal component is smaller in Canada than it is in Mexico, despite the fact that the magnitude of the magnetic field is larger in Canada. The horizontal component is zero right at the magnetic poles, where the magnitude of the magnetic field is largest.

The Earth is a big magnet, but its type "S" pole is in the Arctic, and the "N" end of a compass needle points toward this type "S" pole.

The Earth's magnetic poles are not located exactly at the geographic poles. The "S" pole is located in northern Canada, about 1300 km (800 miles) from the geographic north pole, and the "N" pole is on the Antarctic continent, about 1300 km from the geographic south pole.

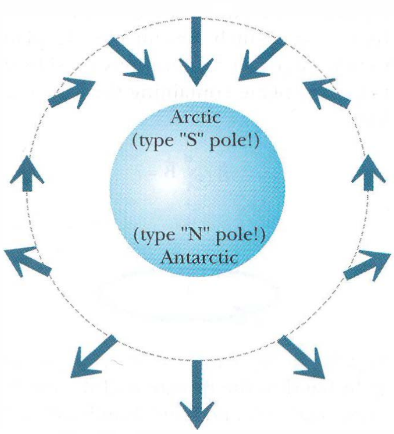


Figure 17.38 The Earth acts like a magnet. Its type "N" pole is in the Antarctic, and its type "S" pole is in the Arctic.

Dependence on distance

The horizontal component of the Earth's magnetic field, which is the component that affects a horizontally held compass, is different at different latitudes, depending on the distance from the "magnetic poles" of the Earth. Here are measurements of the magnitude of the horizontal component of the Earth's magnetic field at a few selected locations in the United States:

Location	Horizontal component of Earth's magnetic field
Maine	about 1.5×10^{-5} tesla
Much of the United States	about 2×10^{-5} tesla
Florida, Hawaii	about 3×10^{-5} tesla

Knowing the horizontal component of the Earth's magnetic field, we can use the deflection of the compass needle to measure the magnitude of the magnetic field of the magnet. We can then study both the distance dependence and the directional pattern of the magnetic field of the magnet.

A bar magnet is a magnetic dipole

The pattern of directions of magnetic field around a bar magnet is very similar to the pattern of directions of the magnetic field around a current loop, and to the pattern of the electric field around a permanent electric dipole seen in Chapter 13 (Figure 17.39). Also, along the axis of a magnet or a current loop or a permanent electric dipole, the magnetic or electric field varies like $1/r^3$ as can be seen in Experiment 17.EXP.25. Moreover, the magnitude of the field off to the side of an electric dipole or a magnet is half as large as it is at the same distance along the axis. Because of these strong similarities, a magnet is often called a "magnetic dipole." In Problem 17.P.41 you can use experimental data to determine the magnetic dipole moment of a bar magnet.

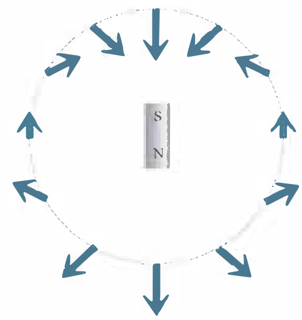


Figure 17.39 A bar magnet is a magnetic dipole. The field pattern is that of a dipole, and the magnitude of the field is proportional to $1/r^3$.

Example: Determining the magnetic dipole moment of a magnet

A compass originally points north. A bar magnet whose mass is 69.5 g is aligned east-west and placed near a compass as shown in Figure 17.40. When the distance between the center of the magnet and the center of the compass is 23.3 cm, the compass deflects 70 degrees. What is the magnetic dipole moment of the bar magnet?

$$\vec{B}_{net} = \vec{B}_{Earth} + \vec{B}_m$$

$$B_{magnet} = B_{Earth}\tan 70°$$

$$B_{magnet} = (2\times10^{-5}\ \text{T})\tan 70° = 5.5\times10^{-5}\ \text{T}$$

$$B_{magnet} = \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$$

$$\mu = \frac{B_m r^3}{2\frac{\mu_0}{4\pi}} = \frac{(5.5\times10^{-5}\ \text{T})(0.233\ \text{m})^3}{2\left(1\times10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)} = 3.5\ \text{A}\cdot\text{m}^2$$

Related experiments:
17.EXP.25-17.EXP.27 on pages 615-616

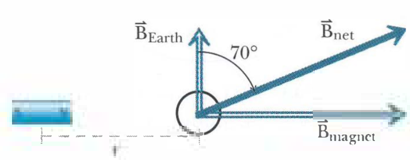


Figure 17.40 Determining the magnetic dipole moment of a bar magnet.

Magnetic monopoles

There is one dramatic difference between electric and magnetic dipoles. The individual positive and negative electric charges ("monopoles") making up an electric dipole can be separated from each other, and these point charges make outward-going or inward-going electric fields (which vary like $1/r^2$). One might expect a magnetic dipole to be made of positive and negative magnetic monopoles, but no one has ever found an individual magnetic monopole. Such a magnetic monopole would presumably make an outward-going or inward-going magnetic field (which would vary like $1/r^2$), but such a pattern of magnetic field has never been observed.

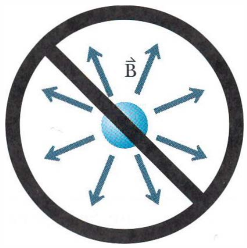


Figure 17.41 No magnetic monopoles. This pattern of outward-going magnetic field has never been observed.

Figure 17.41 is an example of the sort of magnetic field pattern that many scientists have diligently looked for but not found. If you cut a magnet in two, you don't get two magnetic monopoles—you just get two magnets!

Example: A circuit in the Antarctic

In a research station in the Antarctic, a circuit containing a partial loop of wire, of radius 5 cm, lies on a table. A top view of the circuit (looking down on the table) is shown in Figure 17.42. A current of 5 amperes runs in the circuit in the direction shown. You have a bar magnet with magnetic moment 1.2 A·m^2. How far above location A, at the center of the loop, would you have to hold the bar magnet, and in what orientation, so that the net magnetic field at location A would be zero? The magnitude of the Earth's magnetic field in the Antarctic is about 6×10^{-5} T.

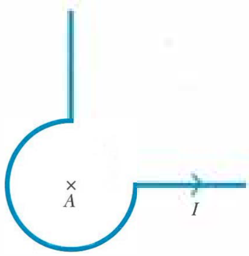


Figure 17.42 Top view of a circuit lying on a table (looking down at the table).

Orientation:

$\vec{B}_{\text{Earth}}$ points out of the page (out of the ground, in the Antarctic)

$\vec{B}_{\text{circuit}}$ points out of the page (out of the table)

Bar magnet must be held with its north pole downward (into page), to make a magnetic field into the ground.

Components of magnetic field out of page at location A:

$B_{\text{Earth}} = 6\times10^{-5}$ T

$B_{\text{straight wires}} = 0$

The straight wires contribute nothing to the magnetic field at A, because for each wire $d\vec{l}\times\hat{r} = 0$.

B due to 3/4 loop: $\vec{B} = \int d\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\hat{r}}{r^2}$

Origin: center of the loop.

$|\vec{r}| = R$ (constant)

At all locations $d\vec{l}\perp\hat{r}$, so $|d\vec{l}\times\hat{r}| = dl\sin 90° = dl = Rd\theta$, as shown in Figure 17.43. So

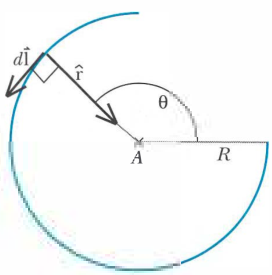


Figure 17.43 $d\vec{l}\perp\hat{r}$ for every step around the loop. The magnitude of r is constant and equal to the radius of the loop.

$$|\vec{B}_{\text{loop}}| = \int_{\pi/2}^{2\pi}\frac{\mu_0}{4\pi}\frac{I(Rd\theta)}{R^2} = \frac{\mu_0}{4\pi}\frac{IR}{R^2}\int_{\pi/2}^{2\pi}d\theta = \frac{\mu_0}{4\pi}\frac{I}{R}\theta\Big|_{\pi/2}^{2\pi} = \frac{\mu_0}{4\pi}\frac{3\pi I}{2R}$$

$$|\vec{B}_{\text{loop}}| = \left(1\times10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)\frac{(3\pi)(5\text{ A})}{2(0.05\text{ m})} = 4.7\times10^{-5}\text{ T}$$

$$B_{\text{Earth}} + B_{3/4\text{ loop}} = 6\times10^{-5}\text{ T} + 4.7\times10^{-5}\text{ T} = 1.06\times10^{-4}\text{ T}$$

So we need

$$B_{\text{magnet}} = 1.06\times10^{-4}\text{ T} \approx \frac{\mu_0}{4\pi}\frac{2\mu}{z^3}$$

We model the magnet as a dipole, located at the center of the magnet.

$$z = \left[\frac{\frac{\mu_0}{4\pi}(2\mu)}{B_{\text{magnet}}}\right]^{1/3} = \left[\frac{\left(1\times10^{-7}\frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2)(1.2\text{ A}\cdot\text{m}^2)}{(1.1\times10^{-4}\text{ T})}\right]^{1/3} = 0.13\text{ m}$$

So we hold the magnet with its center about 13 cm above the circuit, with its north pole toward the ground.

17.11 THE ATOMIC STRUCTURE OF MAGNETS

We have seen that the magnetic field of a circular current loop looks suspiciously similar to the magnetic field of a magnet. The pattern of directions of the magnetic field looks the same, and it is even the case that for both magnets and current loops the magnitude of the field at large distances along the axis falls off like $1/r^3$. Both magnets and current loops can be described in terms of magnetic dipole moment $\vec{\mu}$.

A single atom can be a magnetic dipole

Inside every atom there are moving charged particles. In an atom in a magnetic material, these subatomic "currents" may produce tiny magnetic moments that add up to a nonzero magnetic moment for each atom. (In contrast, in a non-magnetic material, these tiny magnetic moments would add up to zero.)

There are three possibilities for the source of the subatomic "currents" that might give a single atom a nonzero magnetic dipole moment:

- An electron orbiting around the nucleus (Figure 17.44)
- An electron spinning on its own axis (Figure 17.44)
- Rotational motion in the nucleus (protons and neutrons have spin about their own axes and can also orbit around inside the nucleus)

In all of these situations the object has angular momentum, and the magnetic dipole moment turns out to be proportional to the angular momentum L of the particle:

$$\mu = (\text{factor})L$$

We can use a simple model to estimate this proportionality factor.

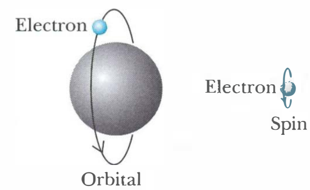


Figure 17.44 Electrons have spin as well as orbital motion that can contribute to the magnetic dipole moment.

Magnetic dipole moment of an orbiting electron

It is easiest to estimate this proportionality factor by considering the magnetic dipole moment of an orbiting electron, using the simple Bohr model of an atom to get an order of magnitude estimate. As indicated in Figure 17.45, let's suppose that each atom has one unpaired electron that orbits the nucleus along a circular path, at constant speed. Each orbiting electron can be considered to be a tiny current loop, which makes a dipole magnetic field.

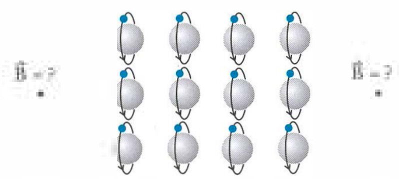


Figure 17.45 A simple model for a magnet might involve electrons in circular atomic orbits.

? In Figure 17.45, what is the direction of the resulting magnetic field to the left and right of the atoms, and of the magnetic dipole moment of one atom? (Remember that electrons are negative.)

The magnetic field due to each orbiting electron would point to the left, so the magnetic dipole moment of each atom, and of the whole object, would point to the left.

For a current loop, we know that

$$\mu = I(\pi R^2)$$

? What would the current I be for one orbiting electron (Figure 17.46)?

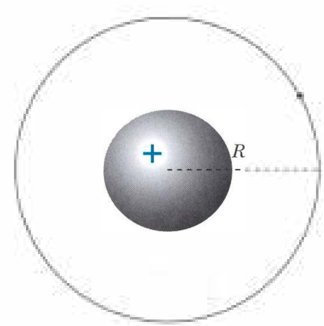


Figure 17.46 Simple model of an atom in which an outer electron orbits a positive inner core.

The current I is a measure of how much charge passes one location per second. The charge on the electron is $-e$, and the time it takes the electron to go around once is $T = 2\pi R/v$, where v is the speed of the electron, so:

$$I = \frac{e}{\left(\frac{2\pi R}{v}\right)} = \frac{ev}{2\pi R}$$

So the magnetic dipole moment of a single orbiting electron would be:

$$\mu = I(\pi R^2) = \left(\frac{ev}{2\pi R}\right)(\pi R^2) = \frac{1}{2}eRv$$

Relating magnetic dipole moment to angular momentum

The magnitude of the angular momentum of an electron in a circular orbit is:

$$|\vec{L}| = |\vec{r} \times \vec{p}|$$

Since in a circular orbit $\vec{r} \perp \vec{p}$, this reduces to:

$$L = Rp\sin 90° = Rmv \ (v \ll c)$$

We can express the magnetic dipole moment of the orbiting electron in terms of angular momentum by multiplying by 1:

$$\mu = \left(\frac{m}{m}\right)\left(\tfrac{1}{2}eRv\right) = \frac{1}{2}\frac{e}{m}(Rmv) = \frac{1}{2}\frac{e}{m}L$$

So our estimate of the proportionality factor relating magnetic dipole moment to angular momentum for a particle in an atom is this:

$$(\text{factor}) \approx \frac{1}{2}\frac{e}{m}$$

We will assume that this expression for the proportionality factor is valid for orbiting electrons, electrons spinning on their axes, and nucleons (protons and neutrons) spinning on their axes. How big is this factor?

$$\text{For an electron: } \frac{1}{2}\frac{e}{m} = \frac{1}{2}\frac{(1.6\times 10^{19}\ \text{C})}{(9\times 10^{31}\ \text{kg})} = 8.9\times 10^{10}\ \text{C/kg}$$

$$\text{For a nucleon: } \frac{1}{2}\frac{e}{m} = \frac{1}{2}\frac{(1.6\times 10^{-19}\ \text{C})}{(1.7\times 10^{27}\ \text{kg})} = 4.7\times 10^{7}\ \text{C/kg}$$

This factor is nearly 2000 times smaller for a nucleon. Angular momentum is quantized, with similar values for electrons and nucleons, so the magnetic dipole moment of a single atom comes almost entirely from orbiting electrons or electrons spinning on their own axes.

Angular momentum of an orbiting or spinning electron

You may recall, from Chapter 10, that both translational angular momentum (like the orbital angular momentum of an electron) and rotational angular momentum (like the angular momentum of an electron spinning on its axis) are quantized in very small systems such as atoms; only certain discrete values are possible. For an electron orbiting a nucleus, the angular momentum has to be an integer multiple of Planck's constant h divided by 2π, written as $\hbar$:

$$L = N\hbar, \text{ where } N = 0, 1, 2, 3, \text{ etc., and } \hbar = 1.05\times 10^{34}\ \text{J·s}$$

The electron's spin angular momentum is always one-half of $\hbar$. (If you are not familiar with angular momentum quantization, you may wish to read Section 17.12, which uses more familiar classical reasoning about circular motion to estimate the orbital angular momentum of an electron. The answer is the same.)

Assuming one quantum of angular momentum, we have the following:

Although the spin angular momentum has a factor of $1/2$, there is a compensating factor of 2 in its relation to the magnetic dipole moment.

$$L = \hbar, \text{ so } \mu \approx \frac{1}{2}\frac{e}{m}\hbar$$

Evaluating this expression, we find that:

$$\mu \approx \left(\tfrac{1}{2}\right)\frac{(1.6\times 10^{-19}\ \text{C})}{(9\times 10^{31}\ \text{kg})}(1.05\times 10^{34}\ \text{J·s})$$

$$\mu \approx 1\times 10^{23}\ \text{ampere·m}^2 \text{ per atom}$$

Comparing with experiment

If you have a bar magnet, a compass, and a meter stick, you can determine the magnetic dipole moment of the bar magnet experimentally. The procedure for doing this is detailed in Problem 17.P.41.

In the example on page 603, we found that a particular magnet whose mass was 69.5 g has a magnetic dipole moment of $3.5\ \mathrm{A \cdot m^2}$. Assume that the magnetic dipole moment of this magnet is due to the fact that each atom has one unpaired electron contributing a magnetic dipole moment of $\mu \approx 1\times10^{-23}$ ampere·m^2 (due either to orbital or spin angular momentum), and that the magnetic dipole moments of all the atoms are aligned in the same direction. What would the predicted magnetic dipole moment of this bar magnet be?

Example: Estimated magnetic dipole moment of a bar magnet

The bar magnet in the example on page 603 has a mass of 69.5 g. Assume that almost all of the atoms in this magnet are iron atoms. The mass of one mole of iron is 56 g.

$$n = \left(\frac{69.5\ \text{g}}{56\ \text{g/mol}}\right)(6.02\times10^{23}\ \text{atoms/mol}) = 7.5\times10^{23}\ \text{atoms}$$

$$\mu = n\mu_{\text{atom}} = (7.5\times10^{23}\ \text{atoms})\left(1\times10^{-23}\frac{\text{A}\cdot\text{m}^2}{\text{atom}}\right) = 7.5\ \text{A}\cdot\text{m}^2$$

? The estimated value of the magnetic dipole moment of this magnet is $7.5\ \mathrm{A\cdot m^2}$, but the observed value is only $3.5\ \mathrm{A\cdot m^2}$. Is this good agreement or poor agreement?

This is excellent agreement! The values differ by a factor of only about 2. The fact that the prediction and measurement are this close is evidence that the basic assumptions of our very simple model—that each atom in a magnet is itself a small dipole, and that the field produced by the magnet is the sum of the fields of all the atoms—are reasonable assumptions. If the predicted and measured values had differed by a factor of a thousand or a million, we would have had to re-examine our model.

Related problem: 17.P.42 on p. 618

Simplified models

We made a number of simplifying assumptions in our model of an atomic dipole. You probably know, from chemistry courses, that more sophisticated atomic models do not assume that electrons orbit the nucleus in well-defined circular paths. In these models, there is only a probability for finding the electron at a particular place. Moreover, many electrons in atoms have spherically symmetric probability distributions ("s orbitals"), and such a symmetric distribution has a zero average angular momentum. However, electrons in non-spherically-symmetric probability distributions ("p, d, or f orbitals") have nonzero angular momentum, and can contribute a non-zero magnetic dipole moment. Additionally, we assumed that in every atom there is just one electron that contributed a magnetic field, but in some materials two or more unpaired electrons may contribute.

Nonetheless, the fact that our simplified model gave a prediction with an appropriate order of magnitude shows the power of reasoning with simple models.

The "spinning ball" model of the electron isn't really adequate, because the most sensitive experiments are consistent with the notion that an electron is a true point charge and has zero radius! It is nevertheless an experimental fact that an electron does make a magnetic field as though it were a spinning ball of charge, and it can be useful to think of it literally as a spinning ball.

The modern theory of magnets

Modern theories of the nature of magnets take into account not just the contribution of individual atoms in a solid magnet, but the effects of their interactions with each other. These more complex theoretical treatments suggest that the magnetic dipole moments due to the rotational angular momentum of spinning electrons contribute most of the magnetic field of a magnet, although the orbital angular momentum of electrons does make some contribution.

As we saw above, because the factor e/m is so much smaller for a proton than for an electron, we can ignore nuclear contributions to the magnetic field of your bar magnet. Nuclear magnetic dipole moments do play an important role in the phenomenon of nuclear magnetic resonance (NMR), and in the technology of magnetic resonance imaging (MRI) used in medicine, which is based on NMR.

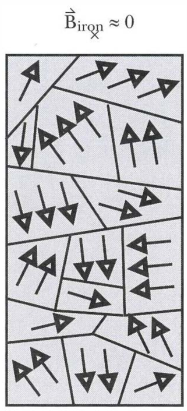


Figure 17.47 Disordered magnetic domains—the net magnetic field produced by the iron is nearly zero.

Alignment of the atomic magnetic dipole moments

In many materials, an atom has no net orbital or spin magnetism. In most materials whose individual atoms do make a magnetic field, the orbital and spin motions in different atoms don't line up with each other, so the net field of the many atoms in a piece of the material averages out to zero. But in a few materials, notably iron, nickel, cobalt, and some alloys of these elements, the orbital and spin motions in neighboring atoms line up with each other and therefore produce a sizable magnetic field. These unusual materials are called "ferromagnetic." The reason for the alignment can be adequately discussed only within the framework of quantum mechanics; basically the alignment is due to electric interactions between the atoms, not to the much weaker magnetic interactions.

Magnetic domains

In an ordinary piece of iron that isn't a magnet, the iron is a patchwork of small regions, called "magnetic domains," within which the alignment of all the atomic magnetic dipole moments is nearly perfect. But normally these domains are oriented in random directions, so the net effect is that this piece of iron doesn't produce a significant net magnetic field. Figure 17.47 shows a picture of the situation, in which the arrows indicate the magnetic dipole moments of the atoms.

If you use a coil to apply a large magnetic field to the iron, domains that happen to be nearly aligned with the applied field tend to grow at the expense of domains that have a different orientation (Figure 17.48). Also, with a sufficiently large applied magnetic field, the magnetic dipole moments of the atoms in a domain tend to rotate toward aligning with the applied field, just as a compass needle turns to align with an applied field. The result of both of these effects is to partially align the magnetic dipole moments of most of the atoms, which produces a magnetic field $\vec{B}_{iron}$ which may be much larger than the applied magnetic field. There is a kind of multiplier effect.

In the case of very pure iron, when you turn off the current in the coil, the piece of iron goes back nearly to its original disordered patchwork of domains and doesn't act like a magnet any more. But in some other ferromagnetic materials, including some alloys of iron, when you remove the applied field the domains remain nearly aligned. In that case you end up with a permanent magnet, which is commonly made of the alloy Alnico (Alnico V contains 51% iron, 8% aluminum, 14% nickel, 24% cobalt, and 3% copper). Hitting a magnet a hard blow may disorder the domains and make the metal no longer act like a magnet. Heating above a critical temperature also destroys the alignments.

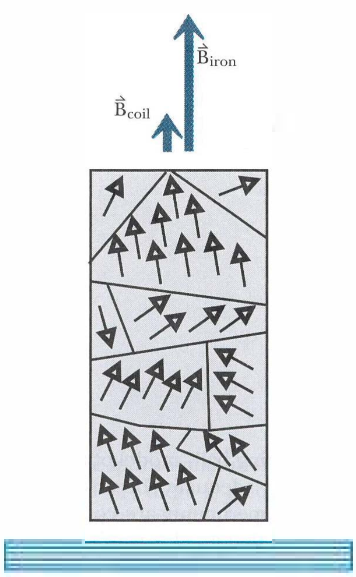


Figure 17.48 The magnetic field of a coil causes a partial ordering of the magnetic domains, which produces a significant nonzero magnetic field contributed by the iron.

Iron inside a coil

It is possible to observe this multiplier effect. A small magnetic field created by current-carrying loops of wire wrapped around a piece of iron can lead to a large observed magnetic field being contributed by the iron, due to the alignment of the magnetic dipole moments in the iron. Thanks to this multiplier effect, putting an iron core inside a current-carrying coil of wire makes a powerful "electromagnet" that can pick up iron objects.

Why are there multiple domains?

Within one domain of a ferromagnetic material such as iron, strong electric forces between neighboring atoms make the atomic magnetic dipole moments line up with each other. Why don't all the atoms in a piece of iron spontaneously align with each other? Why does the piece divide into small magnetic domains with varying magnetic orientations in the absence of an applied magnetic field?

There is a simple way to get some insight into why this happens. Consider two possible patterns of magnetic domains in a bar of iron in the absence of an applied magnetic field, a single domain or two opposed domains (Figure 17.49).

You can simulate this situation by taking two permanently magnetized bar magnets and holding them in one of these two positions, as shown in Figure 17.50.

? Which of these two magnet configurations do you find to be more stable, the parallel or the antiparallel alignment? By analogy, which is more likely for a bar of iron, that it have just one domain or that it split into two domains in the absence of an applied magnetic field?

Related experiments:
17.EXP.28 on page 616
17.EXP.29 on page 616

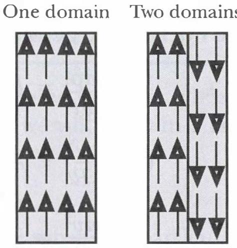


Figure 17.49 Possible arrangements—one magnetic domain or two.

You will find that if you start with the parallel alignment the magnets tend to flip into the antiparallel alignment. The domain structure of iron can be a compromise between a tendency for neighboring atoms to have their magnetic dipole moments line up with each other (a strong but short-range electric interaction), and a competing tendency for magnetic dipole moments to flip each other (a weaker but longer-range magnetic interaction).

However, the net effect depends critically on the details of the geometric arrangement. Although two long bar magnets that are side by side tend to line up with their magnetic dipole moments in opposite directions, two thin disk-shaped magnets tend to line up with their magnetic dipole moments in the same direction. What happens inside a magnetic material depends on the details of the geometrical arrangement of the atoms and on the details of the electric interactions between neighboring atoms.

