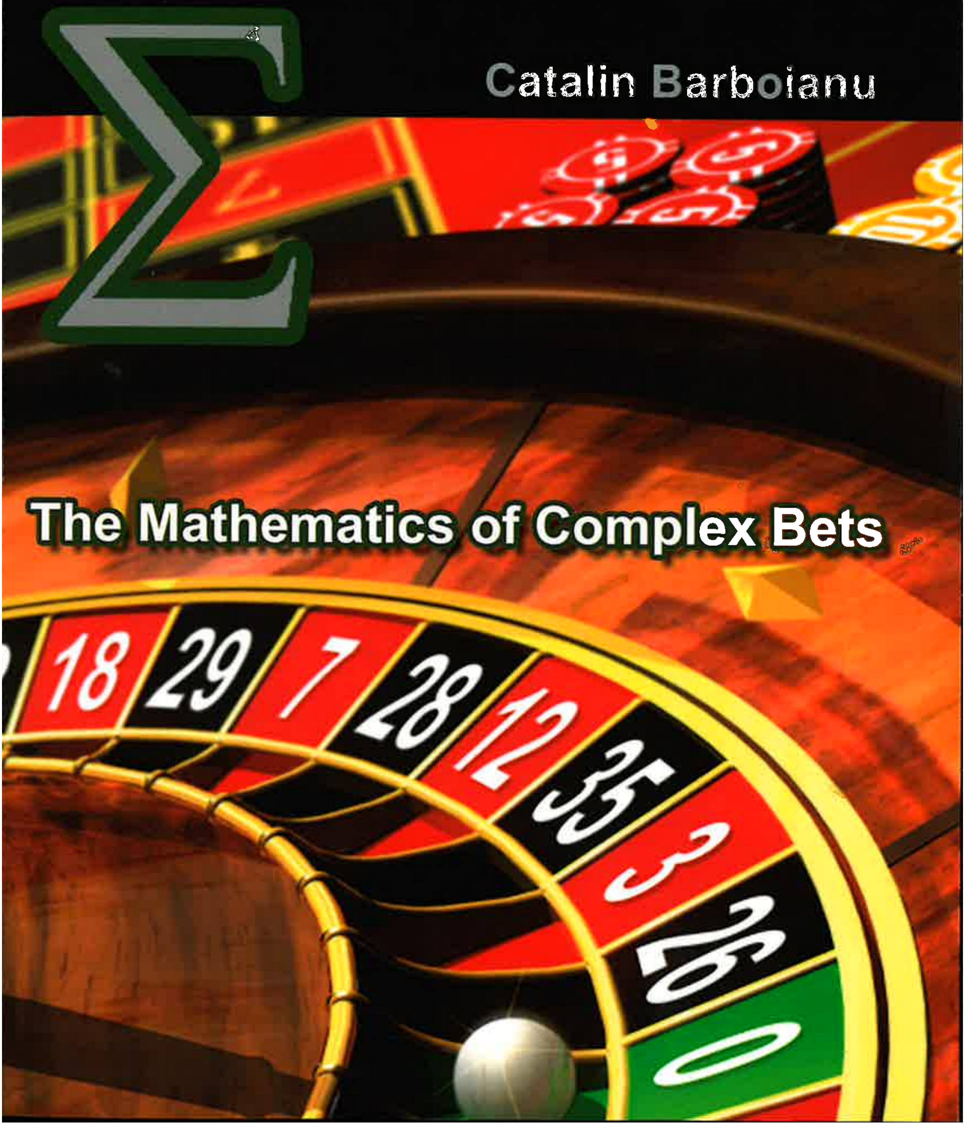
ROULETTE ODDS AND PROFITS

Catalin Barboianu



The Mathematics of Complex Bets

ROULETTE ODDS AND PROFITS

The Mathematics of Complex Bets

$$\Sigma$$

$$\Pi$$

Cătălin Bărboianu

Cătălin Bărboianu
PO Box 1 – 230
Craiova, Romania
http://probability.infarom.ro

INFAROM Publishing
Pure and applied mathematics
office@infarom.com
http://books.infarom.ro
http://www.infarom.com
http://www.infarom.ro

ISBN-10: 973-87520-7-8
ISBN-13: 978-973-87520-7-8

Publisher: **INFAROM**
Author and translator: **Cătălin Bărboianu**
Correction Editor: **Sara M. Stohl**

Contents

Introduction

Roulette has long been the most popular game of chance. Its popularity comes not only from its history and rules, but is also related to its mathematics.