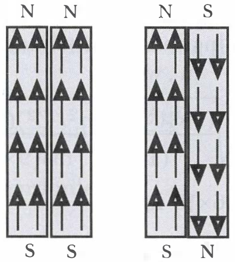


Figure 17.50 Arrange two bar magnets side by side, aligned parallel or antiparallel.

17.12 *ESTIMATE OF ORBITAL ANGULAR MOMENTUM OF AN ELECTRON IN AN ATOM

We can estimate the orbital angular momentum of an electron in an atom by using the simple Bohr model of an atom, in which an electron orbits a nucleus at constant speed in a circular orbit, and applying the Momentum Principle. In this model (Figure 17.51), an outer electron is held in a circular orbit of radius R by the electric attraction to the rest of the atom (which has a net charge of $+e$).

We want to find the magnitude of the angular momentum of the electron:

$$|\vec{L}| = |\vec{r} \times \vec{p}|$$

Since in a circular orbit $\vec{r} \perp \vec{p}$, this reduces to:

$$L = Rp = Rmv \quad (v \ll c)$$

We know the radius of an atom (about 1×10^{-10} m) and the mass of an electron (9×10^{-31} kg), so we need to find the speed of the electron.

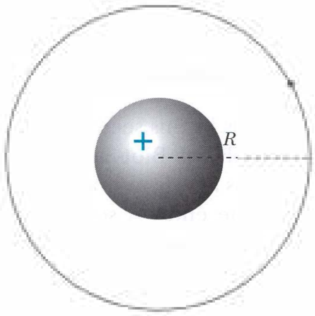


Figure 17.51 Simple model of an atom in which an outer electron orbits a positive inner core.

System: electron, charge $-e$
Surroundings: nucleus and outer electrons, with charge $+e$

Momentum principle:

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

Parallel component (constant speed, so parallel component of momentum is not changing):

$$\frac{dp}{dt}\hat{p} = \vec{0}$$

Perpendicular component (electric force on electron is toward the nucleus, perpendicular to momentum):

$$p\left|\frac{d\hat{p}}{dt}\right| = F\perp$$

$$(mv)\left(\frac{v}{R}\right) = e\left(\frac{1}{4\pi\varepsilon_0}\frac{e}{R^2}\right)$$

$$\frac{mv^2}{R} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}$$

$$v = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{Rm}}$$

Plugging in the electron charge and mass, and assuming an approximate atomic radius of 1×10^{10} m, we get:

$$v = \sqrt{\frac{(9\times10^9\ \mathrm{N\cdot m^2/C^2})(1.6\times10^{-19}\ \mathrm{C})^2}{(1\times10^{-10}\ \mathrm{m})(9\times10^{31}\ \mathrm{kg})}}$$

$$v \approx 1.6\times10^6\mathrm{m/s}$$

This gives a value for angular momentum of:

$$L = Rmv = (1\times10^{-10}\ \mathrm{m})(9\times10^{-31}\ \mathrm{kg})(1.6\times10^6\mathrm{m/s})$$

$$L \approx 1.4\times10^{-34}\ \mathrm{kg\cdot m^2/s^2}$$

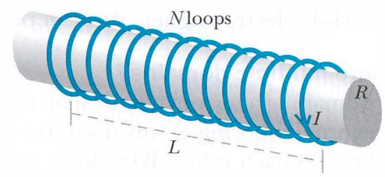
This is close to the quantum mechanical value of $\hbar = 1.05\times10^{34}$ J·s. If we had reproduced Bohr's full calculation we would have obtained a slightly different value for R, and L would be exactly $\hbar$.

17.13 *MAGNETIC FIELD OF A SOLENOID

The analysis in this section offers one more example of how to apply the Biot-Savart law. It shows that the magnetic field is nearly uniform along the axis of a solenoid, far from the ends. A solenoid is a coil which is much longer than its radius.

Step 1: Cut up the distribution into pieces and draw $\Delta\vec{B}$

We consider a solenoid of length L that is made up of N circular loops wound tightly right next to each other, each of radius R. We consider each loop as one piece. The conventional current in the loops is I.



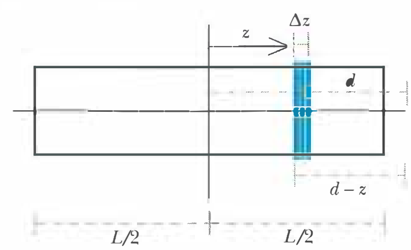
A solenoid of length L with N loops of radius R carrying current I.

? Given what you know about individual current loops, what will be the direction of the magnetic field $\Delta\vec{B}$ contributed by each of the loops at *any* location along the axis of the solenoid in the figure above?

All of the contributions inside the solenoid will point to the right in the diagram.

Step 2: Write an expression for the magnetic field due to one piece

origin: Center of solenoid
location of one piece: Given by z, so integration variable is z
distance from loop to observation location: $d - z$.



Defining an integration variable z for summing the contributions of the many loops.

? How many closely packed loops are contained in a short length Δz of the solenoid?

There are N/L loops per meter, so the number of loops in a length Δz is $(N/L)\Delta z$. We know the magnetic field made by each loop along its axis.

? Show that the loops contained in the section Δz of the solenoid contribute a magnetic field at the observation location in the z direction of this amount:

$$\Delta B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{[(d-z)^2+R^2]^{3/2}}\frac{N}{L}\Delta z$$

Step 3: Add up the contributions of all the pieces

The net magnetic field lies along the axis and is the summation of all the ΔB_z contributed by all the loops:

$$B_z = \sum \Delta B_z = \sum \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\frac{\Delta z}{[(d-z)^2+R^2]^{3/2}}$$

Many of these quantities are the same for every piece and can be taken outside the summation as common multiplicative factors:

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\sum\frac{\Delta z}{[(d-z)^2+R^2]^{3/2}}$$

This can be turned into an integral, with z ranging from $-L/2$ to $+L/2$:

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\int_{z=-L/2}^{z=+L/2}\frac{dz}{[(d-z)^2+R^2]^{3/2}}$$

Let $u = d-z$, in which case $dz = -du$, and therefore the limits on the integration run from $u = d-(-L/2)$ to $u = d-(+L/2)$:

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\int_{u=d+L/2}^{u=d-L/2}\frac{-du}{[u^2+R^2]^{3/2}}$$

$$= \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\int_{u=d-L/2}^{u=d+L/2}\frac{du}{[u^2+R^2]^{3/2}}$$

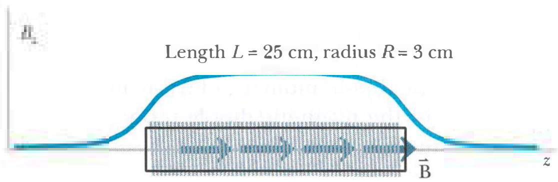
Fortunately, this integral can be found in standard tables of integrals:

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi R^2 IN}{L}\left[\frac{u}{R^2\sqrt{u^2+R^2}}\right]_{u=d-L/2}^{u=d+L/2}$$

$$B_z = \frac{\mu_0}{4\pi}\frac{2\pi IN}{L}\left[\frac{d+L/2}{\sqrt{(d+L/2)^2+R^2}} - \frac{d-L/2}{\sqrt{(d-L/2)^2+R^2}}\right]$$

In this expression, d is the distance from the center of the solenoid.

Here is a plot of this expression for B_z for a particular solenoid, as a function of the distance from the center of the solenoid. Notice that the magnetic field is nearly uniform inside the solenoid, as long as you are not too near the ends. The field is even more uniform for solenoids that are longer and/or thinner than this one.



? If the radius R is small compared with the length L ($R << L$), show that at the center of the solenoid the magnetic field has a remarkably simple value:

$$B_z \approx \frac{\mu_0 NI}{L}$$

With $d = 0$ and $R << L$, the quantity in square brackets above reduces to

$$\frac{L/2}{\sqrt{(L/2)^2}} - \frac{-L/2}{\sqrt{(-L/2)^2}}$$

which is equal to 2, from which follows the simple form $B_z \approx \mu_0 NI/L$.

The function plotted above shows that this is also the approximate magnetic field inside the solenoid along much of the axis, not too close to the ends. Outside the solenoid, the magnetic field falls off rapidly. At a large distance the field falls off like $1/r^3$, since the solenoid then looks like a simple collection of current loops.

MAGNETIC FIELD INSIDE A LONG SOLENOID

$$B_z \approx \frac{\mu_0 NI}{L} \text{ inside a long solenoid}$$

(radius of solenoid << L)

The same result can be obtained with much less effort by using Ampere's law, which we will study in a later chapter.

Step 4: Check the result

Does our final result make sense? In particular, do we have the right units? Comparing with the magnetic field for a single current element,

$$\frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2}$$

we easily verify that our answer does have the right units, since

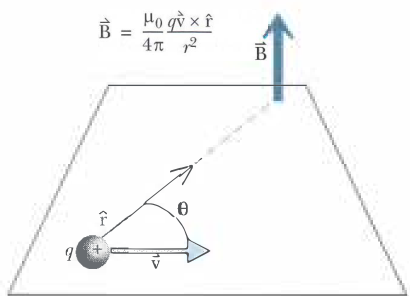
$$\frac{NI}{L} \text{ has the same units as } \frac{I\Delta l}{r^2}$$

(Remember that $\hat{r}$ is dimensionless.)

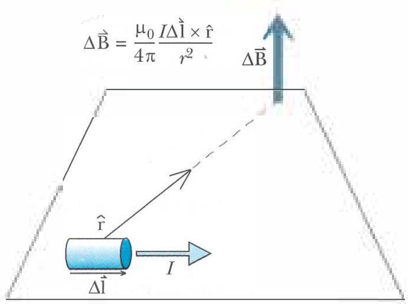
17.14 SUMMARY

The Biot-Savart law

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \text{ (single moving particle)}$$



$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2} \text{ (current distribution)}$$



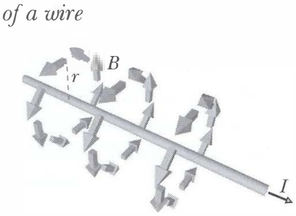
$$\frac{\mu_0}{4\pi} = 10^{-7}\frac{\text{tesla}\cdot\text{m}^2}{\text{coulomb}\cdot\text{m/s}} \textit{ exactly}$$

$\hat{r}$ is a unit vector that points *from* the source charge *toward* the observation location.

Electron current: $i = nA\bar{v}$
Conventional current: $I = |q|nA\bar{v}$

A magnetic field can be detected by a compass; horizontal component of $B_{Earth} \approx 2\times10^{5}$ tesla in much of the continental United States.

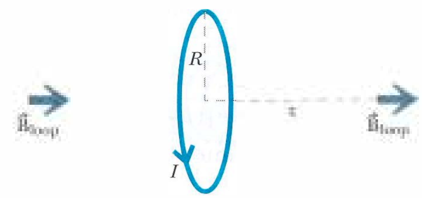
Magnetic field of a wire



$$B_{wire} = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ for } r << L$$

for a straight wire of length L and conventional current I, at a perpendicular distance r from the center of the wire

Magnetic field of a loop

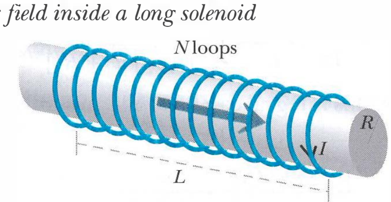


$$B_{loop} = \frac{\mu_0}{4\pi}\frac{2\pi R^2 I}{(z^2+R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2\mu}{z^3} \text{ for } z >> R$$

for a circular loop of radius R and conventional current I, at a distance z from the center, along the axis

Magnetic dipole moment of a loop: $\mu = (\pi R^2)I$

Magnetic field inside a long solenoid



$$B_z \approx \frac{\mu_0 NI}{L}$$

inside a long solenoid (radius of solenoid << L)

Note that a solenoid is very long compared to its radius—it is not the same thing as a coil composed of a few loops close together.

Atomic model of magnets

An atom can be a magnetic dipole, whose magnetic dipole moment is primarily due to the spin (rotational angular momentum) of unpaired electrons. The orbital angular momentum of electrons can also contribute.

The magnetic dipole moment of a bar magnet is the sum of the magnetic dipole moments of all its constituent atoms.

Ferromagnetic material

Organized into domains of aligned atomic magnetic dipole moments, and an applied field can orient these domains. Removal of the applied field may leave the material partially aligned, forming a permanent magnet.

17.15 EXPERIMENTS

Equipment

In the study of magnetic field you will need the following equipment:

- two flashlight batteries in a battery holder

- light bulbs of two kinds
- screw-in bulb sockets
- several short copper wires with clips on the ends ("clip leads" used as connecting wires)
- a long wire (about 2 m in length)
- a liquid-filled magnetic compass
- unmagnetized nails for 17.EXP.28 on page 616

Simple circuits

To observe the magnetic effects of electric currents, it is useful to construct simple circuits containing wires, light bulbs, and batteries. These are the simplest examples of systems in which we can observe the fundamental electric and magnetic properties of continuous electric currents.

The equipment needed for the experiments in this chapter is the following: two D cells (and it useful to have a battery holder for them), flashlight bulbs (#48 and #14 if possible), sockets for these bulbs, insulated "hookup" wire, some clip leads (wires with alligator clips on the end), a liquid-filled compass (air-filled compasses typically don't work well because the needle tends to get stuck on its pivot), and a bar magnet (a magnet with a north end and a south end).

Suitable equipment may be available in a laboratory or you can purchase experiment kit EM-8675 from http://www.pasco.com.

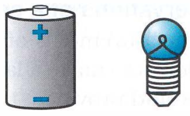
Light bulbs and sockets

The filament of a light bulb (the very thin metal wire that glows) is made of tungsten, a metal that does not melt until reaching a very high temperature. A glowing tungsten wire would rapidly oxidize and burn up in air, so there is a vacuum or an inert gas such as argon inside the bulb.

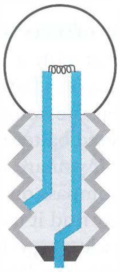
The thin tungsten filament in the bulb strongly resists the passage of electrons. When the electron sea is forced to move through the tungsten, the mobile electrons collide with the positive cores (nuclei plus inner electrons), and this "friction" makes the metal get hot and glow.

17.EXP.17 Experiments with simple circuits

(a) Using one battery, some connecting wires (insulated wires with clips on the ends, not bare Nichrome wire), and a round bulb (#14) *but no socket*, make the bulb light up. If the bulb glows with a steady light, this is a "steady state." Make a diagram showing how you connected the circuit.

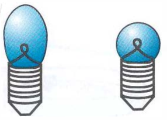


(b) Examine a light bulb carefully, and imagine slicing the bulb in half lengthwise. Here is a cutaway sketch:



On a copy of the sketch, label the important parts and connections, indicating which parts of the bulb you think are metal conductors, and which parts are insulators. Using a different color if available, show the conducting path that electric current follows through the bulb. What happens if you switch the connections to the battery? Does the light bulb still light?

(c) Closely examine the long #48 bulb and the round #14 bulb.

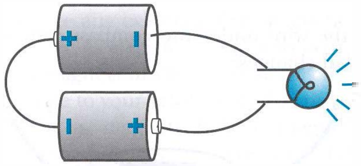


A long #48 bulb (left) and a round #14 bulb (right).

The tungsten filaments in both bulbs are about the same length, but perhaps you can see even with the naked eye that the filament in the long bulb is extremely thin—thinner than the filament in the round bulb. Through which bulb would you guess it would be easier to push electric current, through a thin filament (as in the long bulb) or through a thick filament (as in the round bulb)? Why? (At this point this is mostly just a guess, but we'll study this in detail later.)

(d) Examine a bulb *socket* (the receptacle into which a bulb is screwed), and imagine slicing the socket in half lengthwise. Make a cutaway sketch, and label the important parts and connections. Indicate which parts are metal conductors and which parts are insulators. On your sketch, trace the path (in a different color if available) along which electrons will move through the socket when there is a bulb in it. Screw a bulb into the socket and connect the socket to the battery with two connecting wires. Make sure the bulb lights.

(e) Connect a round bulb in a socket to two batteries "in series" (that is, one after the other) using connecting wires.



Two batteries and a round bulb in series. The socket is not shown.

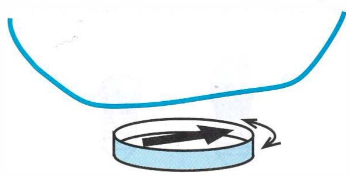
To connect two batteries in series, put them in the battery holder in opposite directions, and connect them as shown (note that "+" is connected to "–"). Compare the brightness of the bulb with one battery and with two batteries in series. Because of this difference we'll usually use two batteries in series in our experiments.

17.EXP.18 The magnetic effects of currents

Make a two-battery circuit with a round bulb in a socket. Place your magnetic compass on a flat surface under one of the wires as shown below. Keep the compass away from steel objects, such as the steel-jacketed batteries, and the alligator clips on the ends of your wires. If you are working on a steel table, you may need to put the compass on a thick book. (For flexibility in placement, you may find it useful to make a long wire by connecting two of your wires together.)

Connect the circuit so that the bulb glows, and do the following:

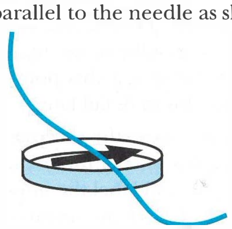
- Lift the wire up above the compass.
- Orient the wire to be horizontal and lined up with the compass needle. (Using a long wire may make it easier to do this.)
- Bring the aligned wire down onto the compass:



Wire aligned with the compass needle.

(a) What is the effect on the compass needle as you bring the wire down on top of the compass?

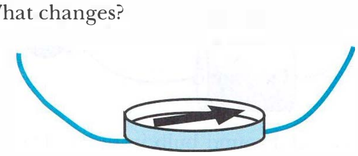
(b) What happens when the wire is initially aligned perpendicular instead of parallel to the needle as shown below?



Wire perpendicular to the compass needle.

(c) Reverse the connections to the batteries, or reverse the direction of the wire over the compass, in order to force electrons through the circuit in the opposite direction. Again make the compass needle deflect. How is the deflection of the compass needle affected by changing the direction of the current?

(d) Run the wire under the compass instead of over the compass. What changes?



Wire under compass.

(e) To make sure that it is the current in the wire, and not the metal wire itself, that affects the compass, disconnect the batteries. When you bring the wire down on the compass, is there a deflection?

(f) Record the observed compass deflection in the following cases:

1: Two batteries and the (bright) round bulb.

2: Two batteries and the (dim) long bulb.

3: Two batteries, and just a long wire (no bulb). This is called a "short circuit", and it puts a large drain on the batteries, so you should not leave this connected for many minutes.

The effect you have just observed was discovered by accident by the Danish scientist Oersted in 1820 while doing a lecture demonstration in a physics class. The phenomenon is often called "the Oersted effect." From your experiments, you should have drawn the following conclusions:

- the magnitude of the magnetic field produced by a current of moving electrons depends on the amount of current
- a wire with no current running in it produces no magnetic field
- the magnetic field due to the current appears to be perpendicular to the direction of the current
- the direction of the magnetic field due to the current under the wire is opposite to the direction of the magnetic field due to the current above the wire

17.EXP.19 Electron current and battery

Use the right-hand rule to predict whether the compass needle should deflect to the left or right, assuming that electron current flows out of the negative end of the battery and into the positive end. Check to see that the needle does deflect in the predicted direction.

17.EXP.20 The magnetic field of a long straight wire

Use clip leads to connect a wire about 2 meters long to a battery, but don't make the final connection yet. Lay the wire down on the floor or a table, making a straight length as long as possible, heading north and south, as far from the steel battery as possible.

- Hold the compass above the wire at a height where the compass deflection is 20° when you turn the current on and off. Mark on a vertically held piece of paper this height r_{20° of the compass needle above the wire, and record this height.
- Mark the paper at twice this height and use the second mark to place the compass at a height $2(r_{20^\circ})$ above the paper, and record the compass deflection.

(a) Are your data consistent with the theoretical prediction that the magnitude of the magnetic field near a long straight wire is proportional to $1/r$, where r is the perpendicular distance to the wire?

(b) Using your value of r_{20}, determine the amount of current I in the wire.

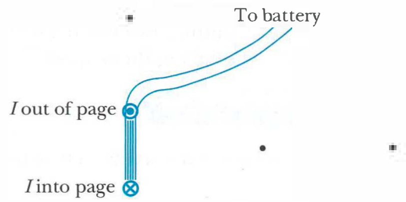
Experimental observations of a coil of wire

Next we will make measurements of a coil of wire, consisting of multiple loops to make a larger magnetic field than a single loop would provide.

17.EXP.21 The magnetic field of a coil of wire

Take an insulated wire about 2 meters in length and wrap a coil of about $N = 20$ turns loosely around two fingers. Twist the ends together to help hold the coil together, and remove the coil from your fingers. Use clip leads to connect the coil to a battery.

By appropriate placement and orientation of your compass relative to your N-turn current loop, measure the experimental magnetic field direction and relative magnitude at the locations indicated. Remember that you must always position the compass in such a way that the coil's magnetic field is perpendicular to the Earth's magnetic field.



Measure the magnetic field at the indicated locations.

On a diagram like that above, draw the magnetic field vectors at all marked locations. Record the number of turns N, the approximate radius of the coil R, the distances to the measurement locations, and the compass deflections at those locations.

(a) Do your measurements agree with the predicted pattern of magnetic field around a coil of wire?

(b) Determine the amount of current I in the wire.

17.EXP.22 A coil is a compass

For reasons that we will discuss in a later chapter, the magnetic dipole moment vector $\vec{\mu}$ acts just like a compass needle. In an applied magnetic field, a current-carrying loop rotates so as to align the magnetic dipole moment $\vec{\mu}$ with the field.

It is hard to observe this twisting with your own hanging coil, because the rather stiff wires prevent the coil from freely rotating, unless you suspend the entire apparatus from a thread, batteries and all. Or do this:

If you have access to kitchen equipment, you may be able to float your batteries and coil in a bowl or large glass or aluminum pan, or on a block of wood. (Avoid steel containers!) Then you may be able to observe the axis of the coil line up with the Earth's magnetic field, just as though the magnetic dipole moment of the coil were a compass needle.

17.EXP.23 Directions of the magnetic field of a bar magnet

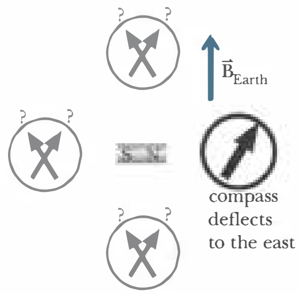
From its name, you might guess that a magnet makes a magnetic field. We say that a magnetic field is present if we see a compass needle twist, so let's see whether your bar magnet does make your compass needle deflect.

Place your compass to the right of your bar magnet, with the magnet oriented in such a way as to make the compass deflect to the east as shown in the accompanying figure. Make a similar diagram in your notebook.

> We define the direction of a magnetic field $\vec{B}$ at a particular location as the direction that the "north" end of a compass points to when placed at that location.

(a) Assuming that the bar magnet makes a magnetic field along the axis of the magnet, draw a vector in your notebook to show the direction of the compass needle, and another arrow to show the direction of the magnetic field $\vec{B}_m$ of your bar magnet at the present location of the compass. For future reference, write "N" for "north" on the end of the magnet that the magnetic field points out of (attach tape to write on if necessary).

(b) Move the compass to the left of your magnet, above your magnet, and below your magnet, as shown. For each location, record the direction of the compass needle, and also record on your diagram the direction of the magnetic field due to the bar magnet at that location.



Place your compass at each of the locations shown above, relative to your bar magnet, and note the deflection of the compass needle. What is the direction of the magnet's magnetic field at these locations?

(c) Does this pattern of field look familiar? Where have we seen a similar pattern of field directions in space before?

(d) Suspend your magnet from a thread or a hair, using a piece of tape, or float the magnet on a dish or plate in water. Note which end points toward the north. Evidently a compass consists simply of a magnet in the shape of a needle, mounted on a pivot.

17.EXP.24 Magnets and matter

Obtain another magnet by working with a partner or finding a kitchen magnet. Make sure that you have labeled the north and south ends of both magnets.

(a) Describe briefly the interactions that the two magnets have with each other.

(b) You probably know that magnets don't interact strongly with anything but iron or steel objects (steel is mostly iron). Magnets also interact with nickel or cobalt objects, but these aren't so readily available. Check to see for yourself that your magnet doesn't interact with aluminum (aluminum foil) or copper (a penny, or an electrical wire), or any nonmetal. Also notice that your magnet is strongly attracted to steel parts of your electricity kit.

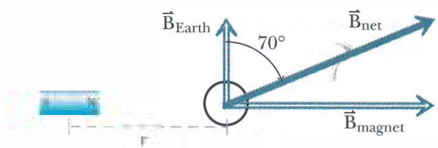
(c) Check what happens with charged invisible tape, and see that while there is the usual attraction of a charged tape for any uncharged object, there doesn't seem to be any magnetic interaction, because either end of the magnet acts the same.

(d) Can you give any evidence that magnetic interactions pass through matter, as electric interactions do?

(e) Place two magnets in various positions near your compass to demonstrate to your satisfaction that the superposition principle holds for the magnetic fields of magnets; that is, the net field of the two magnets is the vector sum of the two fields. Give one example of your observation of the superposition principle for magnetic fields.

17.EXP.25 Dependence on distance for a bar magnet

Again place the compass to the right of your bar magnet, oriented in such a way as to make the compass deflect 70° to the east. *Do not work on a table that contains steel components*, which can interfere with the measurements. The best place to make the measurements is outdoors, away from iron and steel objects.



Place the magnet at a center-to-center distance r such that the compass needle deflection is 70°.

(a) At the location where you get a 70° deflection, measure the distance r from the center of the magnet to the center of the compass. A centimeter ruler is provided at the back of the book.

(b) Use vector analysis and the magnitude of the Earth's field to calculate the magnitude B_r of the magnetic field of the magnet at this distance r from the center of the magnet.

(c) Now move the magnet farther away by a factor of two; that is, place the magnet so that the distance from the center of the magnet to the center of the compass is $2r$. Record the new distance and the new compass deflection angle. Calculate the magnitude B_{2r} of the magnetic field of the magnet at this new distance from the magnet.

(d) By what ratio did the magnetic field of the magnet decrease when the distance from the magnet was doubled?

(e) The magnetic field of the magnet gets smaller with distance, and it is plausible to guess that the magnetic field of the magnet might vary as $1/r^n$, where n is initially unknown. According to your measurements, what is n? (You may wish to make measurements at some other distances to give further support to your analysis.)

(f) Extrapolating to a location near one end of your bar magnet, approximately how strong is the magnetic field in tesla near one end of your magnet? (Remember, measure r to the center of the magnet.)

(g) Place the compass north or south of the magnet, at the original distance r from the center of the magnet, and determine the magnitude of the magnetic field at this location. How does this magnitude compare to your result in (b)?

Place the compass north or south of the bar magnet and note the deflection of the compass needle.

17.EXP.26 Coils attract and repel like magnets

There is yet another way in which the behavior of a magnet and of a current-carrying coil of wire is very similar. Hang your coil over the side of the table and bring your bar magnet near the hanging coil, along the axis of the coil. Reverse the magnet. Repeat from the other side of the coil. Is this behavior similar to the interaction you have observed between two bar magnets?

17.EXP.27 "Cutting a magnet in two"

Without actually cutting a magnet into two pieces, you can simulate such an operation by putting two bar magnets together north end to south end, then taking them apart.

Place two magnets together end-to-end: do they behave like a single magnet?

Do you in fact find that the put-together longer magnet acts just like a single magnet, and that after pulling them apart the two pieces still act like magnets?

If you have the equipment available, you can magnetize a soft iron nail by stroking it with a magnet; then use a hacksaw to cut it in half and investigate the properties of each half.

17.EXP.28 The magnetic multiplier effect

Note: You need unmagnetized nails for this experiment. If the nails have ever been near a magnet, they may have become strongly magnetized. Check this by bringing a nail near the compass, first one end, then the other. If the nail is unmagnetized, both ends of the nail will affect the compass equally. If however the nail is strongly magnetized, the two ends of the nail will affect the compass quite differently.

If a nail is strongly magnetized, bring your bar magnet near it (but not touching the nail) to magnetize the nail in the opposite direction. Check again with the compass and repeat as necessary, trying to reduce the magnetization to zero. Do not proceed until you have at least three unmagnetized nails.

Now you can study the magnetic multiplier effect. Take an insulated wire about 2 meters in length and wrap a coil of about 20 turns loosely around two fingers. Twist the ends together to help hold the coil together and remove it from your fingers. Connect the coil to a battery and position your compass to the east or west of the coil so that the compass needle deflects away from north by about 5 degrees. Turn off the current, so the compass deflection goes to zero.

(a) Insert an unmagnetized iron nail into the coil and record the compass deflection (if any). Now start the current again and observe what happens to the compass deflection. Add additional nails and observe and record what happens to the compass deflection.

(b) Explain briefly why each nail produces a sizable increase in the compass deflection when you turn on the current.

(c) To check whether the nails are permanently magnetized after being inserted into the coil, turn off the current. If the compass still deflects significantly without any current in the coil, the nails have become strongly magnetized. Are they?

(d) Bring one of these nails near the compass, first one end and then the other. Is the nail slightly magnetized?

17.EXP.29 An electromagnet

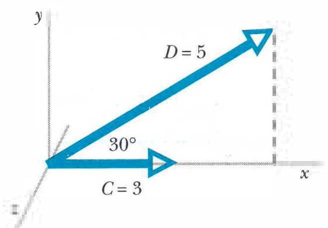
Next, construct an "electromagnet." Take an insulated wire about 2 meters in length and wrap it tightly around an iron nail, all along its length. You can make multiple layers of windings, back and forth along the nail, to increase the effects, but leave the ends of the nail sticking out so that you can touch the nail to other objects. Twist the ends of the wire together to help hold the coil together. Connect the coil to a battery and bring it near another nail.

Can you pick up the other nail? (If the battery is not fresh, you may not get enough current to lift the other nail, but you should be able to lift a paper clip!) Why does the other nail fall when you cut the current? Is there any evidence that the electromagnet nail retains some magnetization?

17.16 REVIEW QUESTIONS

Cross product

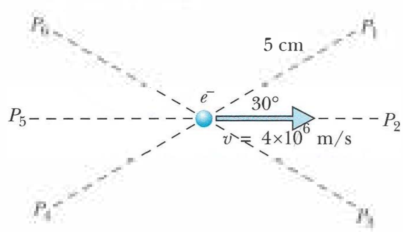
17.RQ.30 A vector $\vec{C}$ of magnitude 3 lies along the x axis, and a vector $\vec{D}$ of magnitude 5 lies in the xy plane, 30 degrees from the x axis. What is the magnitude and direction of the cross product $\vec{C} \times \vec{D}$? What is the magnitude and direction of the cross product $\vec{D} \times \vec{C}$? Draw both vectors on a diagram.



The angle between these two vectors is 30 degrees.

The Biot-Savart law

17.RQ.31 An electron is moving horizontally to the right with speed 4×10^6 m/s. What is the magnetic field due to this moving electron at the indicated locations? (Each location is 5 cm from the electron.) Give both magnitude and direction of the magnetic field at each location.



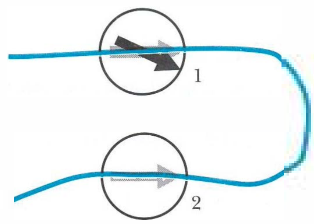
An electron moves to the right with speed 4×10^6 m/s.

17.RQ.32 What is the direction of the magnetic field at the indicated locations inside and outside this current-carrying rectangular coil of wire shown? Explain briefly. (Direction of conventional current is shown.)

Measuring magnetic field with a compass

17.RQ.33 A current-carrying wire oriented north-south is laid on top of a compass. If the compass deflection is 17°, what is the magnitude of the magnetic field due to the current?

17.RQ.34 Consider the portion of a circuit shown. When no current is running, both compasses point north (direction shown by the gray arrows). When current runs in the circuit, the needle of compass 1 deflects as shown. What direction will the needle of compass 2 point? Draw a sketch indicating its deflection.



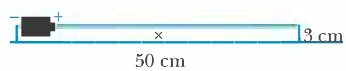
The wire rests on top of two compasses.

Field of a straight wire

17.RQ.35 In a circuit consisting of a long bulb and two flashlight batteries in series the conventional current is about 0.1 ampere. What is the magnetic field 5 mm from the wire? (This is about how far away the compass needle is when you place the wire on top of the compass.) Is this a big or a small field?

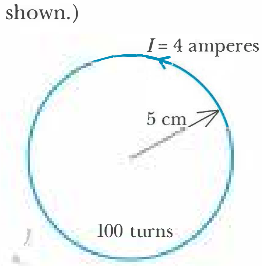
Two straight wires

17.RQ.36 A battery is connected to a Nichrome wire and a conventional current of 0.3 ampere runs through the wire. The wire is laid out in the form of a rectangle 50 cm by 3 centimeters. What is the magnetic field at the center of the rectangle? Give the direction as well as the magnitude.



Coil of wire

17.RQ.37 A thin circular coil of wire of radius 5 cm consists of 100 turns of wire, as shown. If the conventional current in the wire is 4 amperes, what are the magnitude and direction of the magnetic field at the center of the coil? (Direction of conventional current is shown.)



A current of 4 A runs through a thin circular coil of wire with 100 turns.

Dependence on distance

17.RQ.38 (a) How can you produce a magnetic field that is nearly uniform in a region?

(b) How can you produce a magnetic field that falls off like $1/r$?

(c) How can you produce a magnetic field that falls off like $1/r^2$? (Note that you *cannot* use just a short piece of current-carrying wire, because the other parts of the wire also contribute.)

(d) How can you produce a magnetic field that falls off like $1/r^3$?

Magnetic materials

17.RQ.39 An iron bar magnet makes a pattern of magnetic field that looks just like the pattern of magnetic field outside a long current-carrying coil of wire. Are there currents in the iron? Explain briefly.

17.RQ.40 Suppose you have two Alnico bar magnets, one with a mass of 100 grams and one with a mass of a kilogram. At a distance of a meter from the center of either one, how would the magnetic field differ? Why?

17.17 PROBLEMS

17.P.41 Magnetic dipole moment of your bar magnet
The magnetic field along the axis of a bar magnet can be written like this:

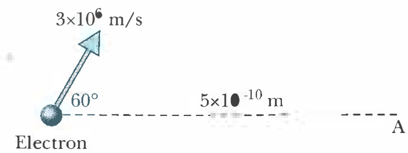
$$B_{\text{axis}} \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$$

Use your own data from Experiment 17.EXP.25 on page 615 to determine the magnetic moment m of your bar magnet. If you have mislaid those data, quickly repeat the measurements now. You will use the value of your magnetic moment in later work. The magnetic moment describes how strong a magnet is: the bigger the magnetic moment, the bigger the magnetic field at some distance r.

17.P.42 Predicting the magnetic dipole moment of your magnet
Determine the mass of your magnet and thereby determine the number N of atoms in your magnet. Although your magnet is probably made of some alloy such as Alnico V (51% iron, 8% aluminum, 14% nickel, 24% cobalt, and 3% copper), for simplicity assume it is made just of iron, which has a density of 8 grams/cm^3 and an atomic mass of 56 (that is, 6×10^{23} atoms weigh a total of 56 grams).

Assuming that each of the atoms has a magnetic moment with a value estimated in Section 17.11, see how well the atomic model for a magnet fits the value of the magnetic moment of your bar magnet determined in the previous problem.

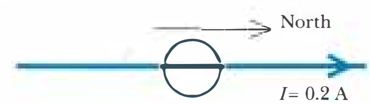
17.P.43 Fields of an electron
The electron is traveling with a speed of 3×10^6 m/s.



(a) At location A, what are the directions of the electric and magnetic fields contributed by the electron?

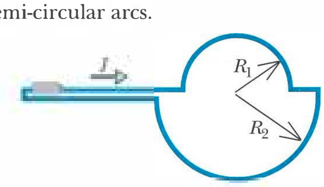
(b) Calculate the magnitudes of the electric and magnetic fields at location A.

17.P.44 Wire on compass
You place a long straight wire on top of your compass, and the wire is a height of 5 millimeters above the compass needle. If the conventional current in the wire is $I = 0.2$ ampere and runs left to right as shown, calculate the approximate angle the needle deflects away from north and draw the position of the compass needle.

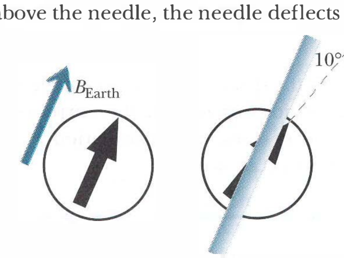


Wire on top of a compass.

17.P.45 Magnetic field of arcs
Two long wires lie very close together and carry a conventional current I as shown and each wire has a semi-circular kink, one of radius R_1 and the other of radius R_2. Calculate the magnitude and direction of the magnetic field at the common center of the two semi-circular arcs.



17.P.46 Deflecting a compass needle
When you bring a current-carrying wire down onto the top of a compass, aligned with the original direction of the needle and 5 mm above the needle, the needle deflects by 10 degrees.



Deflecting a compass needle.

(a) Show on a diagram the direction of conventional current in the wire, and the direction of the additional magnetic field made by the wire underneath the wire, where the compass needle is located. Explain briefly.

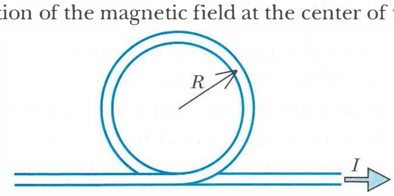
(b) Calculate the amount of current flowing in the wire. The measurement was made at a location where the horizontal component of the Earth's magnetic field is $B_{\text{Earth}} \approx 2\times10^{-5}$ tesla.

17.P.47 Using magnetic field to measure current

You can use measurements of the magnetic field of a coil to determine how much current your battery is supplying to the coil. Using your value of B (Experiment 17.EXP.21 on page 614), determine the conventional current I through your coil. If this current is less than 3 ampere, you should replace the battery.

17.P.48 Wire with a loop in it

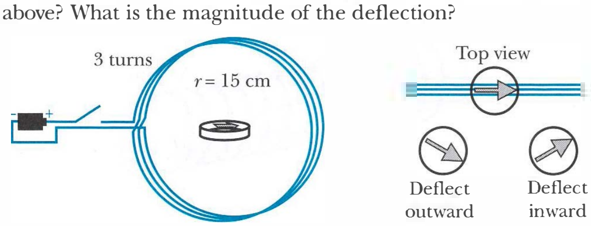
A very long wire carrying a conventional current I is straight except for a circular loop of radius R. Calculate the magnitude and direction of the magnetic field at the center of the loop.



17.P.49 Deflecting a compass needle with a coil

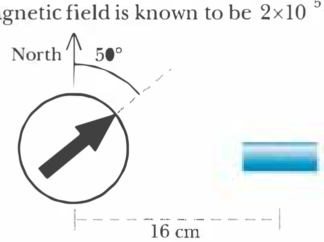
A thin circular coil of radius r = 15 cm contains N = 3 turns of Nichrome wire. A small compass is placed at the center of the coil, as shown below. With the battery disconnected, the compass needle points to the right, in the plane of the coil. Assume that the horizontal component of the Earth's magnetic field is about $B_{\text{Earth}} \approx 2\times10^{-5}$ tesla.

When the battery is connected, a current of 0.25 ampere runs through the coil. Predict the deflection of the compass needle. If you have to make any approximations, state what they are. Is the deflection outward or inward as seen from above? What is the magnitude of the deflection?



17.P.50 Magnetic moment of a bar magnet

A bar magnet is aligned east-west, with its center 16 cm from the center of a compass. The compass is observed to deflect 50° away from north as shown, and the horizontal component of the Earth's magnetic field is known to be 2×10^{5} tesla .

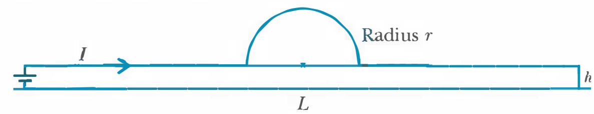


(a) Label the N and S poles of the bar magnet and explain your choice.

(b) Determine the magnetic dipole moment of this bar magnet, including correct units.

17.P.51 Magnetic field in a circuit

A circuit consists of a battery and a Nichrome wire, through which runs a current I.

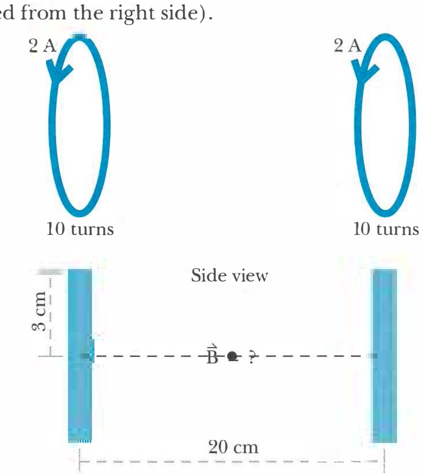


(a) At the location marked × (the center of the semicircle), what is the direction of the magnetic field?

(b) At the location marked × (the center of the semicircle), what is the magnitude of the magnetic field? If you have to make any approximations, state what they are.

17.P.52 Magnetic field of coils

Two thin coils of radius 3 cm are 20 cm apart and concentric with a common axis. Both coils contain 10 turns of wire with a conventional current of 2 amperes that runs counterclockwise as viewed from the right side).



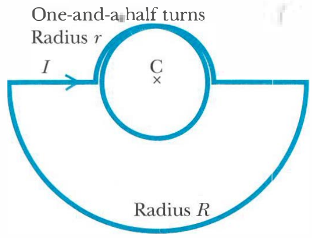
(a) What is the magnitude and direction of the magnetic field on the axis, halfway between the two loops, without making the approximation $z \gg r$? (For comparison, remember that the horizontal component of magnetic field in the United States is about 2×10^{-5} tesla).

(b) In this situation, the observation location is not very far from either coil. How bad is it to make the $1/z^3$ approximation? That is, what percentage error results if you calculate the magnetic field using the approximate formula for a current loop instead of the exact formula?

(c) What is the magnitude and direction of the magnetic field midway between the two coils if the current in the right loop is reversed to run clockwise?

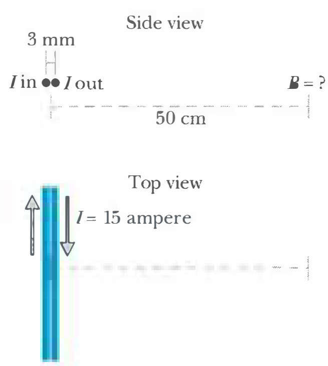
17.P.53 A bent wire

A conventional current I runs in the direction shown. Determine the magnitude and direction of the magnetic field at point C, the center of the circular arcs.



17.P.54 Magnetic fields in the home

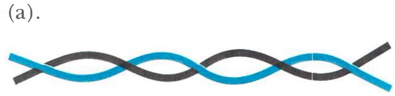
At one time, concern was raised about the possible health effects of the small alternating (60-hertz) magnetic fields created by electric currents, in houses and near power lines. In a house, most wires carry a maximum of 15 amperes (there are 15-ampere fuses that melt and break the circuit if this current is exceeded). The two wires in a home power cord are about 3 millimeters apart as shown, and at any instant they carry currents in opposite directions (both of which change direction 60 times per second).



An appliance power cord consists of two wires side by side.

(a) Calculate the maximum magnitude of the alternating magnetic field, 50 cm away from the center of a long straight power cord that carries a current of 15 amperes. Both wires are at the same height as the observation location.

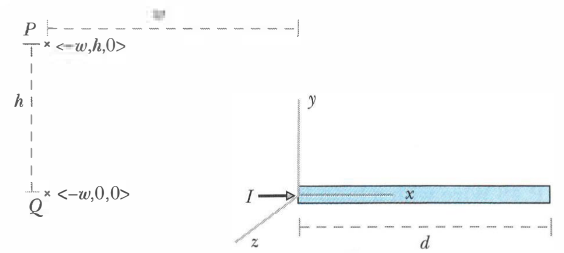
(b) Explain briefly why twisting the pair of wires into a braid as shown would minimize the magnetic field at the location discussed in (a).



(The magnitude of the field that you calculate is very small compared to the Earth's magnetic field, but there were questions as to whether a very small alternating magnetic field might have health effects. After many detailed studies, the consensus of most scientists now seems to be that these small alternating magnetic fields are not a hazard after all.)

17.P.55 Magnetic field of a current

A thin wire is part of a complete electrical circuit which carries a current I. For this problem consider only the piece of wire of length d as shown in the figure below. Answer the following questions based on this figure.



A thin wire is part of an electrical circuit.

(a) What are the magnitude and direction of the magnetic field due to the wire at Q (location $< -w, 0, 0 >$?

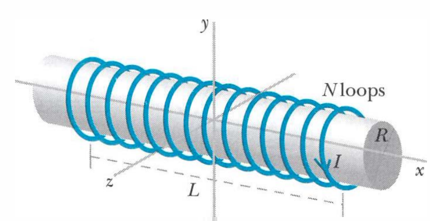
(b) Set up the integrals necessary to determine the x, y, and z components of the magnetic field at P (location $< -w, h, 0 >$). The integrals must be in a form which can be evaluated (no cross products in the integrand), but you do not need to evaluate them.

(c) What is the direction of the magnetic field at location P?

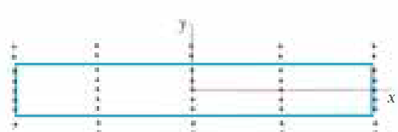
17.P.56 Calculating the magnetic field of a solenoid

This problem requires calculations in three dimensions and will familiarize you with the details of the magnetic field made by a solenoid (a long coil).

A solenoid of length $L = 0.5$ meter and radius $R = 3$ cm is wound with $N = 50$ turns of wire carrying a current $I = 1$ ampere. Its center line lies on the x axis, with the origin at the center of the solenoid.



(a) Calculate and display magnetic field vectors at the locations in the xy plane, inside and outside of the solenoid, as shown below. It is acceptable to approximate the helix as simple loops, if you find that is easier to do. (If you have a helix instead of loops, $\Delta\vec{l}$ also has a z component.) Do the pattern and direction of magnetic field make sense?



Locations at which to calculate the magnetic field of a solenoid.

(b) Display the numerical value of the magnitude of the magnetic field at one location, the center of the solenoid. Also display the theoretical numerical value of the magnetic field, in the approximation that this is a very long solenoid ($B \approx \mu_0 NI/L$). What is the minimum number of steps around one loop (or around one turn of the helix) that are necessary to obtain good agreement between the theoretical value and your numerical integration? What is your criterion for "good agreement"?

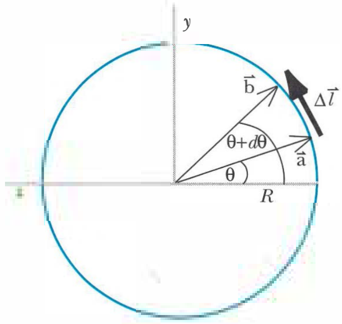
(c) Vary the number of loops. What is the minimum number of loops (or turns of the helix) that are necessary to get an approximately uniform field inside the solenoid?

(d) How do the magnitude and direction of the magnetic field outside the solenoid compare to the magnitude and direction of the magnetic field inside the solenoid?

Suggestions

Consider $\vec{r}$ to be a vector from the midpoint of $\Delta\vec{l}$ to the observation location.

One way to find $\Delta\vec{l}$ is to find two vectors $\vec{a}$ and $\vec{b}$ to the endpoints of $\Delta\vec{l}$, then subtract to get $\Delta\vec{l}$, as shown.



Finding $\Delta\vec{l}$.

One way to get started is to calculate the magnetic field for a single loop located at the origin, then extend the program to include 50 loops.

In debugging your program, you may find it useful to display the $\Delta\vec{l}$ vector for each step.

17.18 ANSWERS TO EXERCISES

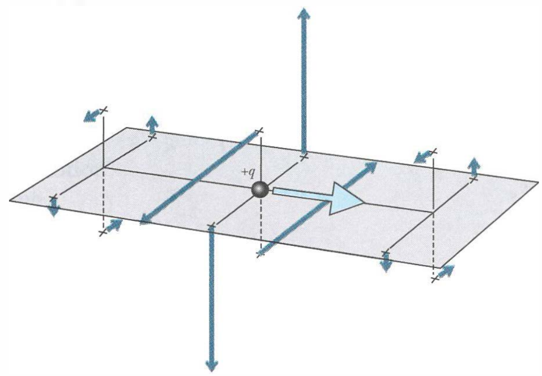
17.X.1 (page 588) 6×10^{18} electrons/s

17.X.2 (page 588) 5.4×10^{21} electrons

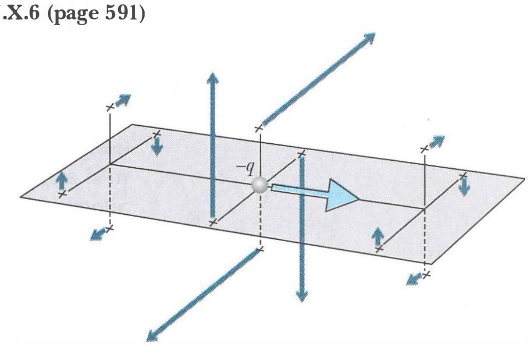
17.X.3 (page 589) 5.4×10^{-6} T

17.X.4 (page 591) $\frac{\langle (v_y z - v_z y), (v_z x - v_x z), (v_x y - v_y x)\rangle}{\sqrt{x^2+y^2+z^2}}$

17.X.5 (page 591)



17.X.6 (page 591)



17.X.7 (page 591) Cross product involves $\sin\theta$, which is zero.

17.X.8 (page 591) $1/r^2$

17.X.9 (page 591) zero magnetic field

17.X.10 (page 591) 14 tesla, which is an extremely large magnetic field

17.X.11 (page 592) Electron current flows to the left, conventional current to the right.

17.X.13 (page 594) 5×10^{-5} m/s

17.X.14 (page 595) 100 minutes

17.X.15 (page 595) 0.54 ampere

17.X.16 (page 602) $9\ \text{A}\cdot\text{m}^2$

CHAPTER 22

FARADAY'S LAW

Key concepts

- A curly electric field accompanies a time-varying magnetic field.
- Faraday's law is a quantitative relationship between the rate of change of magnetic field and the curly electric field.
- Superconductors have zero resistance at low temperatures, and they exclude magnetic field (Meissner effect).
- Coils in circuits oppose change of current, determined by the amount of their "inductance."
- There is energy in a magnetic field.
- Optional: There is a differential form of Faraday's law involving "curl."

Charges make electric fields, and electric fields affect charges. Moving charges make magnetic fields, and magnetic fields affect moving charges. In Chapter 20 we glimpsed a deeper relationship among these phenomena in the oddly different electric and magnetic forces observed by a stationary Jack and a moving Jill, and in field transforms between reference frames.