The first form of roulette was devised in 18th century France.

The game has been played in its current form since as early as 1796 in Paris. The earliest description of the roulette game in its current form is found in a French novel "La Roulette, ou le Jour" by Jaques Lablee, which describes a roulette wheel in the Palais Royal in Paris in 1796. The book was published in 1801. An even earlier reference to a game of this name was published in regulations for French Quebec in 1758.

In 1842, Frenchmen François and Louis Blanc added the "0" to the roulette wheel in order to achieve a house advantage. In the early 1800s, roulette was brought into the U.S. where, to further increase house odds, a second zero, "00", was introduced. In some forms of early American roulette wheels, there were numbers 1 through 28, plus a single zero, a double zero, and an American Eagle. The payout on any of the numbers including the zeros and the eagle was 27 to 1. In the 1800s, roulette spread all over both Europe and the U.S., becoming one of the most famous and most popular casino games.

A legend tells François Blanc supposedly bargained with the devil to obtain the secrets of roulette. The legend is based on the fact that the sum of all the numbers on the roulette wheel (from 1 to 36) is 666, which is the "Number of the Beast".

Of course, this history has nothing to do with the mathematics of roulette, although roulette offers the most relevant experiments in demonstrating the basic applications of probability theory. Added to which, the interpretations of probability regarding predictions have always carried a sense of the mystical.

One thing that makes roulette so popular with gamblers is the game's transparency. All its parts are in view: the numbers on the table, on the wheel, the ball spinning and landing; no hidden cards to guess, no opponents to read their intentions, no strategy to influence the course of each game. We just place our bet and wait for the ball to land.

This transparency also allows for easy calculation of the odds involved, which also contributes to the game's popularity.

Unlike poker, in which only mathematicians can calculate the odds of some categories of events, in roulette any player can quickly calculate and memorize the probabilities of winning and losing any simple bet, and even some complex bets.

Still, for a proper betting system (one that is non-contradictory and profitable), not only the odds must be known, but also the right structure of the complex bets and the stake management.

And that is what this book deals with.

Any roulette player who plays regularly knows that each payout is approximately inversely proportional to the probability of winning that bet, regardless of the house edge. This makes roulette a fair enough game.

Another important attraction is the unrestricted possibility of combinations for the bets placed on the table.

A player may place bets wherever he or she wants, according to his or her own betting systems and criteria.

Some of the improved complex bets can increase the winning probability to over 90%, as you will see in this book. This is a winning probability you won't see in other game of chance.

All these factors make roulette the most popular game and a possible way to make living for many gamblers, even if only for brief periods of time.

Statistically, the most regular gamblers playing the same game are among roulette players.

It has been mathematically proved that, in ideal conditions of randomness, no long-run regular winning is possible for players of games of chance; therefore, gambling is not a good option as a way of making a living.

Most gamblers accept this premise, but still work on strategies to make them win over the long run.

Play using a long-run strategy to achieve a cumulated positive result means ignoring the randomness and skipping the experiments giving negative results.

This strategy is possible only if a player has access to some paranormal information—someone has to have prior knowledge and be able to tell the player when to play and when not to!

Until this magic help becomes possible, probability theory remains the only tool that provides some information about gaming events, even as an idealized relative frequency.

Gamblers also may be interested in isolated winnings, running the risk in a single game or in short-run play, with or without a gaming strategy.

No matter the chosen options and strategies, they will be interested in the amount of that risk and this means probability.

All these factors also mean that there is no optimal long-term strategy for playing roulette. Moreover, any strategy must include a player's personal criteria (playing time, amount of money at the player's disposal, level of acceptable risk, and so on), all of which make it subjective.

Any betting system will fail in the long run (even if *long run* theoretically means *infinity*). Whether you bet by using an elaborate system or just by placing simple bets, you might win in the short or even medium run, but you will cumulatively lose in the long run.

The best approach is to win enough early on with one great hit, then use an appropriate money management formula to give you the time and money to make the next big hit.

If this does not happen, a player may try to run complex bets by correlating their parameters (probabilities, basic stakes, profits and losses), and by factoring in the player's personal playing criteria to achieve regular profits in the short and medium terms.

We take that approach in this book: identify the complex bets that increase the overall winning probability, find the proper correlations between their parameters for the bets being non-contradictory and profitable and list all the numerical results in tables. From those tables, a player can choose the bets that fit the best his or her playing criteria.