In this chapter we will see another fundamental aspect of the relationship between the electric and magnetic fields. It turns out that a time-varying magnetic field can produce an electric field! So there are two different ways to produce an electric field: by charges according to Coulomb's law, or by time-varying magnetic fields. In the latter case, we say that the time-varying magnetic field "induces" an electric field, and the phenomenon is referred to as "magnetic induction."

No matter how an electric field is produced, it has the same effect on a charge q (that is, $\vec{F} = q\vec{E}$), but the "non-Coulomb" electric field induced by a time-varying magnetic field has a different pattern in space than the "Coulomb" electric field due to charges. In particular, a round-trip path integral of the non-Coulomb electric field is not zero, unlike the situation with the Coulomb electric field. Because of this, time-varying magnetic fields can induce an emf around a circuit loop.

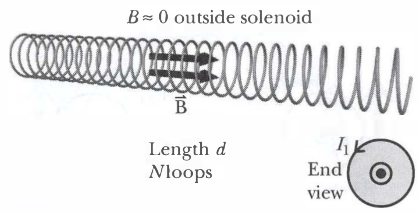


Figure 22.1 The magnetic field inside a long solenoid is $B_1 = \mu_0 N I_1/d$.

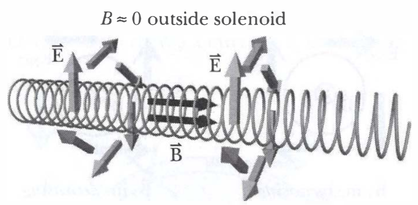


Figure 22.2 A time-varying magnetic field in the solenoid generates a curly electric field!

22.1 CHANGING MAGNETIC FIELDS AND CURLY ELECTRIC FIELDS

Consider a long solenoid, a long hollow coil of current-carrying wire (Figure 22.1). In Chapter 17 using the Biot-Savart law and Chapter 21 using Ampere's law we found that with a current I_1 in the solenoid, the magnetic field B_1 inside a tightly-wound solenoid (not too near the ends) is $B_1 = \mu_0 N I_1/d$, where N is the number of turns of wire and d is the length of the solenoid. The magnetic field outside the solenoid (not too near the ends) is very small and for a very long solenoid can be taken to be zero.

If the current is constant, then B_1 is constant in time. In that case a charge that is in motion somewhere outside the solenoid will not experience an electric or magnetic force, because the electric and magnetic fields outside the solenoid are essentially zero. But something remarkable happens if we vary the current, so that the magnetic field B_1 inside the solenoid varies with time (Figure 22.2). There is still almost no magnetic field outside the solenoid, but we observe a curly electric field both inside and outside of the solenoid! This peculiar electric field curls around the axis of the solenoid (end view; Figure 22.3). The electric field is proportional to dB_1/dt, the rate of change of the magnetic field.

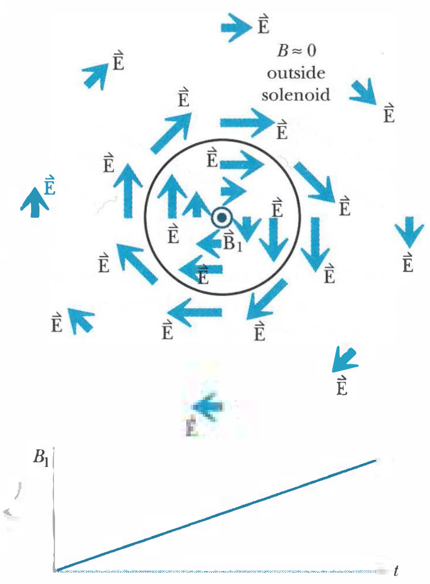


Figure 22.3 There is a curly electric field in the presence of a time-varying magnetic field. In this case B_1 is increasing with time.

Inside the solenoid the electric field is proportional to r, the distance from the axis (smaller near the axis). Outside the solenoid the curly electric field is proportional to $1/r$; the electric field gets smaller as you go farther away from the solenoid. The curly electric field has the usual effect on charges: a charge q experiences a force $\vec{F} = q\vec{E}$. For example, a proton placed above the solenoid in Figure 22.3 will be initially pushed to the right.

While the curly electric field affects charges in the usual way, it isn't produced by charges according to Coulomb's law. Rather, this electric field is associated with the time-varying magnetic field, and we call such an electric field a "non-Coulomb" field $\vec{E}_{NC}$.

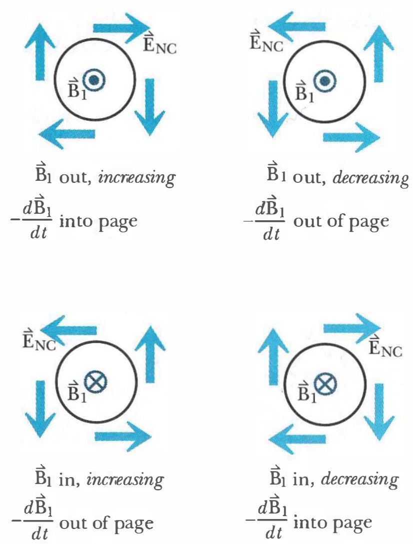


Figure 22.4 Four cases: magnetic field out or in, increasing or decreasing.

? Explain how you know that this pattern of non-Coulomb electric field cannot be produced by an arrangement of stationary charges.

Traveling in a complete loop around the solenoid, $\oint \vec{E}_{NC} \bullet d\vec{l} \neq 0$. Since the round-trip integral of the electric field due to stationary charges is always zero, this electric field cannot be produced by stationary charges.

TWO WAYS TO PRODUCE ELECTRIC FIELD

- A Coulomb electric field is produced by charges according to Coulomb's law:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

- A non-Coulomb electric field $\vec{E}_{NC}$ is associated with time-varying magnetic fields $d\vec{B}/dt$. Outside of a long solenoid inside of which the magnetic field is B_1, the induced electric field is proportional to dB_1/dt and decreases with distance like $1/r$.

No matter how an electric field $\vec{E}_1$ is produced, it has the same effect on a charge q_2: $\vec{F}_{21} = q_2\vec{E}_1$.

Figure 22.4 shows what is observed experimentally in four different cases, where the magnetic field points out of or into the page, and increases or decreases with time. From these results you can see that it is not the direction of $\vec{B}_1$ that determines the direction of $\vec{E}_{NC}$, but rather the direction of the rate of change of $\vec{B}_1$. Here is a right-hand rule that summarizes the experimental observations, and which you should memorize:

DIRECTION OF THE CURLY ELECTRIC FIELD

With the thumb of your right hand pointing in the direction of $-d\vec{B}_1/dt$, your fingers curl around in the direction of $\vec{E}_{NC}$.

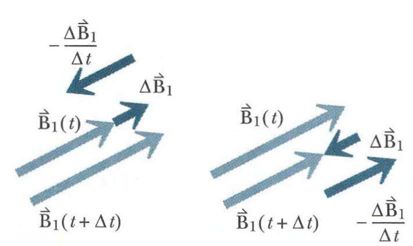


Figure 22.5 Find the change in the magnetic field as a basis for determining the direction of $-d\vec{B}_1/dt$.

Hints on using this right-hand rule

In order to use this right-hand rule, you need to be able to determine the direction of the vector quantity $-d\vec{B}_1/dt$. It helps to think about this quantity in the form $-\Delta\vec{B}_1/\Delta t$, where Δt is a small, finite time interval. From this form you can see that the direction of $-d\vec{B}_1/dt$ is the same as the direction of $-\Delta\vec{B}_1$, the negative of the change in direction of the magnetic field during a short time.

So a good way to find the direction of $-d\vec{B}_1/dt$ is to draw the magnetic field $\vec{B}_1(t)$ at a time t, and the magnetic field $\vec{B}_1(t+\Delta t)$ at a slightly later time $t+\Delta t$, and observe the change $\Delta\vec{B}_1$ (Figure 22.5). Then $-\Delta\vec{B}_1$ is the direction of $-d\vec{B}_1/dt$.

22.X.1 A magnetic field near the floor points up and is increasing. Looking down at the floor, does the non-Coulomb electric field curl clockwise or counterclockwise?

22.X.2 A magnetic field near the ceiling points down and is decreasing. Looking up at the ceiling, does the non-Coulomb electric field curl clockwise or counterclockwise?

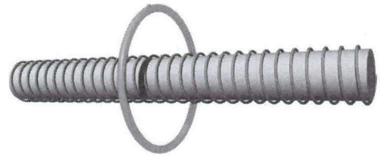


Figure 22.6 A metal ring is placed around the solenoid.

Driving current with a non-Coulomb electric field

Suppose we place a circular metal ring of radius r_2 around a solenoid (Figure 22.6), with the magnetic field in the solenoid increasing with time (Figure 22.7). The non-Coulomb electric field inside the metal will drive conventional current clockwise around the ring ($-d\vec{B}_1/dt$ points into the page; point your right thumb into the page and see how your fingers curl clockwise). The technological importance of this effect is that it can make current run in a wire just as though a battery were present.

The current I_2 in the ring is proportional to the electric field E_{NC} inside the metal, as in an ordinary circuit. However, in ordinary circuits there are charges on the surface of the wires that, together with charges on the battery, produce the electric field inside the metal that drives the current.

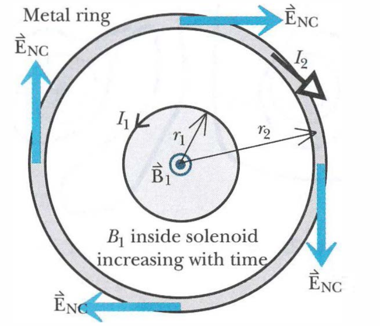


Figure 22.7 End view: The non-Coulomb electric field drives a current I_2 in the ring.

? Think about a possible pattern of surface charge on this ring. Why is it impossible to draw a plausible gradient of surface charge on the ring in this situation?

The symmetry of the ring makes it impossible to have a surface charge gradient along the ring. We reason by contradiction: If you pick one point and draw positive surface charge there, then gradually decrease the amount of positive charge and increase the amount of negative surface charge, you find that when you get back to the starting point there is a huge change in surface charge (from – to +), which would produce a huge E, in the wrong direction (Figure 22.8). But there is nothing special about the point you picked, so it can't have a different electric field from all other points on the ring. So there cannot be a varying surface charge around the ring. (There will be a small pile-up of electrons on the outside of the ring, which then provides the radially-inward force that turns the electron current.)

The emf is the (non-Coulomb) energy input per unit charge. The (non-Coulomb) force per unit charge is the (non-Coulomb) field E_{NC}.

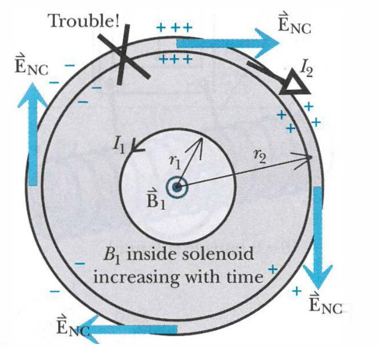


Figure 22.8 This pattern of surface charge is impossible, because it would imply a huge E at the marked location, and in the wrong direction!

? Therefore, what is the emf in terms of E_{NC} for this ring, if the ring has a radius r_2?

We have emf $= \oint \vec{E}_{NC} \bullet d\vec{l} = E_{NC}(2\pi r_2)$, since E_{NC} is constant and parallel to the path. The current in the ring of radius r_2 is $I_2 = \text{emf}/R$, where R is the resistance of the ring. It is as though we had inserted a battery into the ring.

? If the metal ring had a radius r_2 twice as large, what would the emf be, since emf $= \oint \vec{E}_{NC} \bullet d\vec{l}$? Remember that the experimental observations show that the non-Coulomb electric field outside the solenoid is proportional to $1/r$.

Double the radius implies half the electric field, so the product $E_{NC}(2\pi r_2)$ stays the same. Apparently the emf in a ring encircling the solenoid is the same for any radius. (In fact, we get the same emf around any circuit surrounding the solenoid, not just a circular ring.)

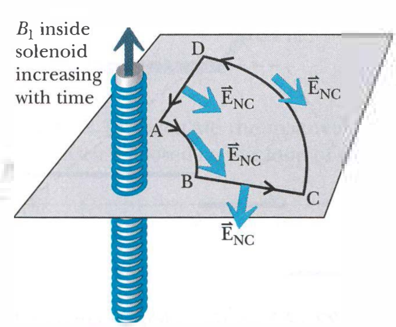


Figure 22.9 A path that does not encircle the solenoid.

? Consider a round-trip path that does not encircle the solenoid (Figure 22.9). The electric field is shown all along the path A-B-C-D-A. Use the fact that $E_{NC} = \text{emf}/(2\pi r)$ to explain why we have the result emf $= \oint \vec{E}_{NC} \bullet d\vec{l} = 0$ around this path that does not encircle the solenoid.

As you make a round trip around the path A-B-C-D-A, the contribution to $\oint \vec{E}_{NC} \bullet d\vec{l}$ is positive along A-B, zero along B-C (the parallel component of electric field is zero), negative along C-D, and zero along D-A. Moreover, the magnitude of the contribution along C-D is equal to that along A-B, because the longer path is compensated by the smaller electric field (which is proportional to $1/r$). Therefore we have a zero emf along this path. The non-Coulomb electric field would polarize a wire that followed this path but would not drive current around the loop.

We see that in order to drive current in a wire the wire must encircle a region where the magnetic field is changing. If the wire doesn't encircle the region of changing magnetic field, there is no emf.

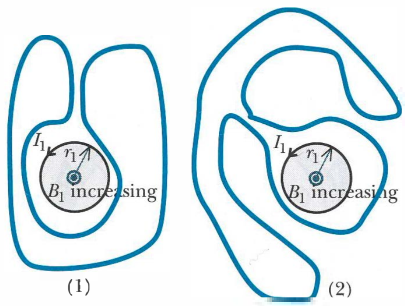


Figure 22.10 Will current run in these wires?

22.X.3 In Figure 22.10, will current run in wire (1)? In wire (2)?

22.X.4 On a circular path of radius 10 cm in air around a solenoid with increasing magnetic field, the emf is 30 volts. What is the magnitude of the non-Coulomb electric field on this path?

22.X.5 A wire with resistance 4 ohms is placed along the path in the previous exercise. What is the current in the wire?

22.2 FARADAY'S LAW

So far we have a right-hand rule for determining the direction of the curly electric field, but we don't have a way to determine the magnitude of the non-Coulomb electric field. Faraday's law is a quantitative relationship between the rate of change of the magnetic field and the magnitude of the non-Coulomb electric field. To establish this quantitative relationship, in principle we could vary the magnetic field and measure E_{NC} by observing the effect of the curly electric field on an individual charged particle, but it can be difficult to track the path of an individual charged particle. Alternatively, we can construct a circuit following a path where we expect a non-Coulomb electric field and measure the current in the circuit as a function of dB/dt. We will describe circuit experiments that lead to Faraday's law.

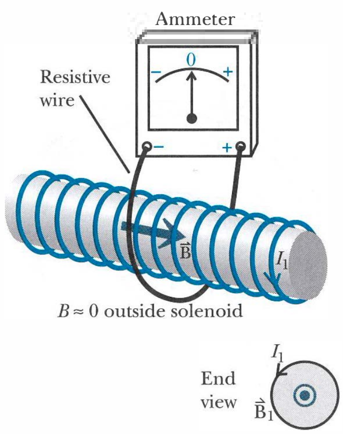


Figure 22.11 An ammeter measures current in a loop surrounding the solenoid. Initially I_1 is constant, so B_1 is constant, and no current runs through the ammeter.

Observing current caused by non-Coulomb electric fields

We can use an ammeter to measure the induced current in a circuit that encircles a solenoid. In Figure 22.11 we omit showing the power supply and connections to the solenoid. Initially the solenoid current I_1 is constant, so B_1 is constant, and no current runs through the resistive wire and the ammeter.

If we vary the magnetic field B_1 by varying I_1, we can infer something about the resulting non-Coulomb electric field E_{NC} from its integral around the circuit, which is the emf. Unfortunately you can't observe the phenomenon with the circuit equipment you have been using, because you need a more sensitive ammeter than is provided by your compass. Perhaps your instructor will demonstrate the effects or arrange for you to experiment with appropriate equipment.

Suppose that we vary the current I_1 in the long solenoid, thus causing the magnetic field B_1 to vary, and we observe the current I_2 in the outer wire. In the solenoid we first increase the current rapidly, then hold the current constant, and then slowly decrease the current, at half the rate we used at first (Figure 22.12). If we know the resistance R of the circuit containing the ammeter we can determine the emf from the ammeter reading, since emf $- RI_2 = 0$; the wire acts as though a battery were inserted.

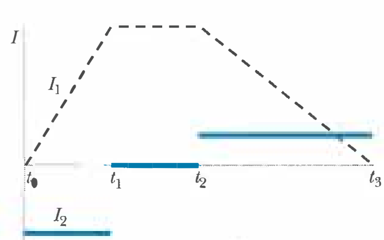


Figure 22.12 Vary the solenoid current I_1 and observe the current I_2 that runs in the outer wire, through the ammeter.

1) While the solenoid current I_1 is increasing from t_0 to t_1 (Figure 22.12), B_1 is increasing, and the current I_2 runs clockwise in the circuit, out of the "+" terminal of the ammeter. The ammeter is observed to read a negative current. Remember that conventional current flowing into the positive terminal of an ammeter gives a positive reading.

2) While the solenoid current I_1 is held constant from t_1 to t_2 (Figure 22.12), the magnetic field in the solenoid isn't changing, and there is no emf in the circuit. The ammeter reading is zero:

dB/dt makes E_{NC} (and associated emf).
No dB/dt, no E_{NC}, even if there is a large (constant) B.

3) While the solenoid current I_1 is decreasing from t_2 to t_3 (at half the initial rate; see Figure 22.12), B_1 is decreasing, and the current I_2 runs counterclockwise in the circuit, into the "+" terminal of the ammeter. The ammeter is observed to read a positive current. The current I_2 is observed to be half what it was during the first interval.

E_{NC} (and associated emf) outside the solenoid are proportional to dB/dt inside the solenoid.

4) There is one more crucial experiment. If we use a solenoid with twice the cross-sectional area but the same magnetic field, we find that the current I_2 is twice as big (which means that the emf is twice as big).

E_{NC} (and associated emf) outside the solenoid are proportional to the cross-sectional area of the solenoid.

Magnetic flux

Putting together these experimental observations, and being quantitative about the emf, we find experimentally that the magnitude of the induced emf is numerically equal to this:

$$|\text{emf}| = \left|\frac{d}{dt}(B_1\pi r_1^2)\right|$$

The quantity $(B_1\pi r_1^2)$ is called the "magnetic flux" Φ_{mag} on the area encircled by the circuit (Φ is the capital Greek letter Phi). Magnetic flux is calculated in the same way as the electric flux introduced in Chapter 21 on Gauss's law.

The magnetic flux on a small area ΔA is $\vec{B} \bullet \hat{n}\Delta A = B_\perp \Delta A$, where $\hat{n}$ is a dimensionless unit vector perpendicular to the area ΔA ($\hat{n}$ is called the "normal" to the surface); $B_\perp$ is the perpendicular component of magnetic field. We add up all such contributions over an extended surface to get the magnetic flux over that surface (Figure 22.13):

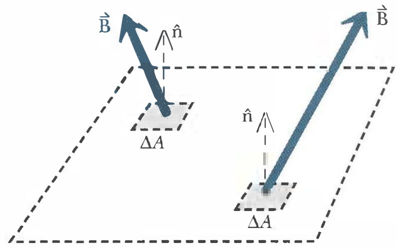


Figure 22.13 The magnetic flux on an area is the sum of the magnetic flux on each small subarea.

DEFINITION OF MAGNETIC FLUX

$$\Phi_{mag} \equiv \int \vec{B} \bullet \hat{n}\, dA = \int B_\perp\, dA$$

Note that magnetic flux can be positive, negative, or zero, depending on the orientation of the magnetic field $\vec{B}$ relative to the normal $\hat{n}$.

22.X.6 A uniform magnetic field of 3 tesla points 30° away from the perpendicular to the plane of a rectangular loop of wire 0.1 m by 0.2 m (Figure 22.14). What is the magnetic flux on this loop?

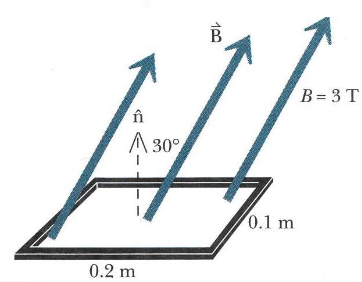


Figure 22.14 Calculate the magnetic flux on the area enclosed by this loop of wire.

Faraday's law—quantitative

The experimental fact that a time-varying magnetic field produces an emf whose magnitude is equal to the rate of change of magnetic flux is called "Faraday's law" (discovered by the British scientist Michael Faraday in 1831). Faraday's law summarizes a great variety of experimental data, not just the data for the experiments we have discussed. Faraday's law is a major physical law concerning time-varying magnetic fields. Unlike the motional emf we studied in Chapter 20, Faraday's law cannot be derived from any of the other fundamental principles we have studied.

FARADAY'S LAW

$$\text{emf} = -\frac{d\Phi_{mag}}{dt}$$

$$\text{where emf} = \oint \vec{E}_{NC} \bullet d\vec{l} \text{ and } \Phi_{mag} \equiv \int \vec{B} \bullet \hat{n}\, dA$$

In words: The induced emf along a round-trip path is equal to the rate of change of the magnetic flux on the area encircled by the path.

Direction: With the thumb of your right hand pointing in the direction of $-d\vec{B}/dt$, your fingers curl around in the direction of $\vec{E}_{NC}$.

Putting these pieces together, we have the following form:

FORMAL VERSION OF FARADAY'S LAW

$$\oint \vec{E}_{NC} \bullet d\vec{l} = -\frac{d}{dt}\left[\int \vec{B} \bullet \hat{n}\, dA\right] \text{ (sign given by right-hand rule)}$$

Faraday's law summarizes the experiments we've described so far:

- A faster rate of change of magnetic flux induces a bigger emf (if the magnetic flux is constant there is no induced emf).
- If there is a bigger area with the same perpendicular magnetic field there is a bigger emf.

The meaning of the minus sign

If the thumb of your right hand points in the direction of $-d\vec{B}/dt$ (that is, the opposite of the direction in which the magnetic field is increasing), your fingers curl around in the direction along which the path integral of electric field is positive:

$$\oint \vec{E}_{NC} \bullet d\vec{l} = -\frac{d}{dt}\left[\int \vec{B} \bullet \hat{n}\, dA\right] \text{ (sign given by right-hand rule)}$$

For most of our work it is simplest to calculate the magnitude of the effect, ignoring signs and directions, and then give the appropriate sign or direction based on the right-hand rule. We will include the minus sign in Faraday's law, as a reminder of what we need to do to get directions.

Flux and path

What area exactly do we use when calculating the magnetic flux? Imagine a soap film stretched over the closed path around which we are calculating $\oint \vec{E}_{NC} \bullet d\vec{l}$. We want to compute Φ_{mag} on the area covered by that soap film.

22.X.7 A wire of resistance 10 ohms and length 2.5 m is bent into a circle and is concentric with a solenoid in which the magnetic flux changes from 5 tesla·m^2 to 3 tesla·m^2 in 0.1 seconds. What is the emf in the wire? What is the non-Coulomb electric field in the wire? What is the current in the wire?

The Coulomb electric field can be included in Faraday's law

We will usually write emf $= \oint \vec{E}_{NC} \bullet d\vec{l}$ as a reminder that it is the curly non-Coulomb electric field that has a nonzero round-trip path integral, which we call the emf around that path. However, we could also calculate the emf in terms of the net electric field $\vec{E} = \vec{E}_C + \vec{E}_{NC}$, where $\vec{E}_C$ is the Coulomb electric field due to charges:

$$\text{emf} = \oint \vec{E} \bullet d\vec{l}$$

This is true because we have

$$\text{emf} = \oint \vec{E} \bullet d\vec{l} = \oint (\vec{E}_C + \vec{E}_{NC}) \bullet d\vec{l} = 0 + \oint \vec{E}_{NC} \bullet d\vec{l}$$

since $\oint \vec{E}_C \bullet d\vec{l} = 0$ (round-trip integral of Coulomb electric field is zero).

Application: A circuit surrounding a solenoid

We can apply Faraday's law to analyze quantitatively the situation of a circuit with an ammeter surrounding a solenoid (Figure 22.15; a cross section of the solenoid is shown). There is magnetic field $\vec{B}_1$ only over the circle of radius r_1 (the solenoid), and it points in the same direction as $\hat{n}$, the unit vector perpendicular to the surface.

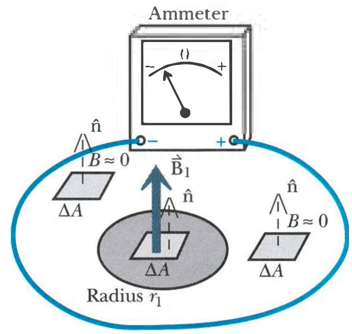


Figure 22.15 A cross section through the solenoid; calculate the flux enclosed by the wire.

? Calculate the magnetic flux Φ_{mag} on the area enclosed by the solenoid, the flux on the other portions of the area encircled by the circuit, and the total flux on the area encircled by the circuit.

On the area enclosed by the solenoid, $\Phi_{mag} = B_1(\pi r_1^2)$, because $B_\perp$ is constant and equal to B_1 throughout the cross-section of the solenoid. On every small area outside the solenoid, the magnetic field is nearly zero, so $\Phi_{mag} = 0$. Therefore the total flux through the outer wire is $\Phi_{mag} = B_1(\pi r_1^2)$.

What counts is the magnetic flux encircled by the circuit, not the magnetic flux on a closed surface such as a box or sphere. Unlike electric flux, the total magnetic flux on a closed surface is always zero, because "magnetic monopoles" seem not to exist.

Experimentally we find that the emf around the circuit is numerically equal to $d\Phi_{mag}/dt$, as predicted by Faraday's law. For example, suppose the magnetic field in the solenoid increases from 0.1 tesla to 0.7 tesla in 0.2 seconds, and the area of the solenoid is 3 cm^2.

? What is the emf around the circuit?

Faraday's law predicts the following average emf:

$$\text{emf} = \frac{\Delta\Phi_{mag}}{\Delta t} = \frac{(0.6\text{T})(3\times10^{-4}\ \text{m}^2)}{(0.2\ \text{s})} = 9\times10^{-4}\ \text{volts}$$

Ammeter reading

? If the resistance of the wire plus ammeter is 0.5 ohms, what current will the ammeter display?

The circuit acts as though a battery were inserted, $\text{emf} - RI = 0$, so

$$I = \frac{\text{emf}}{R} = \frac{(9\times10^{-4}\ \text{volts})}{(0.5\text{ohm})} = 1.8\times10^{-3}\ \text{ampere}$$

One more experiment: let's reconnect the wire and ammeter so that they don't encircle the solenoid (Figure 22.16).

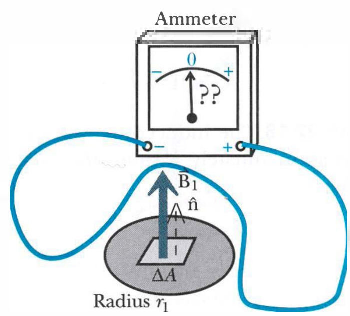


Figure 22.16 What happens if the circuit does not encircle the solenoid?

? As we increase the current (and magnetic field) in the solenoid, what would you predict from Faraday's law about the ammeter reading? Why?

Within the area bounded by the wire outside the solenoid, there is practically no magnetic field, so no magnetic flux Φ_{mag}, and no rate of change of magnetic flux $d\Phi_{mag}/dt$. When we try the experiment, we do indeed find that the ammeter shows little or no current.

Voltmeter readings

Since a voltmeter acts like an ammeter with a large resistance in series, a voltmeter may give a puzzling reading in the presence of time-varying mag-

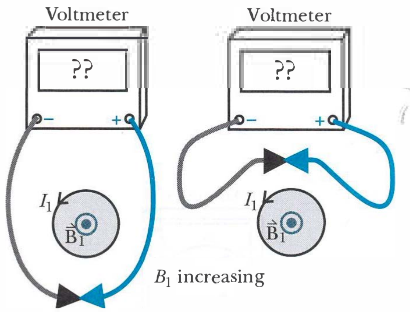


Figure 22.17 What do these voltmeters read?

netic flux. Consider the two voltmeters shown in Figure 22.17. You normally expect that when voltmeter leads are connected to each other, the voltmeter must read zero volts.

? But do these voltmeters read zero?

The voltmeter leads on the left of Figure 22.17 encircle a region of changing magnetic field, so the voltmeter will read an emf equal to $d\Phi_{mag}/dt$. The leads of the other voltmeter don't encircle a region of changing magnetic field, so the voltmeter reads zero.

Because of the effect of time-varying magnetic fields, you have to be a bit careful in interpreting a voltmeter reading when there are varying magnetic fields around. For example, when there are sinusoidally alternating currents ("AC") there are time-varying magnetic fields due to those time-varying currents. If the leads of an AC voltmeter happen to surround some AC magnetic flux, this will affect the voltmeter reading.

22.X.8 The magnetic field in a solenoid is $B = \mu_0 NI/d$. A circular wire of radius 10 cm is concentric with a solenoid of radius 2 cm and length $d = 1$ meter, containing 10,000 turns. The current increases at a rate of 50 A/s. What is the emf in the wire? What is the non-Coulomb electric field in the wire?

The emf for a coil with multiple turns

In many devices, instead of just one loop of wire surrounding a time-varying flux there is a coil with many turns, which increases the effect: the emf in a coil containing N turns is approximately N times the emf for one loop of the coil. This point needs some discussion.

In the space around the increasing flux in the coil, there is a curly pattern of induced (non-Coulomb) electric field. If the loops are tightly wound so that they are very close together, E_{NC} is about the same in each loop (Figure 22.18). The emf from one end of the coil to the other is the integral of the non-Coulomb field:

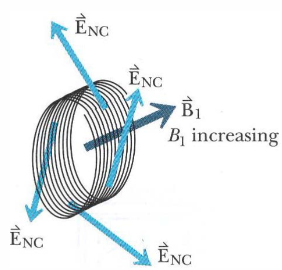


Figure 22.18 The induced emf of a thin coil is approximately N times the emf of one loop.

$$\text{emf} = \int_{N\text{ turns}} \vec{E}_{NC} \bullet d\vec{l} = N(E_{NC}L_{\text{one turn}}) = N(\text{emf}_{\text{one turn}})$$

Because of this, Faraday's law for a coil is written in the following form:

FARADAY'S LAW FOR A COIL

$$\text{emf} = -N\frac{d\Phi_{mag}}{dt} \quad \text{(sign given by right-hand rule)}$$

The induced emf in a coil of N turns is equal to N times the rate of change of the magnetic flux on one loop of the coil.

22.X.9 A thick copper wire connected to a voltmeter surrounds a region of time-varying magnetic flux, and the voltmeter reads 10 volts. If instead of a single wire we use a coil of thick copper wire containing 20 turns, what does the voltmeter read?

22.X.10 A thin Nichrome wire connected to an ammeter surrounds a region of time-varying magnetic flux, and the ammeter reads 10 amperes. If instead of a single wire we use a coil of thin Nichrome wire containing 20 turns, what does the ammeter read?

Faraday's law and moving coils or magnets

Figure 22.19 Move coil 1 toward coil 2, and there is a time-varying magnetic field inside coil 2.

A time-varying magnetic field produces a curly electric field. One way to create a time-varying field is by varying the current in a coil, but this isn't the only way to do produce a time-varying field. With a steady current in one coil in Figure 22.19, you can move that coil closer to a second coil. This increases the magnetic field (and magnetic flux) inside the second coil, and while the magnetic field is increasing there is an emf (and a current in the second coil).

You can also induce an emf by moving a bar magnet toward or away from coil 2, since this creates a time-varying magnetic field (and magnetic flux) inside the coil (Figure 22.20).

You can also get an induced emf by rotating the first coil (or a bar magnet; Figure 22.21), since this changes the flux through the second coil as long as you are rotating and therefore changing the magnetic field (and magnetic flux) in the region of space surrounded by the second coil.

These various experiments give additional confirmation that the emf in one or more loops of wire is due to a time-varying magnetic field, and numerically equal to the rate of change of the magnetic flux enclosed by the loops:

$$\text{emf} = -N\frac{d\Phi_{mag}}{dt} = -N\frac{d}{dt}\left(\sum \vec{B}_1 \bullet \hat{n}\Delta A_2\right)$$

Figure 22.20 Moving a magnet toward coil 2 creates a time-varying magnetic field inside the coil.

Figure 22.21 Rotating a bar magnet (or coil 1) produces a time-varying magnetic field inside coil 2.

22.X.11 Suppose you move a bar magnet toward the coil in Figure 22.22, with the "S" end of the bar magnet closest to the coil. Will the ammeter read positive or negative?

22.X.12 Now move the bar magnet away from the coil, with the "S" end still closest to the coil. Will the ammeter read + or –?

22.X.13 A bar magnet is held vertically above a horizontal metal ring, with the south pole of the magnet at the top (Figure 22.23). If the magnet is lifted straight up, will current run clockwise or counterclockwise in the ring, as seen from above?

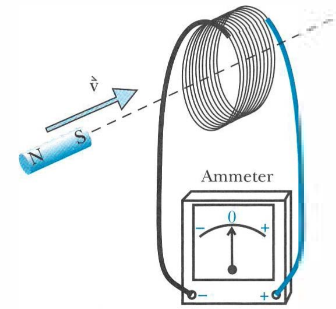


Figure 22.22 Will the ammeter read positive or negative? (Exercises 22.X.11 and 22.X.12.)

An interesting complication

The current I_1 in a coil of wire makes a magnetic field B_1. If I_1 varies with time, there is a time-varying magnetic field dB_1/dt in a second coil that can induce an emf and a current I_2 in the second coil. This induced current I_2 *also* makes a magnetic field B_2, an effect that we have ignored until now. If I_2 is constant (due to a constant dB_1/dt), the additional magnetic field B_2 is constant and so does not contribute additional non-Coulomb electric field.

However, if $d^2B_1/dt^2 \neq 0$ (there is a time-varying rate of change), then I_2 isn't constant, and there is a time-varying magnetic field dB_2/dt that contributes a non-Coulomb electric field and emf of its own. In many cases this effect is small compared to the main effect and can be ignored. If the effect is sizable, you can see that a full analysis could be rather difficult, because you have a changing I_1 making a changing I_2 that contributes to the change, which... Whew!

We will mostly avoid trying to analyze this complicated kind of situation in this introductory textbook, though we will see an example in the discussion of superconductors later in this chapter. A related issue is self-inductance, which we will also study later in this chapter.

Figure 22.23 Will the current run clockwise or counterclockwise? (Exercise 22.X.13.)

Example: Two coils

Two coils of wire are near each other, 20 cm apart, positioned on a common axis, as shown in Figure 22.24. Coil 1 has a radius 8 cm and contains 3000 loops of wire. It is connected to a power supply whose output voltage can be changed, so that the current I_1 in coil 1 can be varied. Coil 2 contains 1500 loops of wire and has radius 5 cm, and is rotated 35° from the axis, as shown in Figure 22.24. The current in Coil 1 changes from 0 to 3 amperes in 4 milliseconds. Calculate the emf in Coil 2.

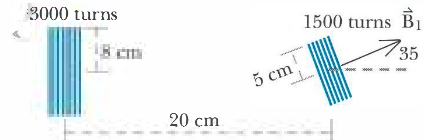


Figure 22.24 Two coils.

Solution

Start from fundamental principles. For a given current I_1 in the first coil, we can find B_1, the magnetic field that this current makes at the location of Coil 2. We know (dI_1/dt), so we can find $(d\Phi_{mag}/dt)$ in Coil 2, which gives us the emf in Coil 2.

Approximations: Assume the coils are far enough apart that the magnetic field of the first coil is approximately uniform in direction and magnitude throughout the interior of the second coil (Figure 22.25), and can be approximated as a magnetic dipole field. Also assume that the length of the coils is small compared to the distance between them (essentially treating all loops as if they were located at the center of the coil).

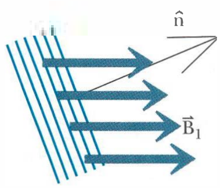


Figure 22.25 Assume the magnetic field made by coil 1 is uniform through the interior of coil 2.

Work out everything symbolically first, plug in numbers at the end.

Magnetic field B_1 at the location of coil 2:

$$B_1 \approx N_1 \frac{\mu_0}{4\pi} \frac{2I_1(\pi r_1^2)}{z^3}$$

Magnetic flux through one loop of coil 2:

$$\Phi_{mag} \approx B_1(\pi r_2^2)\cos\theta$$

$$= N_1 \frac{\mu_0}{4\pi} \frac{2I_1(\pi r_1^2)}{z^3}(\pi r_2^2)\cos\theta$$

I_1 is the only quantity that is changing, so emf around one loop of coil 2 is:

$$\text{emf} = \left|\frac{d\Phi_{mag}}{dt}\right| \approx N_1 \frac{\mu_0}{4\pi} \frac{2(\pi r_1^2)(\pi r_2^2)}{z^3}\cos\theta \frac{dI_1}{dt}$$

total emf for all of coil 2:

$$\text{emf} = N_2\left|\frac{d\Phi_{mag}}{dt}\right| \approx N_1 N_2 \frac{\mu_0}{4\pi} \frac{2(\pi r_1^2)(\pi r_2^2)}{z^3}\cos\theta \frac{dI_1}{dt}$$

$$\frac{(1500)(3000)\left(1\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2)(0.08\ \text{m})^2(\pi^2)(0.05\ \text{m})^2\cos 35°(3\ \text{A})}{(0.20\ \text{m})^3(4\times10^{-3}\ \text{s})}$$

$$= 10.9\ \text{V}$$

Example: Wire loop and a solenoid

A length of wire whose total resistance is R is made into a loop with two quarter-circle arcs of radius r_1 and radius r_2, and two straight radial sections as shown in Figure 22.26. A very long solenoid is positioned as shown, going into and out of the page. The solenoid consists of N turns of wire wound tightly onto a cylinder of radius r and length d, and it carries a counterclockwise current I_s which is increasing at a rate dI_s/dt.

What are the magnitude and direction of the magnetic field at C, at the center of the two arcs? The "fringe" magnetic field of the solenoid (that is, the magnetic field produced by the solenoid outside of the solenoid) is negligible at point C. Show all relevant quantities on a diagram.

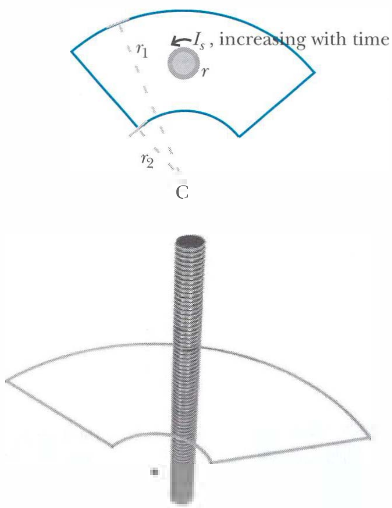


Figure 22.26 Upper: A long solenoid passes through a loop of wire. Lower: A perspective view.

Solution

As usual, start from fundamentals. Initially the only sources of field are the moving charges in the solenoid (which make B_s in the solenoid) and dB_s/dt in the solenoid (which makes a curly non-Coulomb electric field around the solenoid). The curly electric field constitutes an emf that drives current I_{loop}

in the surrounding wire. The current-carrying wires produce a magnetic field at location C.

Minor point: Because the wire is not circular, pile-up leads to tiny amounts of surface charge on the wire. The net electric field in the wire (the superposition of the non-Coulomb and Coulomb fields) follows the wire and is uniform in magnitude throughout the wire.

1) Inside solenoid, $B_s = \frac{\mu_0 N I_s}{d}$ out of page. See Figure 22.27.

2) Flux through *outer* loop =

$$B_s A_s \cos(0°) + (0) A_{\text{rest of loop}} = \frac{\mu_0 N I_s}{d}\pi r^2$$

3) $-d\vec{B}/dt$ into page, so $\vec{E}_{NC}$ and induced current I_{loop} clockwise as shown.

$$|\text{emf}| = \frac{d}{dt}\left(\frac{\mu_0 N I_s}{d}\pi r^2\right) = \frac{\mu_0 N}{d}\pi r^2 \frac{dI_s}{dt}, \text{ so } I_{\text{loop}} = \frac{\text{emf}}{R} = \frac{\mu_0 N \pi r^2}{d}\frac{dI_s}{R\,dt}$$

4) $B_{\text{at C}}$ due to straight segments, inner wire, and outer wire:

$B_{\text{straight segments}} = 0$ because $d\vec{l} \times \hat{r} = 0$ (see diagram)

$$B_{\text{inner wire}} = \left|\int_{\text{quarter loop}} \frac{\mu_0}{4\pi}\frac{I_{\text{loop}}\, d\vec{l} \times \hat{r}}{r_1^2}\right| = \frac{\mu_0}{4\pi}\frac{I_{\text{loop}}}{r_1^2}\int_{\text{quarter loop}} dl \sin(90°)$$

$$B_{\text{inner wire}} = \frac{\mu_0}{4\pi}\frac{I_{\text{loop}}}{r_1^2}\left(\frac{2\pi r_1}{4}\right) = \frac{\mu_0}{4\pi}\frac{\pi I_{\text{loop}}}{2r_1}, \text{ and } B_{\text{outer wire}} = \frac{\mu_0}{4\pi}\frac{\pi I_{\text{loop}}}{2r_2}$$

$B_{\text{inner wire}}$ is out of the page, and $B_{\text{outer wire}}$ is into the page at location C. Since B is proportional to $1/r$, $B_{\text{inner wire}}$ is larger than $B_{\text{outer wire}}$. So the net field is out of the page, with this magnitude:

$$B_{\text{net at C}} = B_{\text{inner wire}} - B_{\text{outer wire}} = \frac{\mu_0}{4\pi}\frac{\pi I_{\text{loop}}}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$B_{\text{net at C}} = \frac{\mu_0}{4\pi}\left(\frac{\pi}{2}\right)\left(\frac{\mu_0 N \pi r^2}{d}\frac{dI_s}{R\,dt}\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \text{ out of the page}$$

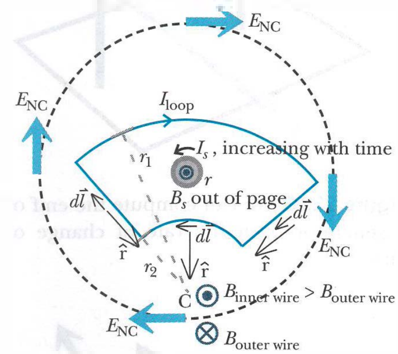


Figure 22.27 Fields and currents.

22.3 FARADAY'S LAW AND MOTIONAL EMF

In Chapter 20 we studied motional emf: when a wire moves through a magnetic field, magnetic forces drive current along the wire. We calculated the motional emf in a moving bar of length L as $(q|\vec{v} \times \vec{B}|)L/q$, the (non-Coulomb) work per unit charge done by the magnetic force $q\vec{v} \times \vec{B}$. If the velocity of the bar is perpendicular to the magnetic field, the motional emf is simply *vBL*. There is another way to calculate motional emf that is often mathematically easier, in terms of magnetic flux.

As shown in Figure 22.28, in a short time Δt the bar moved a distance $\Delta x = v\Delta t$, and the area surrounded by the current-carrying pieces of the circuit increased by an amount $\Delta A = L\Delta x = Lv\Delta t$. There is an increased magnetic flux through the circuit of amount

$$\Delta\Phi_{\text{mag}} = B_\perp \Delta A = B(Lv\Delta t)$$

Dividing by Δt we find this:

$$\frac{\Delta\Phi_{\text{mag}}}{\Delta t} = BLv$$

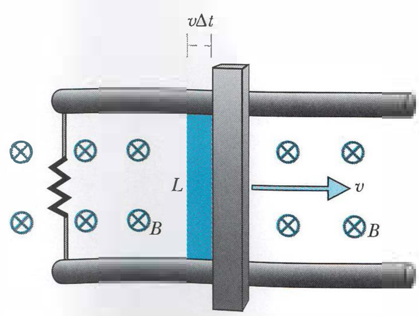


Figure 22.28 There is increased flux through the circuit as the bar moves.

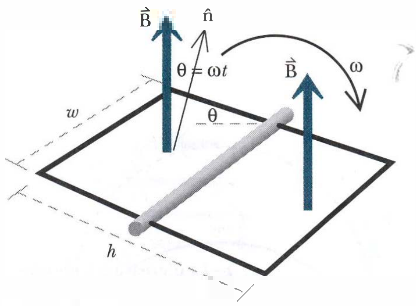


Figure 22.29 We can compute the emf of a generator from the rate of change of flux.

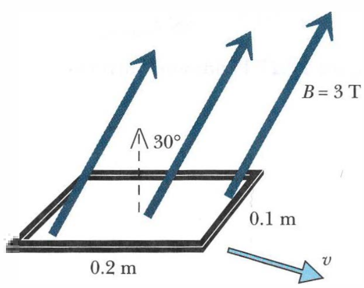


Figure 22.30 Move this rectangular circuit around in a uniform, constant field.

But BLv is equal to the emf vBL generated by the magnetic force, and in the limit of small Δt we have the following:

MOTIONAL EMF = RATE OF CHANGE OF MAGNETIC FLUX

$$|\text{emf}| = \left|\frac{d\Phi_{\text{mag}}}{dt}\right| \text{ (sign given by direction of magnetic force)}$$

Although we have proved this result only for a particular special case, it can be shown that this result applies in general to calculating motional emf—that is, an emf associated with the motion of a piece of a circuit in a magnetic field that is not varying in time.

As an example, consider a generator (Figure 22.29). When the normal to the loop is at an angle $\theta = \omega t$ to the magnetic field, the flux through the loop is $Bhw\cos(\omega t)$, so emf $= |d\Phi/dt| = \omega Bhw\sin(\omega t)$, which is the same result we obtained in Chapter 20 when we calculated the emf directly in terms of magnetic forces acting on electrons in the moving wires (motional emf).

22.X.14 A uniform, non-time-varying magnetic field of 3 tesla points 30° away from the perpendicular to the plane of a rectangular loop of wire 0.1 m by 0.2 m (Figure 22.30). The loop as a whole is moved in such a way that it maintains its shape and its orientation in the uniform magnetic field. What is the emf around the loop during this move?

22.X.15 In 0.1 s the loop is stretched to be 0.12 m by 0.22 m. What is the average emf around the loop during this time?

The two pieces of the flux derivative

A time-varying magnetic field is associated with a curly (non-Coulomb) electric field. We can quantitatively predict the behavior of circuits enclosing a time-varying magnetic field by calculating the rate of change of magnetic flux on the surface bounded by the circuit.

However, in the preceding section we saw that there is another way that magnetic flux can change. If the magnetic field is constant but the shape or orientation of the loop changes, so as to enclose more or less flux, the flux changes with time even though the magnetic field doesn't, and this is the phenomenon of motional emf associated with magnetic forces.

Any change in magnetic flux, whether due to a change in magnetic field or a change in shape or orientation, produces an emf equal to the rate of change of flux. Both effects can be present simultaneously. Suppose that as you drag a metal bar along rails, the magnitude of the uniform magnetic field is also increasing with time. The flux is $B_\perp A$, and the rate of change of the flux is due in part to the change in the magnetic field, and in part to the change in the area enclosed by the changing path:

$$\frac{d\Phi_{\text{mag}}}{dt} = \frac{d}{dt}(B_\perp A) = \frac{dB_\perp}{dt}A + B_\perp\frac{dA}{dt} = \frac{dB_\perp}{dt}A + B_\perp\frac{Ldx}{dt} = \frac{dB_\perp}{dt}A + B_\perp Lv$$

Change of B, fixed path

Change of path, fixed B

In some but not all cases the two effects can be related through a change of reference frame. If you move a magnet toward a stationary coil, the magnetic field in the coil is increasing with time, and there is a curly non-Coulomb electric field that drives current in the coil.

If on the other hand you move a coil toward a stationary magnet, the magnetic field is not changing with time anywhere, but the coil is moving in a magnetic field, so there is a magnetic force on the mobile charges in the wire, and motional emf.

In both situations you can calculate the same emf from the same rate of change of flux, and you observe the same current to run in the coil, though the mechanism looks different in the two cases. You can shift between one situation and the other simply by changing your reference frame (move with the magnet, or move with the coil). The principle of relativity implies that you should predict the same emf and the same current in either frame of reference, and you do.

It is not always possible to relate the two kinds of effects simply by changing reference frame. For example, consider a bar sliding along rails. In the reference frame of the bar, the bar is at rest but the resistor is moving. In neither reference frame is there a time-varying magnetic field.

(A full analysis in the reference frame of the bar is complicated. If the bar is at rest in this frame, why is it polarized? We saw in Chapter 20 that the answer is given by special relativity: when you transform to the frame of the bar in the presence of a uniform magnetic field, a transverse electric field of magnitude vB appears, and in this reference frame it is this new electric field that polarizes the bar.)

22.4 SUMMARY: NON-COULOMB FIELDS AND FORCES

If the magnetic field changes ($d\vec{B}/dt$ nonzero), there is a non-Coulomb electric field $\vec{E}_{NC}$ which curls around the region of changing flux, and emf $= \oint \vec{E}_{NC} \bullet d\vec{l}$ is nonzero. In purely motional emf, however, where the path changes but the magnetic field doesn't change, the non-Coulomb forces are magnetic forces, not electric.

The non-Coulomb electric field $\vec{E}_{NC}$ affects stationary as well as moving charges, but the (non-Coulomb) magnetic forces only affect moving charges, such as electrons in the moving metal bar. It is somewhat odd that the magnitude of the emf in both of these situations is $d\Phi_{mag}/dt$, but that's how it works.

We have identified three different kinds of electric and magnetic effects on the electrons inside a wire:

1) Coulomb electric fields due to surface charges (superposition of contributions of the form $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$);

2) non-Coulomb electric fields associated with time-varying magnetic fields, $\frac{d\vec{B}}{dt}$; and

3) magnetic forces $q\vec{v} \times \vec{B}$ if the wire is moving (yielding a motional $|\text{emf}| = \left|\frac{d\Phi_{mag}}{dt}\right|$ due to change of path).