This is not a roulette strategy book because such a strategy does not exist: only betting systems exist.

It is rather a collection of odds and figures attached to a large range of complex bets, revealed in their entire mathematical structure. This book provides just mathematical facts and not so-called winning strategies.

The structure and content of the major chapters follow.

The Rules of Roulette

This chapter gives readers the entire ensemble of rules of roulette: structure, betting, categories of bets, and payouts.

Supporting Mathematics

Here, the actions of roulette are converted into probability experiments that generate aleatory events. You will see the sample space, the field of events and the probability space in which the numerical probabilities of roulette are worked out.

We also present the probability properties and formulas used, as theoretical support.

In addition, you will see a mathematical model of a bet and the parameters a bet depends on.

We define complex bets and improved complex bets with respect to probability and to all parameters involved in a bet.

Anyone with a minimal mathematical background can follow this chapter because it requires only basic arithmetic and algebraic skills. On the other hand, readers who are only interested in direct results can skip this chapter and go to the tables of results to come.

Further, the next chapters present specific **improved complex bets**, each chapter dealing with one category of such bets.

All numerical results are presented in tables and the probability values are worked out for both American and European style roulette.

Repeated bets

The last chapter deals with repeated bets in the various categories, including the martingale, and lists the probabilities for each of the possible events involved in runs up to 100.

The Rules of Roulette

Roulette is a simple, easy to learn game. It offers a wide variety of bets and combinations of bets.

The Roulette wheel has 36 numbers from 1 to 36, a "0", and usually a "00". Most U.S. casinos have a "00" as well as the "0", so they have 38 numbers. This is called American roulette.

Most European casinos have only "0", without "00", so they have 37 numbers. This is called European roulette.

The players place bets on numbers or groups of numbers; a player's goal is to predict the winning number or other of properties of that number (colour, evenness, size, or place on the roulette table).

Each game consists of placing bets and waiting for a number, which is randomly generated by a spinning ball coming to rest inside a disk on which numbers are inscribed (the roulette wheel) that is also spinning, but in the opposite direction.

The bets are placed on the roulette table, which is designed to allow players to place paid chips on several combinations or sets of numbers.

The numbers are alternately coloured red and black with the "0" and "00" green.

Play begins when the players have placed most of their bets by placing chips on the numbered layout. The dealer then spins the white ball. Bets may be placed until the ball is ready to leave the track and fall onto the spinning wheel. At this point, the dealer will call "No more bets."

The ball then falls onto a number on the wheel, the dealer places a marker on the winning number and bets are paid accordingly. For a bet won, the dealer returns to the player the stake of that bet plus that amount multiplied by payout. For a lost bet, the dealer takes its stake from the table.

Chips (also known as "checks"), range in value and can be bought from the dealer.

Players can make as many bets as they wish at one game, but the stakes have their limits established by each casino in part. The table minimums are posted at each roulette table.

The players bet by placing their chips in one or several fields on the roulette table. There are many different bets that can be made on a roulette table:

Single Number (Straight Up): Any number on the table. (Example: 00, 5, 22, etc.)

2 - Number (Split): Placing a bet on the line dividing two adjoining numbers on the table. When this bet is placed, you are betting that on of the two numbers will come up. (Example: 13 and 14, 22 and 25.)

3 - Number (Street): Placing a bet on any three adjoining numbers on a table. To place this bet, place your chips on the line to the left of the first number in the series. (Example: 16, 17, 18. Bet would be placed on the left line of the box around the 16.)

4 - Number (Corner): Placing a bet on four numbers whose position on the table make a square. To place this bet, place your chips on the line in the center of the square. (Example: 11, 12, 14, 15. Bet would be placed in the middle of the square made up by these four numbers.)

6 - Number (Line): Placing a bet on six numbers made up of two rows of three numbers each. To place this bet, place your chips on the line to the left of the first number in the series and between the two rows of numbers. (Example: 31, 32, 33, 34, 35, 36. Bet would be placed on the line to the left of the 31 and 34 and on the line that divides the two rows.)

12 - Number (Dozen): There are three different ways to make this bet. You can either bet that the number that comes up will be "1st 12", "2nd 12" or "3rd 12". That is that the number will be in the first group of 12 numbers (1 - 12), the second group (13 - 24) or the third group (25 - 36). Note that none of these groups include the "0" or "00". To place this bet, place your chips in the section marked "1st 12", "2nd 12" or "3rd 12".