All of these effects can be present simultaneously. For example, if the magnetic field is changing while you pull a bar along rails, there is a Coulomb electric field due to the charges that build up on the surfaces of the circuit elements, there is a non-Coulomb electric field due to the changing magnetic field, and there are non-Coulomb magnetic forces on electrons inside the moving bar. The round-trip integral of the Coulomb electric field is zero. The round-trip integral of the non-Coulomb electric field and the non-Coulomb magnetic force per unit charge together gives the emf, and the current I is given by $I = \text{emf}/R$, where $|\text{emf}| = |d\Phi_{mag}/dt|$.

22.5 THE CHARACTER OF PHYSICAL LAWS

We certainly haven't explained *why* there is a curly, non-Coulomb electric field around a solenoid when the magnetic field in the solenoid changes. All we have done is show that Faraday's law correctly predicts the observed electric field (and emf) in this admittedly strange situation. That is why Faraday's law is called a physical "law": it summarizes a wide variety of experiments and predicts the outcomes of experiments yet to be done, but in a deeper sense it doesn't "explain" anything. It doesn't tell us "why" there is a non-Coulomb electric field.

However, the status of Faraday's law is really no different than the status of Coulomb's law. Coulomb's law correctly summarizes a wide variety of experiments and predicts the outcomes of electrical experiments yet to be done, but it explains nothing at all about "why" there are "electrically charged particles" or "why" these "charged particles" attract and repel each other with a $1/r^2$ force.

When we ask you to "explain" electric or magnetic phenomena, we ask you to explain a wide range of phenomena in terms of a small number of fundamental physical laws: Coulomb's law or Gauss's law, the Biot-Savart law or Ampere's law, electric and magnetic forces, Faraday's law. We don't and can't ask you to go one level deeper and explain these fundamental laws, because these laws are in fact summaries of a wide variety of experiments and may not have an explanation in terms of principles that are even more basic.

It would be satisfying to find deeper physical laws that would explain the laws we have been studying and explain other laws as well, because that would reduce even further the number of principles required to predict a very wide range of phenomena. In fact, we have seen that in the framework of Einstein's theory of special relativity, magnetic phenomena appear to be essentially electric phenomena as seen from a different reference frame. Quantum electrodynamics goes even further in unifying electricity and magnetism. And recent work by theoretical and experimental physicists has provided the basis for physical laws that encompass both electromagnetism and the so-called "weak force" responsible for certain kinds of radioactivity.

The search goes on for an all-encompassing "theory of everything" that would include gravitation and the physics of quarks, thought to be the building blocks of protons and neutrons. But many physicists suspect that even if a "theory of everything" explained all observable physical phenomena, it wouldn't explain itself.

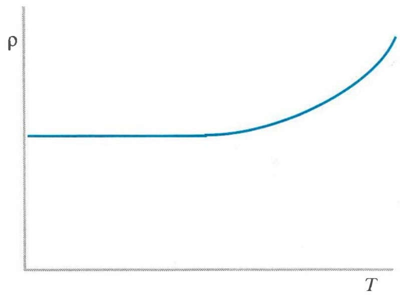


Figure 22.31 Resistivity vs. temperature for an ordinary metal.

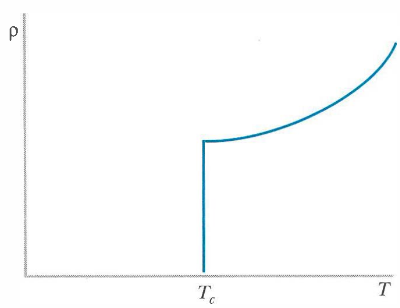


Figure 22.32 Resistivity vs. temperature for a superconductor.

22.6 SUPERCONDUCTORS

As you lower the temperature, the resistivity of an ordinary metal decreases but remains nonzero even at absolute zero (Figure 22.31), while some materials lose all resistance to electric current. Such materials are called "superconductors." It isn't that the resistance merely gets very small. Below a critical temperature T_C that is different for each superconducting material, the resistance vanishes completely, as indicated in Figure 22.32. To put it another way, the mobility is infinite. A nonzero electric field in a superconductor would produce an infinite current! As a result, the electric field inside a superconductor must always be zero, yet a nonzero current can run anyway.

In a ring made of lead that is kept at a temperature below 7.2 degrees Kelvin (that is, only 7.2 degrees above absolute zero), one can observe a current that persists undiminished for years (Figure 22.33)!

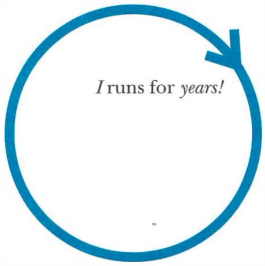


Figure 22.33 A current in a superconducting ring persists for years.

? How can you check that the current in the lead ring hasn't changed since last month, without inserting an ammeter into the ring or touching the ring in any way?

Since the current loop produces a magnetic field, you could use a compass or other more sensitive device to measure the magnetic field produced by the current.

? Does this persistent current violate the fundamental principle of conservation of energy? Why or why not?

No energy is being dissipated in the ring. Since its resistance is zero, the power used up is zero (power = RI^2). Energy conservation is not violated.

? Do the persistent atomic currents in your little bar magnet violate the principle of conservation of energy? Why or why not?

The "currents" in the bar magnet are due to the spin and orbital motion of electrons in the atoms of the magnet. The energy of these electrons remains constant—no energy is used up, so no energy input is required. Energy conservation is not violated.

A fruitful way to think about a superconductor is to say that the persistent current is similar to the persistent atomic currents in a magnet, but the diameter of a persistent current loop in a superconductor can be huge compared to an atom. In both the magnet and the superconductor the persistent currents can be explained only by means of quantum mechanics. In both cases the explanation hinges on the fact that these systems have discrete energy levels. Since the energy levels are separated by a gap, there can be no small energy changes, and hence there is no mechanism for energy dissipation.

Effect of magnetic fields on a superconductor

The lack of resistance in a superconductor offers obvious advantages for power transmission and in electromagnets. But for a long time after 1911, when the Dutch physicist Kamerlingh Onnes discovered superconductivity in mercury below 4.2 degrees Kelvin, all superconductors required very low temperatures. Recently materials have been discovered that are superconducting at temperatures above the boiling point of liquid nitrogen (77 Kelvin, –196 degrees Celsius). Because liquid nitrogen is rather inexpensive to produce, this opens up new opportunities for the use of superconductors. There is even hope that some material might be found that would be a superconductor at room temperature.

Magnetic flux through a superconducting ring

The complete lack of electric resistance has some odd consequences for the effect of magnetic fields on a superconductor. Suppose you try to change the flux through a superconducting metal ring. This would produce a curly electric field (whose round-trip integral is an emf equal to $d\Phi_{mag}/dt$). This curly electric field would drive an infinite current in the ring, because there is no resistance! Because an infinite current is impossible, we conclude that it is not possible to change the flux through a superconducting ring.

What does happen when you try to change the flux through the ring by an amount $\Delta\Phi_{mag}$? Since the flux cannot change, what happens is that a (noninfinite) current runs in the ring in such a way as to produce an amount of flux $-\Delta\Phi_{mag}$, so that the net flux change is zero! We'll consider a specific experimental situation to illustrate the phenomenon.

Start with a ring and a magnet at room temperature, so that there is a certain amount of flux Φ_0 through the ring (Figure 22.34), then cool the ring below the critical temperature at which it becomes superconducting (obviously the ring has to be made of a material that does become superconducting at some low temperature).

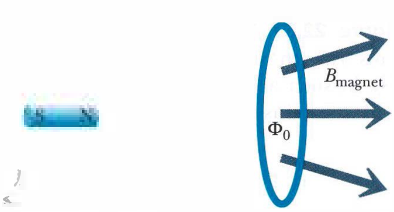


Figure 22.34 Start with a ring with initial flux Φ_0, then cool the ring down to make it become superconducting.

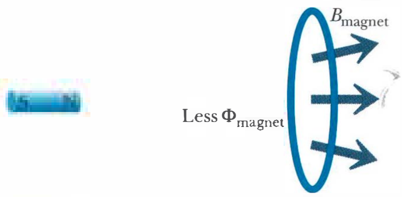


Figure 22.35 At low temperature, move the magnet away from the ring, reducing the flux in the ring contributed by the magnet.

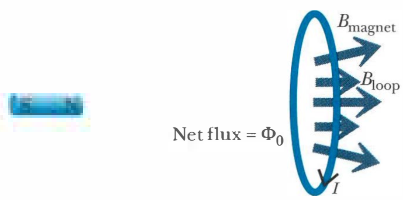


Figure 22.36 Current runs in the superconducting ring in such a way that the net change in the flux in the ring is zero.

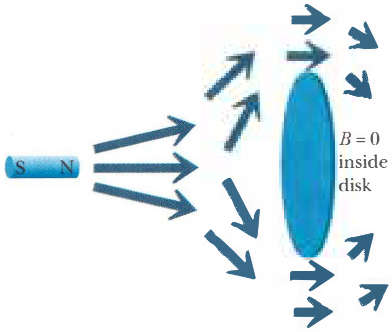


Figure 22.37 The Meissner effect: Currents run in the solid superconducting disk in such a way that the net magnetic field is zero inside the disk.

Next move the magnet away from the ring, thus decreasing the flux made by the magnet inside the ring (Figure 22.35); there is a change in the flux due to the magnet, of some amount $-\Delta\Phi$.

In a nonsuperconducting material, an emf would be induced in the ring, with a current emf = RI. But since the resistance R of the superconducting ring is zero, the emf must be zero (0 times I is 0). Assuming that Faraday's law applies to superconductors, it must be that just enough current runs in the superconductor, in the appropriate direction, to increase the flux by an amount $+\Delta\Phi$. The net change in the flux is zero, and the flux remains equal to the original flux Φ_0.

Experimentally, that's exactly what happens. Enough current runs in the ring to make exactly the right amount of magnetic flux to make up for the decrease in the flux due to moving the magnet farther away, so that the flux through the ring remains constant (Figure 22.36). Even though a current runs, the electric field inside the superconductor is zero.

22.X.16 When the magnet is moved very far away, how much flux is inside the ring compared to the original flux Φ_0? How much of this flux is due to the current in the ring?

The Meissner effect

For many years it was assumed that a similar effect would happen with a solid disk instead of a hollow ring. It was expected that as you cooled the disk below its critical temperature the magnetic flux through the disk would stay the same, and that when you moved the magnet away the induced current in the disk would maintain the original flux. In 1933 physicists were astonished to discover that something else happened, something quite dramatic. As soon as the disk was cooled down below the critical temperature for the onset of superconductivity, the magnetic field throughout the interior of the solid disk suddenly went to zero!

This peculiar phenomenon is called the Meissner effect, and it applies to a particular class of materials called "Type I" superconductors. The configuration of net magnetic field (due to the magnet plus the currents inside the superconductor) looks something like Figure 22.37, with no net magnetic field inside the superconductor. Persistent currents are created in the superconductor which make a magnetic field that exactly cancels the magnetic field due to the magnet, everywhere inside the superconductor. Note that the currents in Figure 22.37 are oriented so that the disk and magnet repel each other.

The Meissner effect was totally unexpected, and it cannot be explained simply in terms of the lack of electric resistance in a superconductor (plus Faraday's law) as we did for the hollow ring. The Meissner effect is a special quantum-mechanical property of superconductors, and its explanation along with the quantum-mechanical explanation of other properties of superconductors was a triumph of the Bardeen-Cooper-Schrieffer (BCS) theory published in 1957.

Given that both the electric field and the magnetic field must be zero inside the superconductor, by using Gauss's law and Ampere's law it can be shown that the superconducting currents can run only on the surface of the superconductor, not in the interior. In particular, current in the interior would produce magnetic field throughout the material.

The electric field in an ordinary conductor in equilibrium goes to zero due to polarization. However, the fact that electric and magnetic fields must be zero in a superconductor even when not in equilibrium is quite a different, quantum-mechanical effect.

Magnetic levitation

The Meissner effect is the basis for a dramatic kind of magnetic levitation. A magnet can hover above a superconducting disk, because no matter how the magnet is oriented, currents run in the superconducting disk in such a way as to repel the magnet. One design for maglev trains uses this superconducting effect.

This is in contrast to the situation with ordinary magnets. If you try to balance a magnet above another magnet, with the slightest misalignment the upper magnet flips over and is attracted downward rather than repelled upward.

The "Levitron" toy achieves levitation with ordinary magnets by giving the upper magnet a high spin, so that the magnetic torque makes the spin direction precess rather than flip over. When a superconductor supports the upper magnet, gyroscopic stabilization is not needed.

22.7 INDUCTANCE, AND ENERGY IN A MAGNETIC FIELD

Changing the current in a coil can induce an emf in a second coil. A related effect happens even with a single coil: an attempt to change the current in a coil induces an emf in the same coil, because the coil surrounds a region of time-varying magnetic field (dB/dt), produced by itself. We can show that this self-induced emf acts in a direction to oppose the change in the current. As a result, there is a kind of sluggishness in any coil of wire: it is hard to change the current, either to increase it or to decrease it. It is not difficult to calculate this self-induction effect quantitatively for a solenoid.

A very long plastic cylinder with radius R, and length $d >> R$, is wound with N closely-packed loops of wire with negligible resistance. The solenoid is placed in series with a power supply and a resistor, and current I runs through the solenoid in the direction shown in Figure 22.38. This current makes a magnetic field B inside the solenoid that points to the right.

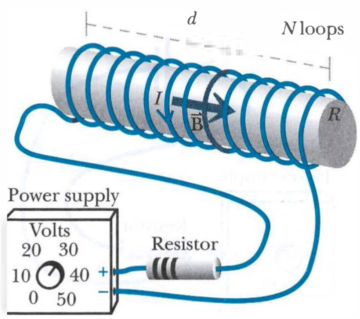


Figure 22.38 A circuit contains a solenoid.

If the current is steady, the electric field inside the metal of the solenoid coil is nearly zero (very small resistance of this wire). Now suppose that you try to increase the current, by turning up the voltage of the power supply. If the current increases with time, there will be an increasing magnetic flux enclosed by each loop of the solenoid. In Chapter 17 we found that the magnetic field inside a solenoid of length d is $B = \mu_0 NI/d$.

? How much emf is generated in one loop of the coil if the current changes at a known rate dI/dt?

We have this:

$$\text{emf} = \left|\frac{d\Phi_{\text{mag}}}{dt}\right| = \frac{d}{dt}\left[\frac{\mu_0 NI}{d}\pi R^2\right] = \frac{\mu_0 N}{d}\pi R^2\frac{dI}{dt} \quad \text{(one loop)}$$

? There are N loops, and each loop encloses approximately the same amount of time-varying magnetic flux. Therefore, what is the net emf induced along the full length of the solenoid?

Evidently we add up all the individual one-loop emf's in series to obtain the emf from one end of the coil to the other:

$$\text{emf} = N\left[\frac{\mu_0 N}{d}\pi R^2\frac{dI}{dt}\right] = \frac{\mu_0 N^2}{d}\pi R^2\frac{dI}{dt} \quad \text{(entire solenoid)}$$

? If we increase the current I, what is the direction of the induced curly electric field, clockwise or counterclockwise as seen from the right end of the solenoid? Does this induced electric field act to assist or oppose the change in I?

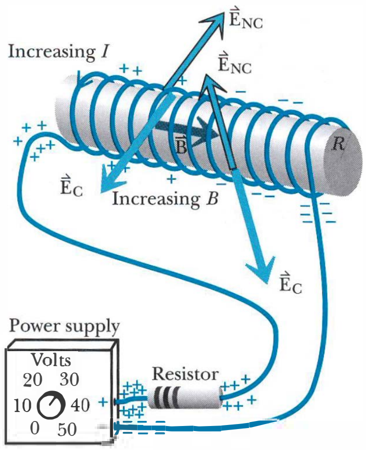


Figure 22.39 The non-Coulomb electric field opposes the increase in the current, and polarizes the solenoid.

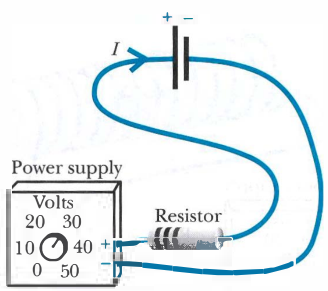


Figure 22.40 The coil acts temporarily like a battery inserted to oppose the change.

If the current is increasing, the non-Coulomb electric field $\vec{E}_{NC}$ curls clockwise around the solenoid, opposing the increase in the current and polarizing the solenoid (Figure 22.39). The new surface charges produce a Coulomb electric field $\vec{E}_C$ which follows the wire and points opposite to the non-Coulomb field. If the resistance of the solenoid wire is very small, the magnitude of the two electric fields is nearly equal ($E_C \approx E_{NC}$).

Note the similarity to a battery: a non-Coulomb emf_{ind} (the induced emf) maintains a charge separation, and there is a potential drop ΔV_{sol} along the solenoid that is numerically equal to emf_{ind}. If the wire resistance r_{sol} is significant, we have $\Delta V_{sol} = \text{emf}_{ind} - r_{sol}I$, just like a battery with internal resistance. You can think of this self-induced emf as making the solenoid act like a battery that has been put in the circuit "backwards," opposing the change in the current(Figure 22.40).

It is standard practice to lump the many constants together and write that the self-induced emf is proportional to the rate of change of the current, with proportionality constant L:

$$|\text{emf}_{ind}| = L\left|\frac{dI}{dt}\right|$$

The proportionality constant L is called the "inductance" or "self-inductance" of the device, which is called an "inductor."

? Use the results above to express the inductance L in terms of the properties of a solenoid.

Evidently the (self-)inductance of a solenoid is this:

$$L = \frac{\mu_0 N^2}{d}\pi R^2$$

The unit of inductance (volt-seconds/ampere) is called the "henry" in honor of the 19th-century American physicist Joseph Henry, who simultaneously with Michael Faraday in 1831 discovered the effects of time-varying magnetic fields.

? What if you decrease the current through the solenoid? What happens to the induced emf? Does it assist or oppose the change?

If you go through the previous analysis but with a decreasing current, you find that the emf goes in the other direction, tending to drive current in the original current direction and therefore opposing the decrease. The solenoid introduces some sluggishness into the circuit: it is more difficult to increase the current or to decrease the current. The sign of the effect should not be surprising, since if the self-induced emf assisted rather than opposing an increase in the current, the current would grow without limit!

22.X.17 To get an idea of the order of magnitude of inductance, calculate the self-inductance in henries for a solenoid with 1000 loops of wire wound on a rod 10 cm long with radius 1 cm.

Energy density

In Chapter 16 we showed that there is an energy density associated with electric field, by expressing the energy in a capacitor in terms of the electric field in the capacitor:

$$\frac{\text{Electric energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Now we can show that there is energy density associated with magnetic field, by expressing the energy in an inductor in terms of the magnetic field in the inductor. We obtain an important result about magnetic energy that is quite general, despite the fact that the derivation is for the specific case of an inductor.

It is easy to show that the magnetic energy stored in an inductor is $\frac{1}{2}LI^2$. The power going into an inductor is $I\Delta V$ as for any device, and

$$P = I\Delta V = I(\text{emf}) = I\left(L\frac{dI}{dt}\right)$$

Integrating over time, we have

$$\text{energy input} = \int(I\Delta V)dt = L\int_{t_i}^{t_f} I\frac{dI}{dt}dt = L\int_{I_i}^{I_f} IdI = L[\tfrac{1}{2}I^2]_{I_i}^{I_f} = \Delta(\tfrac{1}{2}LI^2)$$

Since $B = \frac{\mu_0 NI}{d}$ and $L = \frac{\mu_0 N^2}{d}\pi R^2$, we have

$$\text{Magnetic energy} = \frac{1}{2}\left(\frac{\mu_0 N^2}{d}\pi R^2\right)\left(\frac{Bd}{\mu_0 N}\right)^2$$

$$\text{Magnetic energy} = \frac{1}{2}\frac{1}{\mu_0}(\pi R^2 d)B^2$$

The magnetic field of the solenoid is large in the volume $(\pi R^2 d)$. Therefore we have the following result for the electric and magnetic energy densities:

ELECTRIC AND MAGNETIC ENERGY DENSITIES

$$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2\text{, measured in joules/m}^3$$

Although we have calculated the magnetic energy density for the specific case of a long solenoid, this is a general result. The interpretation is that where there are electric or magnetic fields in space, there is an associated energy density, proportional to the square of the field (E^2 and/or B^2).

*Current in an RL circuit

A circuit containing a resistor R and an inductor L is called an "RL" circuit. Figure 22.41 shows a series RL circuit some time after a switch was closed. The energy-conservation loop rule for this series circuit is this:

$$\Delta V_{\text{battery}} + \Delta V_{\text{resistor}} + \Delta V_{\text{inductor}} = 0$$

$$\text{emf}_{\text{battery}} - RI - L\frac{dI}{dt} = 0$$

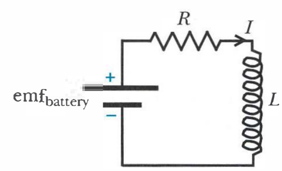


Figure 22.41 An RL circuit containing a resistor and an inductor.

The term $-LdI/dt$ in the loop equation has a minus sign because, as we have seen, the emf of the inductor opposes the attempt to increase the current when we close the switch. The properties of this differential equation are similar to those of the differential equation of a resistor-capacitor (RC) circuit analyzed in Chapter 19.

Let the time when we closed the switch be $t = 0$. The current was of course zero just before closing the switch, and it is also zero just after closing the switch, since the sluggishness of the inductor does not permit an instantaneous change from $I = 0$ to I = nonzero. In fact, such an instantaneous change would mean that dI/dt would be infinite, which would require an infinite battery voltage to overcome the infinite self-induced emf in the inductor.

? Prove that

$$I = \frac{\text{emf}_{\text{battery}}}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

by substituting I and dI/dt into the energy-conservation equation. What is the final current in the circuit (that is, after a very long time)? Does that make sense?

For large t, the exponential goes to zero, and we have $I = \text{emf}_{\text{battery}}/R$. This makes sense, because after a long time there is a steady-state current, and the flux does not change any more. With no change in the flux, no emf is generated in the inductor, and the current has the value that it would have if the inductor were not in the circuit. The current builds up slowly to this value. Figure 22.42 shows a graph of the current vs. time. One measure of the time it takes for the current to build up is the "time constant" L/R; at this time the exponential factor has dropped to $1/e$:

$$e^{-\left(\frac{R}{L}\right)t} = e^{-\left(\frac{R}{L}\right)\left(\frac{L}{R}\right)} = e^{-1} = \frac{1}{e}$$

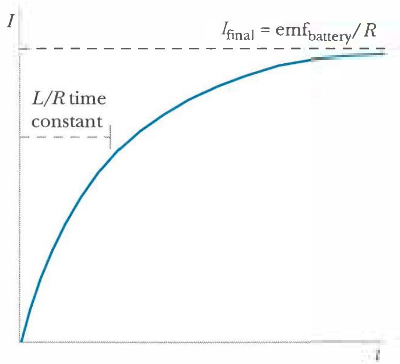


Figure 22.42 A graph of current vs. time in the RL circuit. The switch was closed at $t = 0$.

You see that having an inductor in a circuit makes the circuit somewhat sluggish. It takes a while for the current to reach the value that it would have reached right away if the circuit had consisted just of a battery and a resistor.

? It is interesting to see what happens when you try to open the switch, after the steady current has been attained. If the current were to drop from I to zero in a short time Δt, explain why the emf induced along the solenoid is very large, and can be much larger than the battery emf.

The induced emf depends on dB/dt, which is proportional to dI/dt. If the time interval is very short, then dI/dt is very large, and the induced emf is very large. As a result of this large emf, you may see a spark jump across the opening switch. This phenomenon makes it dangerous to open a switch in an inductive circuit if there are explosive gases around.

*Current in an LC circuit

We have studied many examples of equilibrium and of steady-state currents. We have also seen examples of a slow approach to equilibrium (an RC circuit) or to a steady-state current (the RL circuit we just examined).

There is another possibility: a circuit might oscillate, with charge sloshing back and forth forever—the system never settles down to a final equilibrium or a steady state. Of the systems we've analyzed or experimented with, none of them oscillated continuously, because there was always some resistance in the system that would have damped out any such oscillatory tendencies. It is possible that during the first few nanoseconds when surface charge is rearranging itself in a circuit, there may be some sloshing of charge back and forth, but this oscillation dies out due to dissipation in the resistive wires.

A circuit containing an inductor L and a capacitor C is called an LC circuit (Figure 22.43). Such a circuit can oscillate if the resistance is small. In this circuit, the connecting wires are low-resistance thick copper wires. Suppose the capacitor is initially charged, and then you close a switch, connecting the capacitor to the inductor.

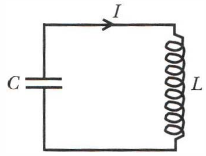


Figure 22.43 An LC circuit consists of an inductor and a capacitor.

At first it is difficult for charge on the capacitor to flow through the inductor, because the inductor opposes attempts to change the current (and the initial current was zero). But the inductor can't completely prevent the current from changing, so little by little there is more and more current, which of course drains the capacitor of its charge.

We'll see that just at the moment when the capacitor runs out of charge, there is a current in the inductor which can't change to zero instantaneously, due to the sluggishness of the inductor. So the system doesn't come to equilibrium but overshoots the equilibrium condition and pours charge into the capacitor. When the capacitor gets fully charged (opposite in sign to the original condition), the capacitor starts discharging back through the inductor.

This oscillatory cycle repeats forever if there is no resistance in the circuit. If there is some resistance, the oscillations eventually die out, but the system may go through many cycles before equilibrium is reached.