12 - Number (Column): There are three different ways to make this bet too. You can bet that the number that comes up will be in the first column (1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34), second column (2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35) or third column (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36). To place this bet, place your chips in the square at the bottom of the column you wish to bet on.

Depending on how the chips should be placed on roulette table, these bets are of two categories: inside and outside bets.

The next two tables note all possible bets, along with their brief description and corresponding payouts.

Inside bet	Description	Payout
Straight Up	A bet directly on any single number	35 to 1
Split Bet	A bet split between any two numbers	17 to 1
Street Bet	A bet on a row of 3 numbers	11 to 1
Corner Bet	A bet on 4 numbers	8 to 1
Line Bet	A bet on 6 numbers over 2 rows	5 to 1

Outside bet	Description	Payout
Column Bet	A bet covering 12 numbers from a column	2 to 1
Dozen Bet	A bet covering a set of 12 numbers low (1-12) mid (13-24) and high (25-36)	2 to 1
Colour Bet	A bet on either red or black	1 to 1
Even/Odd Bet	A bet on either even or odd	1 to 1
Low/High Bet	A bet on either 1-18 or 19-36	1 to 1

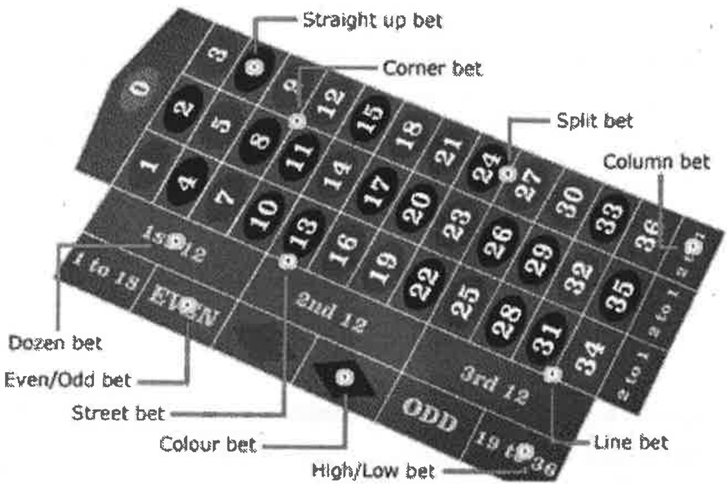
The payout is written in odds format and represents the coefficient to multiply the stake of a won bet.

For example, if you bet $3 on a column (payout 2 to 1) and a number from that column wins, you will receive $3 x 2 = $6, together with your initial $3 stake.

If you place a split bet (payout 17 to 1) of $2 and one of those two numbers wins, you will receive $2 x 17 = $34, together with your initial $2 stake.

If you place a colour bet (payout 1 to 1) and your colour wins, you will get your stake plus an amount equal to that stake.

The next figure gives examples of how the various bets are placed on a roulette table.



The payouts may vary from one casino to another, but usually they are not much different from those noted here.

The Supporting Mathematics

The application of probability theory in gambling is a simple process because a finite sample space can be attached to any game of chance. In some games, probability calculations for some events can become harder because of their structure, but applying the theory is very natural and simple everywhere in this field.

The finite sample space and the randomness of the events (whether it is about rolling dices, drawing cards or spinning a wheel) allow us to build a simple probability model to work within to find the numerical probabilities of the events involved in that game.

This model assumes a finite probability field in which the field of events is the set of parts of the sample space (and, implicitly, is finite) and the probability-function is given by the classical definition of probability.

In this probability field, any event, no matter how complex, can be decomposed into elementary events.

Therefore, finding the probability of a compound event means applying some properties of probability and doing some algebraic calculations.

Among all games of chance, roulette holds the distinction of being the easiest game with respect to probability calculations. This is because the elementary events are one-dimensional elements, the numbers on the roulette wheel. Even dice do not allow easier calculations because games that require the use of dice deal with combinations of numbers.

At the opposite pole, card games (draw poker, for example) are known for their hardest-to-calculate probabilities.

In roulette, anyone with a minimal mathematical background can perform its probability applications and calculations. All the basic calculations involve only arithmetic and basic algebraic operations, but at some point some problems become a matter of math skill, especially those involving repeated events.