The energy-conservation loop rule for this circuit is

$$\Delta V_{\text{capacitor}} + \Delta V_{\text{inductor}} = \frac{1}{C}Q - L\frac{dI}{dt} = 0$$

where Q is the charge on the upper plate of the capacitor, and I is the conventional current leaving the upper plate and going through the inductor.

? Can you explain why $I = -dQ/dt$?

dQ/dt is the amount of charge flowing off of the capacitor plate per second. This is the same thing as the current. Because charge is leaving the plate, dQ is negative, so $I = -dQ/dt$. Since $I = -dQ/dt$, we can rewrite the energy-conservation equation as

$$\frac{1}{C}Q + L\frac{d^2Q}{dt^2} = 0$$

? Show that

$$Q = Q_i\cos\left(\frac{1}{\sqrt{LC}}t\right)$$

is a possible solution of the rewritten energy-conservation equation, by substituting Q and its second derivative into the equation.

Moreover, this solution fits the initial conditions at $t = 0$, just after closing the switch, if Q_i is the initial charge on the upper plate, since this expression reduces to $Q = Q_i\cos(0) = Q_i$. (For other initial conditions, the correct solution is a sine, or a combination of sine and cosine.)

? Now that you know Q on the upper plate of the capacitor as a function of time, calculate the current I through the inductor as a function of time.

The current is given by the following:

$$I = -\frac{dQ}{dt} = \frac{Q_i}{\sqrt{LC}}\sin\left(\frac{1}{\sqrt{LC}}t\right)$$

Now that we have expressions for Q and I as a function of time, we can graph both quantities (Figure 22.44). This is very special: charge oscillates back and forth in the circuit forever (if there is negligible resistance). There is no equilibrium, and there is no steady state. No battery or other source of energy input is needed, because there is no dissipation. The charge just keeps going back and forth on its own. If there is some resistance, the oscillations slowly die out, as shown in Figure 22.45.

Note that the current I does not become large instantaneously (due to the sluggishness of the inductor). Also note that the current, which is the rate of depletion of the capacitor charge, reaches a maximum just when Q goes to zero. This maximum current starts charging the bottom plate of the capacitor positive, which makes the current decrease. When the current becomes zero, the system is in a state very similar to its original state, but

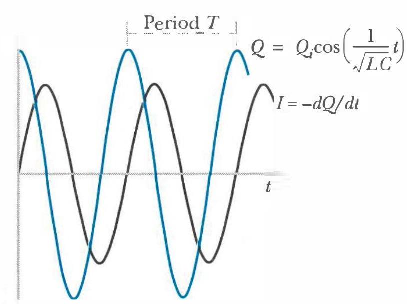


Figure 22.44 Capacitor charge and inductor current in an LC circuit with no resistance.

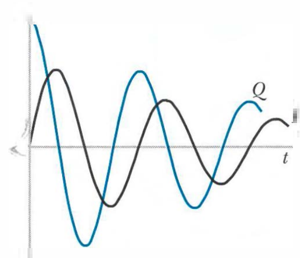


Figure 22.45 An LC circuit with resistance.

inverted (the bottom plate is positive). Now the system runs the other way and eventually gets back after one complete cycle to a state in which the top plate is again positive, and the current is momentarily zero. Then the process repeats.

After one complete cycle, $(1/\sqrt{LC})t = 2\pi$, so the period of the oscillation is $T = 2\pi\sqrt{LC}$. The frequency $f = 1/T = 1/(2\pi\sqrt{LC})$.

22.X.18 What is the oscillation frequency of an LC circuit whose capacitor has a capacitance of 1 microfarad and whose inductor has an inductance of 1 millihenry? (Both of these are fairly typical values for capacitors and inductors in electronic circuits. See page 794 for a numerical example of inductance.)

*Energy in an LC circuit

Another way to talk about an LC circuit is in terms of stored energy. The original energy was the electric energy stored in the capacitor (equal to $Q^2/(2C)$; see Chapter 19). At the instant during the oscillation when the charge on the capacitor is momentarily zero, there is no energy stored in the capacitor; all the energy at that moment is stored in the inductor, in the form of magnetic energy. As the system oscillates, energy is passed back and forth between the capacitor and the inductor.

Earlier in this chapter we showed that the magnetic energy stored in an inductor is $\frac{1}{2}LI^2$. At $t = 0$, the current is zero, and all the energy in the system is the electric energy of the capacitor. At the instant when $Q = 0$ on the capacitor, the current is a maximum, and all the energy in the system is the magnetic energy of the inductor.

? If Q_i is the initial charge on the capacitor, use an energy argument to find the maximum current I_{max} in the circuit.

Initially, all of the stored energy is electric:

$$U = U_{electric} + U_{magnetic} = \frac{1}{2}\frac{Q_i^2}{C}$$

When $I = I_{max}$, $Q = 0$, so all of the stored energy is magnetic:

$$U = U_{electric} + U_{magnetic} = \frac{1}{2}LI_{max}^2$$

Since the total energy in the circuit does not change, because the circuit has no resistance and no energy is dissipated as heat, the following must be true:

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}\frac{Q_i^2}{C}, \text{ and } I_{max} = \frac{Q_i}{\sqrt{LC}}$$

? Show that this result is consistent with $Q = Q_i\cos(t/\sqrt{LC})$ by calculating $I = -dQ/dt$ and seeing what the maximum I is.

Starting from the charge as a function of time, we have this:

$$I = -\frac{dQ}{dt} = -\frac{Q_i}{\sqrt{LC}}\sin\left(\frac{t}{\sqrt{LC}}\right), \text{ so that } I_{max} = \frac{Q_i}{\sqrt{LC}}$$

This is the same result we obtained by using an energy argument.

We have been discussing "free" oscillations, in which the oscillations (perpetual or dying out) proceed with no inputs from the outside. If you try to "force" the oscillations by applying an AC voltage from the outside (with an AC power supply), you find that it is difficult to get much current to run unless you nearly match the "free" oscillation frequency $f = 1/(2\pi\sqrt{LC})$. This is an example of what is called a "resonance" phenomenon (see end of Chapter 6), in which a system (the LC circuit in this case) responds significantly only when the forcing of the system is done at a frequency close to the free-oscillation frequency.

*AC circuits

For completeness, it should be mentioned that an important use of inductors is in AC circuits (sinusoidally alternating current). You have just seen that LC circuits are characterized by sinusoidally alternating currents, as are generators with loops rotating in a magnetic field. Whenever there is an AC current passing through an inductor, there is an AC emf generated in the inductor due to the time-varying flux. If the current is a sine function, the emf is a cosine function, and vice versa.

There is an elaborate mathematical formalism for dealing with AC circuits, including accounting for the fact that the current and voltage in an AC circuit need not reach their maxima at the same time (a "phase shift", which was 90° in the LC circuit we just analyzed). We will not go into these complications here, but it is worth noting that an important element in understanding such circuits is to recognize the role of the self-induced emf in an inductor, and the fact that in an inductor a sinusoidal current is associated with a cosinusoidal voltage.

Figure 22.46 Two light bulbs connected around a long solenoid with varying *B*.

22.8 *SOME PECULIAR CIRCUITS

This section offers examples of surprising aspects of circuits when there is a time-varying magnetic field.

*Two bulbs near a solenoid

We can show you a very simple circuit that behaves in a rather surprising way. Consider two light bulbs connected in series around a solenoid with a varying magnetic field (Figure 22.46). The solenoid carries an alternating current (AC) in order to provide a time-varying magnetic flux through the circuit, so that there is an emf to light the bulbs. If the flux varies as $\Phi_{mag} = \Phi_0 \sin(\omega t)$, in the bulb circuit we have a varying emf = $\text{emf}_0 \cos(\omega t)$, since the emf is proportional to the rate of change of the magnetic flux. If each bulb has a resistance *R*, there is an alternating current through the bulbs of this amount:

$$I = \frac{\text{emf}}{2R} = \frac{\text{emf}_0}{2R}\cos(\omega t)$$

Notice that a uniformly increasing flux ($\Phi_{mag} = \Phi_0 t$) would yield a constant emf, but a sinusoidally varying flux yields a cosinusoidally varying emf. It is very convenient to use AC currents to study the effects of time-varying magnetic fields, because both the inducing current and the induced current vary as sines and cosines, and AC voltmeters and ammeters can be used to measure both parts of the circuit.

Now we alter the circuit by connecting a thick copper wire across the circuit (Figure 22.47). You may find it surprising that the top bulb gets much brighter and the bottom bulb no longer glows. Let's try to understand how this happens.

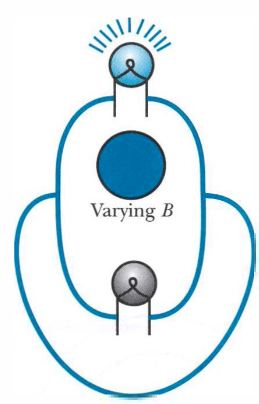


Figure 22.47 Add a thick copper wire to the two-bulb circuit.

As usual, we need to write charge-conservation node equations and energy-conservation loop equations, so we need to label the nodes and loops as in Figure 22.48 (any two of these three loops would be enough, but we'd like to show how to handle all of them).

Here is the key to analyzing this peculiar circuit. The round trip around loop 1 or loop 2 takes in an emf (just as though there were a battery in the loop). But the round trip around loop 3 does not enclose any time-varying flux and so has no emf. Therefore the loop and node rules are the follow-

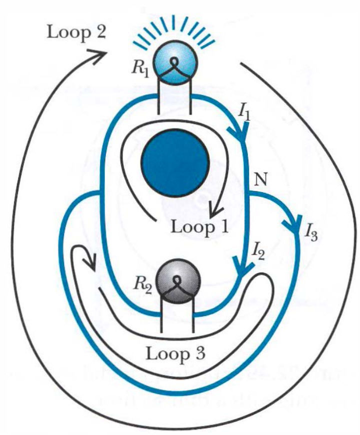


Figure 22.48 Add a thick copper wire to the two-bulb circuit.

ing, where R_1 and R_2 are the resistances of the hot top bulb and cold bottom bulb, and the resistance of the added copper wire is essentially zero:

loop 1: $\text{emf} - R_1I_1 - R_2I_2 = 0$ node N: $I_1 = I_2 + I_3$

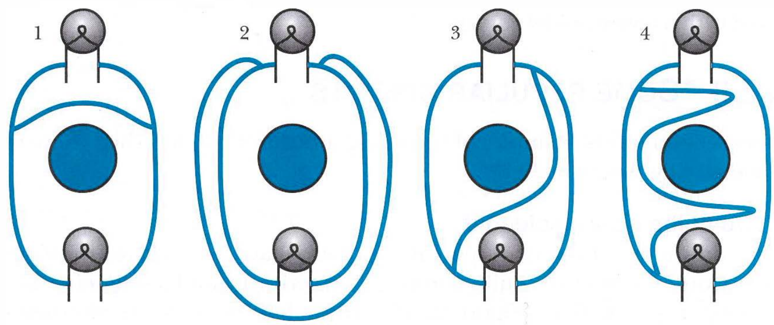
loop 2: $\text{emf} - R_1I_1 = 0$

loop 3: $R_2I_2 = 0$ (no flux enclosed)

From the loop 3 equation we see that $I_2 = 0$, which is why the bottom bulb doesn't glow.

With $I_2 = 0$, the node N rule says that $I_1 = I_3$: all the current in the top bulb goes through the copper wire (and none through the bottom bulb). Both the loop 1 and loop 2 equations reduce to $R_1I_1 = \text{emf}$, so the current through the top bulb is $I_1 = \text{emf}/R_1 = (\text{emf}_0/R_1)\cos(\omega t)$, which is nearly twice what it was before we added the extra copper wire ($R_1 > R$ due to higher temperature).

22.X.19 Here are four different ways to connect the copper wire. Based on the analysis we have just carried out, involving identifying whether or not there is a battery-like emf in a loop, what is the brightness of both bulbs in circuits 1, 2, 3, and 4?



*Coulomb and non-Coulomb electric fields in a nonuniform ring

In a circular ring encircling a solenoid with a time-varying magnetic field, there is current driven by a non-Coulomb electric field. There is no gradient of surface charge along the ring, although there is a small transverse polarization required to turn the electrons in a circle (top of Figure 22.49).

If there is a thin section of the ring, however, charge will pile up at the ends of the thin section, and there will be surface charges all along the ring (bottom of Figure 22.49). These surface charges will produce a Coulomb electric field $\vec{E}_C$ which adds vectorially to the non-Coulomb electric field $\vec{E}_{NC}$ that is due to the changing magnetic flux. The net electric field $\vec{E}_{NC} + \vec{E}_C$ is what drives the electrons around the ring.

Inside the thin section the net electric field is larger than it was for the uniform ring, and in the thick sections the net electric field is smaller than it was for the uniform ring.

? If both rings are made out of the same material, how do you know that the current must be smaller in the ring with the thin section than it was in the uniform ring?

In the thick section of the nonuniform ring, the only thing that changed is that the net electric field is smaller, so the current in the thick section must be smaller. (Of course this same smaller current runs through the thin section of this ring.)

Note that the Coulomb electric field $\vec{E}_C$ is large and goes clockwise in the thin section, but $\vec{E}_C$ is small and goes counterclockwise in the thick section. Since $\vec{E}_C$ is due to point charges, the round-trip integral of $\vec{E}_C$ is zero.

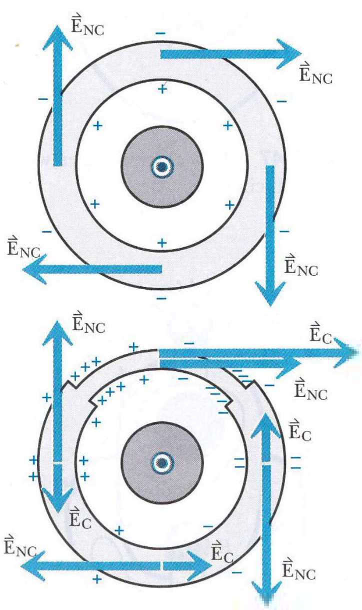


Figure 22.49 A uniform metal ring, and a metal ring with a thin section.

On the other hand, the non-Coulomb electric field $\vec{E}_{NC}$ due to the changing magnetic flux goes clockwise all around the ring, and its round-trip integral is emf $= d\Phi_{mag}/dt$, which is not zero.

*Two competing effects in a shrinking ring

Suppose we stretch a springy metal ring of radius R and place it in a region of uniform magnetic field B pointing out of the page, which is increasing at a rate of dB/dt (Figure 22.50). When we let go of the ring, it contracts at a rate dR/dt (which is a negative number). What is the emf around the metal ring?

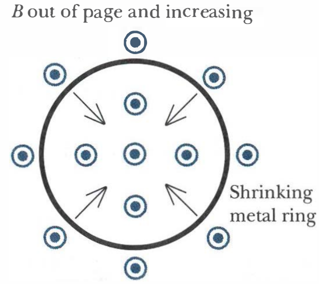


Figure 22.50 A springy metal ring that had been stretched and is now shrinking.

In terms of flux, there are two competing effects here. The increasing magnetic field B tends to increase the flux, but the decreasing area tends to decrease the flux. Whether or not a current will flow depends on the relative magnitudes of these two contributions to the net flux.

Since the magnetic field is the same throughout the ring and is perpendicular to the page, the flux through the ring at any instant is $\Phi_{mag} = B(\pi R^2)$. The emf is the rate of change of the flux:

$$\text{emf} = \frac{d\Phi_{mag}}{dt} = \frac{d}{dt}[B(\pi R^2)] = \frac{dB}{dt}(\pi R^2) + B\frac{d(\pi R^2)}{dt}$$

$$\text{emf} = \frac{dB}{dt}(\pi R^2) + B(2\pi R)\frac{dR}{dt}$$

What is the physical significance of these two contributions to the emf? Because there is a changing magnetic field, there is a clockwise non-Coulomb electric field in the wire. The first term, involving dB/dt, is associated with this non-Coulomb electric field in the wire.

Because the wire is moving inward, there is a magnetic force on the electrons in the wire. In this case, $\vec{v} \times \vec{B}$ (the magnetic force per unit charge) is counterclockwise in the ring. The second term, involving the negative quantity dR/dt, is associated with the magnetic force on the moving pieces of the ring (motional emf). Note that $|dR/dt|$ is the speed of pieces of ring toward the center, and $2\pi r$ is the circumference, so the contribution to the emf is similar to emf $= BLv$ for a rod of length L sliding on rails.

Note that if $(dB/dt)(\pi R^2) = B(2\pi R)|(dR/dt)|$, the emf would be zero even though we had both a changing magnetic field and a changing area. In this case the magnetic force on an electron inside the ring is equal and opposite to the non-Coulomb electric force, due to the non-Coulomb electric field.

There is no surface charge gradient around this symmetrical ring, and no Coulomb electric field around the ring. The forces on electrons in the metal ring are due to the non-Coulomb electric field and to the non-Coulomb magnetic force on the moving pieces of the ring.

22.9 *THE DIFFERENTIAL FORM OF FARADAY'S LAW

As with Gauss's law and Ampere's law, there is a differential form of Faraday's law, valid at a particular location and time. Using properties of vector calculus one can show that the integral form of Faraday's law is equivalent to the following, where the derivative is a "partial derivative" with respect to time (holding position constant):

DIFFERENTIAL FORM OF FARADAY'S LAW

$$\text{curl}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \quad \text{or} \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

22.10 *LENZ'S RULE

With the thumb of your right hand pointing in the direction of $-d\vec{B}_1/dt$, your fingers curl around in the direction of $\vec{E}_{NC}$. Another rule, called "Lenz's rule," can be used to get the direction of the non-Coulomb electric field. We describe Lenz's rule briefly, although it is unnecessary to learn and use Lenz's rule in this introductory textbook on electricity and magnetism, for reasons that we will give below.

Imagine that you place a wire around the changing flux, so that the non-Coulomb electric field drives current in the wire:

LENZ'S RULE

The induced (non-Coulomb) electric field would drive current in a direction to make a magnetic field that attempts to keep the flux constant.

To see how this works, consider the first example in Figure 22.51, where $\vec{B}_1$ points out of the page and is increasing, and the non-Coulomb electric field curls clockwise around the solenoid. If a wire encircles the solenoid, conventional current runs clockwise in the wire, and this induced current produces an additional magnetic field in the region, pointing into the page.

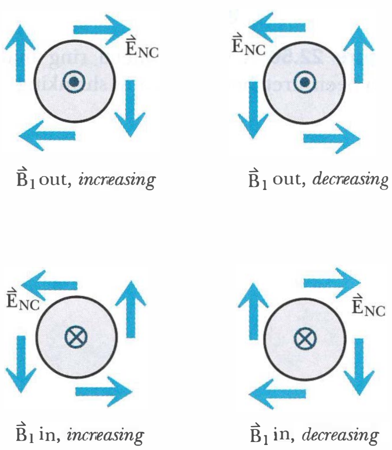


Figure 22.51 Using Lenz's rule to determine the direction of the non-Coulomb electric field.

This additional magnetic field is in the opposite direction to the change in $\vec{B}_1$, so we say that the induced magnetic field attempts to keep the magnetic flux constant despite the increase in B_1. The induced current does not succeed at keeping the net flux constant (unless the wire is made of superconducting material). In particular, if B_1 is increasing at a constant rate, the current I_2 in the ring and the induced magnetic field are constant by Faraday's law, so the net flux does increase.

If on the other hand $\vec{B}_1$ points out of the page and is decreasing (the second example in Figure 22.51), there is a counterclockwise non-Coulomb electric field that would drive conventional current counterclockwise in a wire, which would make a magnetic field out of the page, in the direction to attempt to keep the magnetic flux constant despite the decrease in B_1. (Again, the net flux will nevertheless decrease, unless the ring is superconducting.)

? Go through this analysis for the third and fourth examples shown on the previous page, and convince yourself that Lenz's rule does correctly summarize the experimental data for the direction of the non-Coulomb electric field.

An interesting property of Lenz's rule is that it also correctly gives the direction of current in the case of motional emf, or in situations where both a time-varying magnetic field and motion contribute to a change in the flux enclosed by a part of a circuit.

We have not emphasized Lenz's rule for two reasons. We have emphasized time-varying magnetic field and non-Coulomb electric field (rather than just time-varying flux and emf), to give a stronger sense of mechanism and to distinguish the effects of time-varying magnetic fields from the effects of motion of a wire in a magnetic field. Also, it is very easy to make serious conceptual mistakes using Lenz's rule when first studying the effect of time-varying magnetic fields, due to a natural tendency to try to use it for predicting the magnitude of the induced current flow, not just its direction. In particular, except for superconductors, it is not true that "enough current runs to cancel the change in the magnetic field."

22.11 SUMMARY

Faraday's law

The new phenomenon introduced in this chapter is that a time-varying magnetic field is accompanied by a non-Coulomb electric field that curls around the region of varying field. Unlike the Coulomb electric field produced by point charges, the non-Coulomb electric field has a nonzero round-trip path integral if the path encircles a time-varying flux.

Faraday's law

$$\text{emf} = -\frac{d\Phi_{mag}}{dt}$$

$$\text{where emf} = \oint \vec{E}_{NC} \bullet d\vec{l} \text{ and } \Phi_{mag} \equiv \int \vec{B} \bullet \hat{n}\, dA$$

In words: The induced emf along a round-trip path is equal to the rate of change of the magnetic flux on the area encircled by the path.

Direction: With the thumb of your right hand pointing in the direction of $-d\vec{B}/dt$, your fingers curl around in the direction of $\vec{E}_{NC}$.

Formal version of Faraday's law

$$\oint \vec{E}_{NC} \bullet d\vec{l} = -\frac{d}{dt}\left[\int \vec{B} \bullet \hat{n}\, dA\right]$$

(sign given by right-hand rule)

Faraday's law also covers the case where the flux changes due to changes in the path ("motional emf"), and the non-Coulomb force in this case is a magnetic force.

In general, there can be any combination of three effects acting on electrons in a circuit:

1) Coulomb electric field due to surface charges;
2) Non-Coulomb electric field due to time-varying magnetic field; and
3) Magnetic forces on moving portions of the circuit (motional emf).

Superconductors

Superconductors have zero resistance, and a consequence of this is that a superconducting ring maintains a constant flux in the area enclosed by the ring. The magnetic field inside a superconductor is always zero (the Meissner effect), and this important property cannot be explained merely in terms of zero resistance.

Inductance

When you vary the current in a coil, the varying flux enclosed by the coil induces an emf in the *same* coil which makes it difficult to change the current. This sluggishness plays a role in RL and LC circuits, and it is quantified by the "inductance" L of the coil. The emf is proportional to the rate of change of the magnetic flux, which is proportional to the rate of change of the current:

$$|\text{emf}_{ind}| = L\left|\frac{dI}{dt}\right|$$

We calculated the inductance in one particular case, that of a long solenoid:

$$L_{solenoid} = \frac{\mu_0 N^2}{d}\pi R^2$$

Electric and magnetic energy densities

$$\frac{\text{Energy}}{\text{Volume}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{1}{\mu_0}B^2, \text{ measured in joules/m}^3$$

RL (resistor-inductor) circuits

The current in an RL series circuit varies with time:

$$I = \frac{\text{emf}_{battery}}{R}\left[1 - e^{-\left(\frac{R}{L}\right)t}\right]$$

LC (inductor-capacitor) circuits

An LC circuit oscillates sinusoidally with a frequency $f = 1/(2\pi\sqrt{LC})$:

$$Q = Q_i \cos\left(\frac{1}{\sqrt{LC}}t\right)$$

If there is no resistance, an LC circuit never reaches a final equilibrium or a final steady state.

**The differential form of Faraday's law*

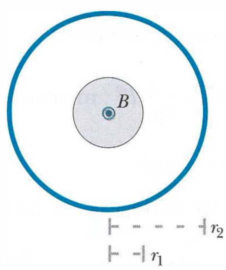
$$\text{curl}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \text{ or } \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

22.12 REVIEW QUESTIONS

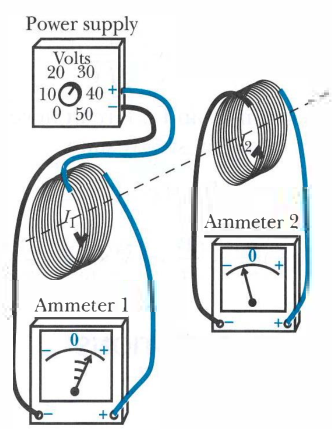
Faraday's law

22.RQ.20 The north pole of a bar magnet points toward a thin circular coil of wire containing 40 turns. The magnet is moved away from the coil, so that the flux through one turn inside the coil decreases by 0.3 tesla·m^2 in 0.2 seconds. What is the average emf induced in the coil during this time interval? Viewed from the right side (opposite the bar magnet), does the induced current run clockwise or counterclockwise? Explain briefly.

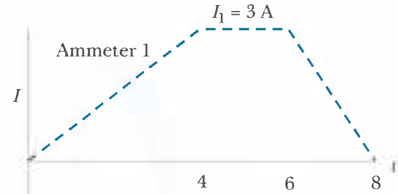
22.RQ.21 In a circular region of radius r_1 there is a uniform magnetic field B pointing out of the paper (the magnetic field is essentially zero outside this region). The magnetic field is *decreasing* at this moment at a rate $|dB/dt|$. A wire of radius r_2 and resistance R lies entirely *outside* the magnetic-field region (where there is no magnetic field). In which direction does conventional current I flow in the ring? What is the magnitude of the current I? Show the pattern of (non-Coulomb) electric field in the ring. What is the magnitude E_{NC} of the non-Coulomb electric field?