For those interested in improving their probability calculus skills and figuring out correct probability results for any game of chance, we recommend the beginner's guide, *Understanding and Calculating the Odds*, which is full of gambling applications.

The math chapter is devoted principally to the mathematical model of a roulette bet: the definition of complex bets, the profit function and its properties, and the equivalence relation between bets and its properties.

Let us see now how probability theory can be applied in Roulette and how the numerical probability results from this book were obtained.

The probability space

As in every game of chance, we are interested in making predictions for the events regarding the outcomes of roulette.

In roulette, there are no opponents or a dealer in the game, so the only events to deal with are the outcomes of the machine—the roulette wheel.

These events can be described as the occurrences of certain numbers or groups or numbers having a specific property (colour, evenness, size, or place on the roulette table).

Every spin of the wheel is an experiment generating an outcome: a number from 1 to 36 plus 0 (in European roulette), or plus 0 and 00 (in American roulette).

The set of these outcomes is the *sample space* attached to this experiment.

The sample space is the set of all elementary events (i.e., events than cannot be decomposed as an union of other non-empty events).

It is natural to take as the elementary events any number that could occur as the result of a spin.

This choice is convenient because it allows us make the following idealization: *the occurrences of the elementary events are equally possible*.

In our case, the occurrence of any number is possible in the same measure (if we assume a random spin and nonfraudulent conditions).

Without this *equally possible* idealization, the construction of a probability model to work within is not possible.

We have established the elementary events and the sample space attached to a spin as being the set of all possible elementary events.

Thus, the sample space in European roulette is the set:
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36}
and the sample space in American roulette is the set:
{00, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36},
which are finite sets.

The field of events is then the set of parts of the sample space and is implicitly finite.

As a set of parts of a set, the field of events is a Boole algebra.

Any event belonging to the field of events, no matter how complex, can be decomposed as a union of elementary events.

Because the events are identified with sets of numbers and the axioms of a Boole algebra, the operations between events (union, intersection, complementary) revert to the operations between sets of numbers.

Therefore, any counting of elementary events (for example, the elementary events a compound event consists of) reverts to counting numbers.

As examples:

The event *red number* or *occurrence of a red number* is the set of elementary events
$A = \{1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34\}$.

The event *even number* is the set of elementary events
$B = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36\}$.

The event *second column* is the set of elementary events
$C = \{2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35\}$.

The event *red and even number* will be the set of elementary events $A \cap B = \{12, 14, 16, 18, 30, 32, 34\}$.

The event *red or even number* will be the set of elementary events $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 32, 34, 36\}$.

The event *even number from second column* is the set of elementary events $B \cap C = \{2, 8, 14, 20, 26, 32\}$.

At this point, we have rigorously established a sample space and a field of events:

- the elementary events are the numbers on the roulette wheel;
- the sample space (Ω) is the set of the numbers on the roulette wheel (Ω could be 37 or 38 elements);
- the field of events is the set of parts of the sample space ($\Sigma = \mathcal{P}(\Omega)$) and has a Boolean structure.

This field of events is suitable as a domain for a function *P* given by the classical definition of probability on a finite field of events with equally possible elementary events:

The probability *P* of event *A* is the ratio between the number of situations favorable for *A* to occur and the number of equally possible situations.

On a finite field of events, *P* is a function $P : \Sigma \to R$ that satisfies the following axioms:

(1) $P(A) \geq 0$ for any $A \in \Sigma$;

(2) $\mathrm{P}(\Omega) = 1$;

(3) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$, for any $A_1, A_2 \in \Sigma$ that $A_1 \cap A_2 = \phi$.

Therefore, *P* is a probability-function and we have built a probability space (field) (Ω, Σ, P) that ensures a rigorous basic probability model on which to work any application for the game of Roulette.

The probability properties and formulas used

Because we deal in all applications with a finite probability space with equally possible elementary events, the probability calculus uses a few basic properties of probability, starting with the classical definition:

(F1) $P = m/n$ (the probability of an event is the ratio between the number of cases favorable for that event to occur and the number of equally possible cases) (the classical definition of probability)

This formula is used on a large scale throughout the book, especially to calculate probabilities of events involving simple bets. It is applied by dividing the number of favorable numbers for a respective event to occur by the number of all possible numbers (37 or 38).