22.RQ.22 Two coils of wire are near each other, positioned on a common axis. Coil 1 is connected to a power supply whose output voltage can be adjusted by turning a knob, so that the current I_1 in coil 1 can be varied, and I_1 is measured by ammeter 1.



Current I_2 in coil 2 is measured by ammeter 2. The ammeters have needles that deflect positive or negative depending on the direction of current passing through the ammeter, and ammeters read positive if conventional current flows into the "+" terminal. Here is a graph of I_1 vs. time.



Draw a graph of I_2 vs. time over the same time interval. Explain your reasoning.

Inductance

22.RQ.23 A thin coil has 12 rectangular turns of wire. When a current of 3 amperes runs through the coil, there is a total flux of 10^{-3} tesla·m^2 enclosed by one turn of the coil (note that $\Phi = kI$, and you can calculate the proportionality constant k). Determine the inductance in henries.

22.RQ.24 Would the inductance of a solenoid be larger or smaller if the solenoid is filled with iron? Explain briefly.

RL and LC circuits

22.RQ.25

(a) When an RL circuit has been connected to a 1.5-volt battery for a very *short* time, what is the potential difference from one end of the resistor to the other?

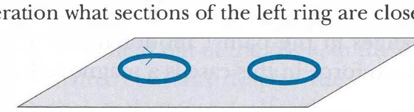
(b) When an RL circuit has been connected to a 1.5-volt battery for a very *long* time, what is the potential difference from one end of the resistor to the other?

(c) Explain briefly the difference between equilibrium, steady-state current, and the behavior of an LC circuit.

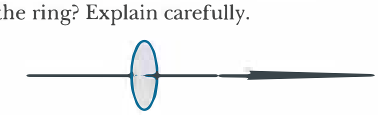
22.13 PROBLEMS

22.P.26 Some small problems on time-varying magnetic fields

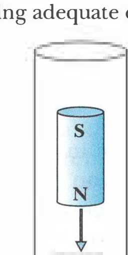
(a) Two metal rings lie side-by-side on a table. Current in the left ring runs clockwise and is increasing with time. This induces a current to run in the right ring. Does this induced current run clockwise or counterclockwise? Explain, using a diagram. (Hint: Think carefully about the direction of magnetic field in the right ring produced by the left ring, taking into consideration what sections of the left ring are closest.)



(b) A very long straight wire (essentially infinite in length) carries a current of 6 ampere. The wire passes through the center of a circular metal ring of radius 4 cm and resistance 2 ohms that is perpendicular to the wire. If the current in the wire increases at a rate of 0.25 ampere/s, what is the current induced in the ring? Explain carefully.

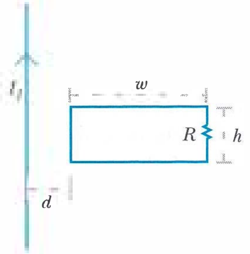


(c) A bar magnet is dropped through a vertical copper tube and is observed to fall very slowly, despite the fact that mechanical friction between the magnet and the tube is negligible. Explain carefully, including adequate diagrams.



22.P.27 A wire and a rectangular loop

A very long wire carries a current I_1 *upward* as shown, and this current is *decreasing with time*. Nearby is a rectangular loop of wire lying in the plane of the long wire, and containing a resistor R; the resistance of the rest of the loop is negligible compared to R. The loop has a width w and height h, and is located a distance d to the right of the long wire.



(a) Does the current I_2 in the loop run clockwise or counterclockwise? Explain, using a diagram.

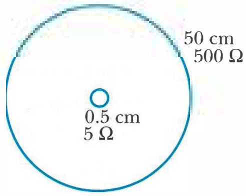
(b) Show that the magnitude of the current I_2 in the loop is this:

$$I_2 = \frac{\mu_0 (2h) \ln[(d+w)/d]}{4\pi R} \left| \frac{dI_1}{dt} \right|$$

(Hint: divide the area into narrow vertical strips along which you know the magnetic field.)

22.P.28 Two concentric metal rings

A very small circular metal ring of radius $r = 0.5$ cm and resistance $x = 5$ ohms is at the center of a large concentric circular metal ring of radius $R = 50$ cm and resistance $X = 500$ ohms. The two rings lie in the same plane. At $t = 3$ seconds, the large ring carries a *clockwise* current of 3 ampere. At $t = 3.2$ seconds, the large ring carries a *counterclockwise* current of 5 ampere.



(a) What is the average electric field induced in the small ring during this time? Give both the magnitude and direction of the electric field. Draw a diagram, showing all relevant quantities.

(b) What are the magnitude and direction of the average current in the small ring?

22.P.29 Design a circuit

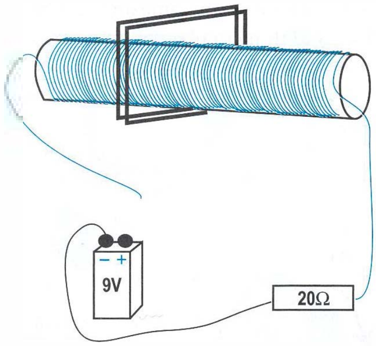
It is now possible to buy capacitors that have a capacitance of one farad.

(a) Design a solenoid so that when it is connected to a charged one-farad capacitor, the circuit will oscillate with a period of one second. Give all the relevant parameters of the solenoid (length, etc.) so that someone could build the solenoid from your design specifications. Assume that there is wire available with low enough resistance that the resistance of the solenoid is negligible, although this may be difficult to achieve in practice unless the wire is superconducting.

(b) If the one-farad capacitor is initially charged to 3 volts, what is the maximum current that will run through the inductor?

22.P.30 Connect a battery to a solenoid

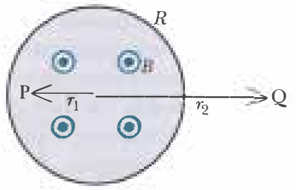
A cylindrical solenoid 40 cm long with a radius of 5 mm has 300 tightly-wound turns of wire uniformly distributed along its length. Around the middle of the solenoid is a two-turn rectangular loop 3 cm by 2 cm made of resistive wire having a resistance of 150 ohms.



One microsecond after connecting the loose wire to the battery to form a series circuit with the battery and a 20-ohm resistor, what is the magnitude of the current in the rectangular loop and its direction (clockwise or counterclockwise in the diagram)?

22.P.31 Induced electric field

The magnetic field is uniform and out of the page inside a circle of radius R. The magnetic field is essentially zero outside the circular region. The magnitude of the magnetic field as a function of time is $(B_0 + bt^3)$. B_0 and b are positive constants, and t = time.



(a) What are the direction and magnitude of the induced electric field at location P, at a distance r_1 to the left of the center ($r_1 < R$)?

(b) What are the direction and magnitude of the induced electric field at location Q, at a distance r_2 to the right of the center ($r_2 > R$)?

22.P.32 Magnetic monopoles

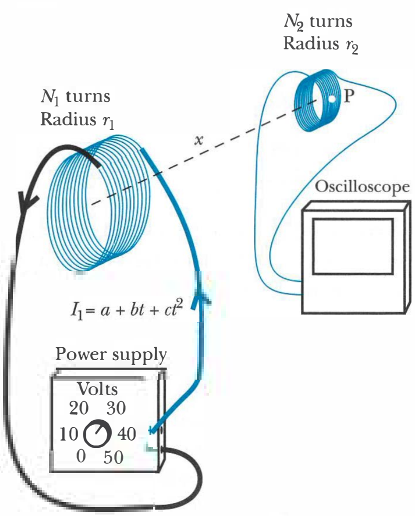
One of the methods physicists have used to search for magnetic monopoles is to monitor the current produced in a loop of wire. Draw graphs of current in the loop vs. time for an electrically uncharged magnetic monopole passing through the loop, and for an electrically uncharged magnetic dipole (such as a neutron) passing through the loop with its north end headfirst. Don't worry too much about the details of the exact moment when the particle goes through the plane of the loop;

concentrate on the times just before and just after this event. Explain the differences in the two graphs.

(A signal corresponding to a magnetic monopole was seen once in such an experiment, but no one has been able to reproduce this result, and most physicists seem to believe that the supposed event was due to extraneous noise in the system or other malfunction of the apparatus.)

22.P.33 A coil attached to an oscilloscope

A thin circular coil of radius r_1 with N_1 turns carries a current $I_1 = a + bt + ct^2$, where t is the time and a, b, and c are positive constants. A second thin circular coil of radius r_2 with N_2 turns is located a long distance x along the axis of the first coil. The second coil is connected to an oscilloscope, which has very high resistance.



Observing with an oscilloscope.

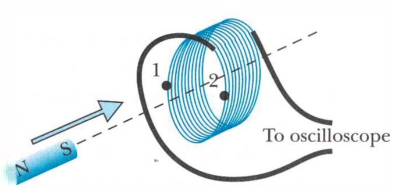
(a) As a function of time t, calculate the magnitude of the voltage that is displayed on the oscilloscope. Explain your work carefully, but you do not have to worry about signs.

(b) At point P on the drawing (on the right side of the second coil), draw a vector representing the non-Coulomb electric field.

(c) Calculate the magnitude of this non-Coulomb electric field.

22.P.34 A bar magnet and a coil

A coil with 3000 turns and radius 5 cm is connected to an oscilloscope. You move a bar magnet toward the coil along the coil's axis, moving from 40 cm away to 30 cm away in 0.2 seconds. The bar magnet has a magnetic moment of 0.8 ampere·m^2.



A bar magnet is moved toward a coil.

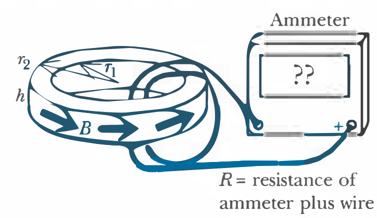
(a) On the diagram, draw the non-Coulomb electric field vectors at locations 1 and 2. Briefly explain your choices graphically.

(b) What is the approximate magnitude of the signal observed on the oscilloscope?

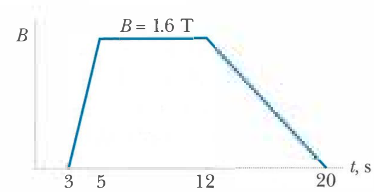
(c) What approximations or simplifying assumptions did you make?

22.P.35 Toroid

A toroid has a rectangular cross section with an inner radius r_1 = 9 cm, an outer radius r_2 = 12 cm, and a height h = 5 cm, and it is wrapped around by many densely-packed turns of current-carrying wire (not shown in the diagram). The direction of the magnetic field inside the windings is shown on the diagram. There is essentially no magnetic field outside the windings. A wire is connected to a sensitive ammeter as shown. The resistance of the wire and ammeter is R = 1.4 ohms.



The current in the windings of the toroid is varied so that the magnetic field inside the windings, averaged over the cross section, varies with time as shown:



Make a careful graph of the ammeter reading, including sign, as a function of time. Label your graph, and explain the numerical aspects of the graph, including signs.

22.P.36 The betatron

Suppose you have an electron moving with speed comparable to the speed of light in a circular orbit of radius r in a large region of uniform magnetic field B.

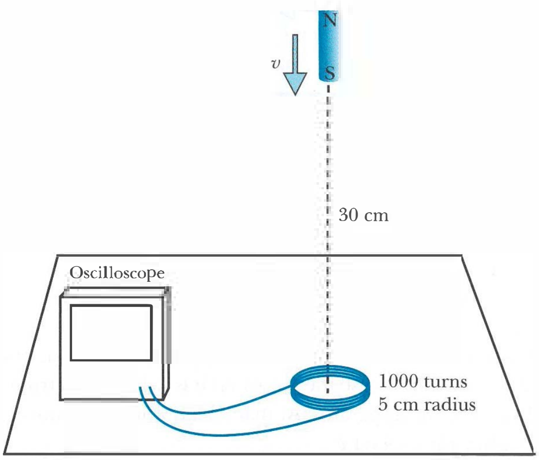
(a) What must be the relativistic momentum p of the electron?

(b) Now the uniform magnetic field begins to increase with time: $B = B_0 + bt$, where B_0 and b are positive constants. In one orbit, how much does the energy of the electron increase, assuming that in one orbit the radius doesn't change very much? (This effect was exploited in the "betatron," an electron accelerator invented in the 1940s.)

Note: it turns out that the electron's energy increases by less than the amount you calculated, for reasons that will become clear in Chapter 23.

22.P.37 Throw a bar magnet

You throw a bar magnet downward with its south end pointing down. The bar magnet has a magnetic dipole moment of 1.2 A·m^2. Lying on the table is a nearly flat circular coil of 1000 turns of wire, with radius 5 cm. The coil is connected to an oscilloscope, which has a very large resistance.

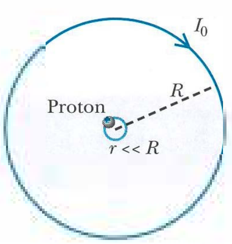


(a) On a diagram, show the pattern of non-Coulomb electric field in the coil. Explain briefly.

(b) At the instant when the magnet is 30 cm above the table, the oscilloscope indicates a voltage of magnitude 2 millivolts. What is the speed v of the magnet at that instant?

22.P.38 A proton in a magnetic field

A single circular loop of wire with radius R carries a large clockwise constant current $I_{loop} = I_0$, which constrains a proton of mass M and charge e to travel in a small circle of radius r at constant speed around the center of the loop, in the plane of the loop. The orbit radius r is much smaller than the loop radius R: $r << R$.



(a) Draw a diagram of the proton orbit, indicating the direction that the proton travels, clockwise or counterclockwise. Explain briefly.

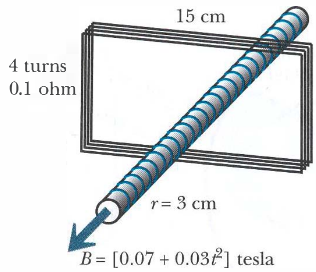
(b) What is the speed v of the proton, in terms of the known quantities I_0, R, r, M, and e? Explain your work, including any approximations you must make.

(c) The current was constant for a while, but at a certain time $t = t_0$ it began to decrease slowly, so that after t_0 the current was $I_{loop} = I_0 - k(t - t_0)$. On your diagram of the proton orbit, draw electric field vectors at four locations (one in each quadrant) and explain briefly.

(d) When the current decreases, does the proton speed up or slow down?

22.P.39 A rectangular loop

A very long, tightly-wound solenoid has a circular cross section of radius 3 cm (only a portion of the very long solenoid is shown in the diagram). The magnetic field outside the solenoid is negligible. Throughout the inside of the solenoid the magnetic field B is uniform, to the left as shown, but varying with time t: $B = [0.07 + 0.03t^2]$ tesla. Surrounding the circular solenoid is a loop of 4 turns of wire in the shape of a rectangle 10 cm by 15 cm. The total resistance of the 4-turn loop is 0.1 ohm.



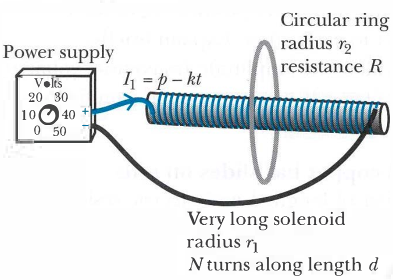
A long solenoid inside a rectangular coil.

(a) At $t = 2$ seconds, what is the direction of the current in the 4-turn loop? Explain briefly.

(b) At $t = 2$ seconds, what is the magnitude of the current in the 4-turn loop? Explain briefly.

22.P.40 Solenoid and ring

A very long solenoid of length d and radius r_1 is tightly wound uniformly with N turns of wire. A variable power supply forces a current to run in the solenoid of amount $I_1 = p - kt$, where p and k are positive constants (so that the current is initially equal to p but decreases with time). A circular metal ring of radius r_2 and resistance R is centered on the solenoid and located near the middle of the solenoid, very far from the ends of the solenoid (so that the solenoid contributes essentially no magnetic field outside the solenoid).



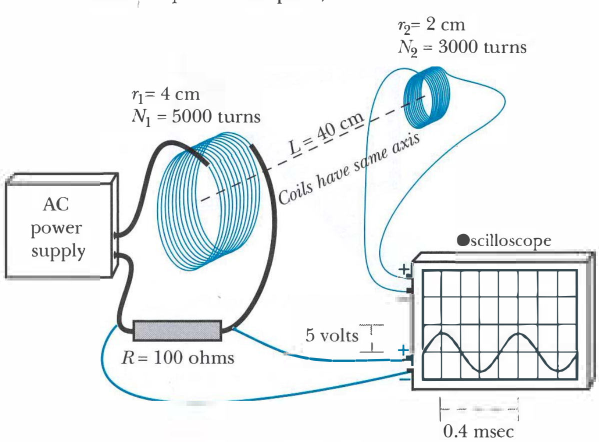
A metal ring around a long solenoid.

(a) What are the magnitude and the direction of the magnetic field at the center of the ring, due to the solenoid and the ring, as a function of the time t? Explain your work; part of the credit will be given for the clarity of your explanation, including clarity of appropriate diagrams.

(b) Qualitatively, how would these results have been affected if an iron rod had been inserted into the solenoid? Very briefly, why?

22.P.41 Oscilloscope and coils

A thin coil of radius r_1 = 4 cm, containing N_1 = 5000 turns, is connected through a resistor R = 100 ohms to an AC power supply running at a frequency f = 2500 hertz, so that the current through the resistor (and coil) is $I_1 \sin(2\pi \times 2500t)$. The voltage across the resistor triggers an oscilloscope which also displays this voltage, which is 10 volts peak-to-peak and therefore has an amplitude of 5 volts (and therefore the amplitude of the current is I_1 = 0.05 ampere).



Oscilloscope and coils.

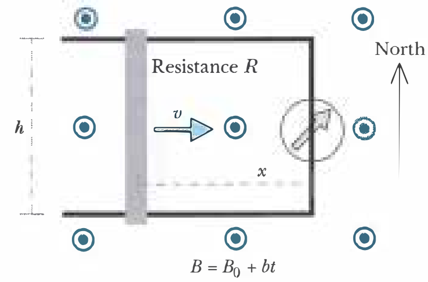
A second thin coil of radius r_2 = 2 cm, containing N_2 = 3000 turns, is a distance L = 40 cm from the first coil. The axes of the two coils are along the same line. The second coil is connected to the upper input of the oscilloscope, so that the voltage across the second coil can be displayed along with the voltage across the resistor.

(a) Assume you have adjusted the VOLTS/DIVISION knob on the upper input so that you can easily see the signal from the second coil. Sketch the second coil voltage along with the resistor voltage on the oscilloscope display, paying careful attention to the shape and positioning of the two voltage curves with respect to each other. Explain briefly.

(b) Calculate the amplitude (maximum voltage) of the second coil voltage. If you must make simplifying assumptions, state clearly what they are.

22.P.42 A copper bar slides on rails

A copper bar of length h and electric resistance R slides with negligible friction on metal rails which have negligible electric resistance. The rails are connected on the right with a wire of negligible electric resistance, and a magnetic compass is placed under this wire (the diagram is a top view). The compass needle deflects to the right of north, as shown on the diagram.



A copper bar slides on rails.

Throughout this region there is a uniform magnetic field B pointing out of the page, produced by large coils which are not shown. This magnetic field is increasing with time, and the magnitude is $B = B_0 + bt$, where B_0 and b are constants, and t is the time in seconds. You slide the copper bar to the right and at time $t = 0$ you release the bar when it is a distance x from the right end of the apparatus. At that instant the bar is moving to the right with a speed v.

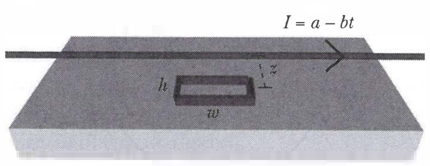
(a) Calculate the magnitude of the initial current I in this circuit.

(b) Calculate the magnitude of the net force on the bar just after you release it.

(c) Will the bar speed up, slow down, or slide at a constant speed? Explain briefly.

22.P.43 A wire and a rectangular loop

A thin rectangular coil lies flat on a low-friction table. A very long straight wire also lying flat on the table, a distance z from the coil, carries a conventional current I to the right as shown, and this current is decreasing: $I = a - bt$, where t is the time in seconds, and a and b are positive constants. The coil has width w and height h, where $h \ll z$. It has N turns of wire with total resistance R.



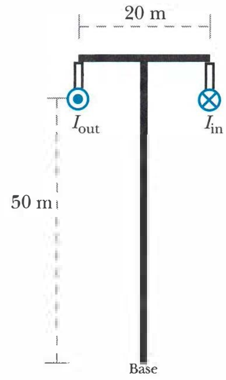
A thin rectangular coil lies on a low-friction table a distance z from a very long current-carrying wire. Not to scale: $h \ll z$.

What are the initial magnitude and direction of the nonzero net force that is acting on the coil? Explain in detail. If you must make simplifying assumptions, state clearly what they are, but bear in mind that the net force is not zero.

22.P.44 Power lines

Tall towers support power lines 50 m above the ground and 20 m apart that run from a hydroelectric plant to a large city, carrying 60-hertz alternating current with amplitude 10^4 ampere.

That is, the current in both of the power lines is $I = (10^4 \text{ A})\sin(2\pi \cdot 60 \text{ hertz} \cdot t)$.



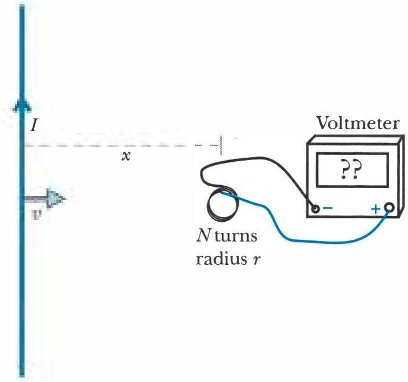
Power lines carrying alternating current.

(a) Calculate the amplitude (largest magnitude) and direction of the magnetic field produced by the two power lines at the base of the tower, when a current of 10^4 ampere in the left power line is headed out of the page, and a current of 10^4 ampere in the right power line is headed into the page.

(b) This magnetic field is not large compared to the Earth's magnetic field, but it varies in time and so might have different biological effects than the Earth's steady field. For a person lying on the ground at the base of the tower, approximately what is the maximum emf produced around the perimeter of the body, which is about 2 meter long by half a meter wide?

22.P.45 A current-carrying wire moves toward a coil

A long straight wire carrying current I is moving with speed v toward a small circular coil of radius r containing N turns, which is attached to a voltmeter as shown. The long wire is in the plane of the coil. (Only a small portion of the wire is shown in the diagram.)

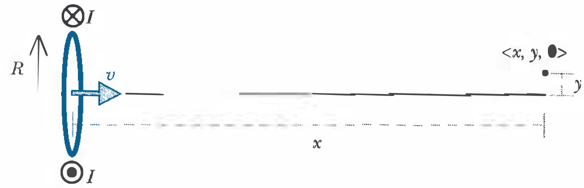


A current-carrying wire moves toward a coil.

At the instant when the long wire is a distance x from the center of the coil, what is the voltmeter reading? Include both magnitude and sign. (Remember that a voltmeter reads "+" if the higher potential is connected to the "+" terminal of the voltmeter.) Explain. State what approximations or simplifying assumptions you make.

22.P.46 A moving superconducting ring

A superconducting ring of radius R has a permanent conventional current I in the direction shown and is oriented with its axis along the x axis. The ring is moving to the right along the x axis with uniform speed v. Calculate the electric field $\vec{E}$ (magnitude and direction) at location $\langle x, y, 0\rangle$ relative to the center of the ring. If you must make any simplifying assumptions or approximations, state them explicitly.



A moving superconducting ring.

22.14 ANSWERS TO EXERCISES

22.X.1 (page 778) Clockwise.

22.X.2 (page 779) Counterclockwise.

22.X.3 (page 780) (1) no current. (2) current.

22.X.4 (page 780) 47.7 V/m

22.X.5 (page 780) 7.5 amperes

22.X.6 (page 781) 0.052 tesla·m^2

22.X.7 (page 782) 20 volts; 8 V/m; 2 amperes

22.X.8 (page 784) 7.9×10^{-4} volts ; 1.3×10^{3} V/m

22.X.9 (page 784) 200 volts

22.X.10 (page 784) 10 amperes; 20 times emf, but 20 times resistance

22.X.11 (page 785) Ammeter reading is positive. Remember that an ammeter reads positive if conventional current flows into the "+" terminal of the ammeter.

22.X.12 (page 785) Ammeter reading is negative.

22.X.13 (page 785) Current in loop runs clockwise, seen from above.

22.X.14 (page 788) 0

22.X.15 (page 788) 0.166 volts

22.X.16 (page 792) The flux must still be equal to Φ_0. Since the magnet is too far away to contribute, all the flux must be due to the induced current in the ring.

22.X.17 (page 794) 4×10^{-3} henry

22.X.18 (page 798) 5000 hertz

22.X.19 (page 800)

In the first three cases, for the bulb labeled "bright" a loop can be drawn which consists of just that bulb plus a wire which encircles the varying magnetic flux; for the bulb labeled "off" a loop can be drawn which consists of just that bulb plus a wire which does not encircle the varying magnetic flux. 1) lower bulb bright, upper off; 2) upper bulb bright, lower off; 3) upper bulb bright, lower off.

For the fourth case, neither bulb is in a loop that doesn't encircle the varying magnetic flux. The current is smaller than in the other circuits, because the induced emf drives two bulbs in series.