(F2) $P(A_1 \cup A_2) = P(A_1) + P(A_2)$, for any $A_1, A_2 \in \Sigma$ with $A_1 \cap A_2 = \phi$.

and its generalization:

(F3) $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$, for any finite family of mutually exclusive events $(A_i)_{i=1}^{n}$ (finite additivity in condition of incompatibility)

These properties of finite additivity were used to calculate the overall winning or losing probabilities for complex bets.

(F4) $P(A^C) = 1 - P(A)$ (probability of a contrary event)

This primary property is used everywhere we know the probability of the event that is contrary to the event to measure.

(F5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (general formula of probability of union of two events) and its generalization:

(F6) $P(A_1 \cup A_2 \cup ... \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{j<i} P(A_i \cap A_j) +$

$+ \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + ... + (-1)^{n-1} P(A_1 \cap A_2 \cap ... \cap A_n)$.

This property is also called the *inclusion-exclusion principle.*

Formula (F6) is applied as follows:

We have *n* events and want to calculate the probability of their union.

We consider in succession all 1-size combinations, 2-size combinations and so on until *n*-size combinations of the *n* events (we have a single 1-size combination and a single *n*-size combination).

We add the probabilities of intersections of each group of same-size combinations. For the groups of combinations having an even number as a dimension, the total result is add with minus (subtracted). For those with an uneven number as a dimension, the total result is add with plus (addition).

We used the inclusion-exclusion principle when evaluating the probability of hitting the coverage of a complex bet.

(F7) $P(A \cap B) = P(A) \cdot P(B)$, if events *A* and *B* are independent (the definition of independent events).

We used this definition to work out the probability of the occurrence of two certain events (or the same event, in the sense of their identical literal definition) after two different spins.

(F8) Bernoulli scheme

Consider that *n* independent experiments are performed. In each experiment, event *A* may occur with probability *p* and does not occur with probability $q = 1 - p$.

We express the probability for event *A* to occur exactly *m* times in the *n* experiments.

Let B_m be the event *A occurs exactly* m *times in the* n *experiments*. We denote by $P_{m,n}$ the probability $P(B_m)$.

We then have:

$$P(B_m) = P_{m,n} = \underbrace{p^m q^{n-m} + \ldots + p^m q^{n-m}}_{C_n^m \ times} = C_n^m p^m q^{n-m}.$$

Probabilities $P_{m,n}$ have the form of the terms from the development of the binomial $(p+q)^n$.

We used the Bernoulli scheme and formula to calculate the probabilities for the various repeated bets in the last section.

(F9) Mathematical expectation

If *X* is a discrete random variable with values x_i and corresponding probabilities p_i, $i \in I$, the sum $M(X) = \sum_{i \in I} x_i \cdot p_i$ is called mathematical expectation, expected value or mean of variable *X*.

The goal of this section is to underline the theoretical results and formulas used in our applications. Their detailed application is presented in the sections in which they occur.

For readers who want to delve deeper into probability theory and its applications, we recommend the beginner's guide *Understanding and Calculating the Odds*.

Simple bets

When betting in roulette, we are interested in the winning or losing probability, and in the amount of profit we can gain or lose. These depend, of course, on the stake we put in. In fact, they depend on the basic stake we put on each placement.

The winning probability, losing probability, and possible profit and loss are objective criteria for a player when deciding the type of bet to make at a certain moment or what betting system to run.

Beside these objective criteria, there are also subjective criteria related to a player's personal gambling behavior. However, in this book we deal only with objective criteria, which are strictly related to mathematics.

We call a *simple bet* a bet that is made through a unique placement of chips on the roulette table.

So, if we place chips in any number or amount in a single place on the table, we have made a simple bet.

If the outcome after the spin is a number we have bet on, we win the bet and the profit on that bet, which is the stake multiplied by the payout. If the outcome is not favorable, we lose the stake. This applies to any simple bet made.

Therefore, all inside and outside bets described in the chapter titled *The Rules of Roulette* are simple bets.

The table below notes the winning probabilities for each category of simple bet, for both European and American roulette:

Simple bet	Probability (odds) European roulette	Probability (odds) American roulette	Payout
Straight Up	1/37 = 2.70% (36 : 1)	1/38 = 2.63% (37 : 1)	35 to 1
Split Bet	2/37 = 5.40% (17.5 : 1)	2/38 = 5.26% (18 : 1)	17 to 1
Street Bet	3/37 = 8.10% (11.3 : 1)	3/38 = 7.89% (11.6 : 1)	11 to 1
Corner Bet	4/37 = 10.81% (8.2 : 1)	4/38 = 10.52% (8.5 : 1)	8 to 1
Line Bet	6/37 = 16.21% (5.1 : 1)	6/38 = 15.78% (5.3 : 1)	5 to 1
Column Bet	12/37 = 32.43% (2 : 1)	12/38 = 31.57% (2.1 : 1)	2 to 1
Dozen Bet	12/37 = 32.43% (2 : 1)	12/38 = 31.57% (2.1 : 1)	2 to 1
Colour Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1
Even/Odd Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1
Low/High Bet	18/37 = 48.64% (1.0 : 1)	18/38 = 47.36% (1.1 : 1)	1 to 1

The probabilities of winning the simple bets are approximately equal in both American and European roulette (the difference varies from 1/1406 in the case of one number bet to about 1/78 in the case of a colour bet).

The probabilities of specific events attached to the experiment of spinning the roulette wheel are easily calculable—each probability is given by the ratio between the number of favorable numbers and 37 or 38.

The table shows 10 categories of simple bets, each containing several specific placements. Let us count all possible placements for simple bets.

Straight Up
American roulette: 38 placements
European roulette: 37 placements, one for each number

Split Bet
The exact number depends on each casino's rules regarding splitting a bet with the number 0 or 00. We make the calculus in our assumption that European roulette allows 1, 2 and 3 in a split with 0, but American roulette allows only 1 with 0 and only 3 with 00.

American roulette: 59 placements

If we ignore 0 and 00, we have 2 possible horizontal placements on each street and 11 possible vertical placements on each column for a split bet. Therefore, we have 12 x 2 = 24 possible horizontal and 11 x 3 = 33 possible vertical placements. Totaling these with 2 placements for the splits with either 0 or 00, we obtain 24 + 33 + 2 = 59 possible placements.

European roulette: 60 placements

If we ignore 0, we still have 24 + 33 = 57 possible placements. By adding 3 more placements for 0, we obtain 60 possible placements.

Street Bet
We have 12 streets made of numbers that are aligned three in a row (the numbers from 1 to 36), so we have 12 possible placements (some casinos allow special street bets like on 0, 1, 2 or 0, 2, 3; these are not counted here).

Corner Bet
We can place a corner bet at every internal node (that is, a node not lying on a margin of the table) of the grid holding the numbers from 1 to 36. We then have 11 x 2 = 22 possible placements.

Line Bet

We can place a line bet at every external node of the grid holding the numbers from 1 to 36, so we have 11 possible placements for a line bet.

Column Bet

Of course, we have 3 possible placements, one for each column.

Dozen Bet

Still 3 possible placements, one for each dozen.

Colour Bet

We have 2 possible placements, one for each colour.

Even/Odd Bet and Low/High Bet each have 2 possible placements, as well.

By totaling, we can state that we have 154 possible simple bets.

This statement is rigorous if we identify a bet with its placement, but a bet is determined not only by the placement of the chips, but also on the amount of the bet. In fact, the number of all possible simple bets is infinite (under the idealization that any chip division in smaller amounts is possible, even when only limited stakes are allowed).

Let us denote by R the set of all roulette numbers. Any placement for a bet is then a subset of R, or an element of $\mathcal{P}(R)$.

Denote by $\mathcal{A}$ the set of the groups of numbers from R allowed for a bet made through a unique placement.

For example, $\{2\} \in \mathcal{A}$ (straight-up bet), $\{16, 17\} \in \mathcal{A}$ (split bet), $\{11, 12, 14, 15\} \in \mathcal{A}$ (corner bet), $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35\} \in \mathcal{A}$ (odd bet), $\{0, 19\} \notin \mathcal{A}$ (the numbers 0 and 19 cannot be covered by an allowed unique placement).

By the previous count, $\mathcal{A}$ has 154 elements; the number of unique placements allowed.

As mentioned earlier, a simple bet is determined solely by the placement and the stake.

We can define a simple bet as being a pair (*A*, *S*), where $A \in \mathcal{A}$ and $S > 0$ is a real number.

A is the placement (the set of numbers covered by the bet) and *S* is the basic stake (the money amount in chips).

Because each simple bet has a payout defined by the rules of roulette, we can also look at a simple bet as at a triple (A, p_A, S), where p_A is a natural number (the coefficient of multiplication of the stake in case of winning), which is determined solely by *A*.

We have that $p_A \in \{1, 2, 5, 8, 11, 17, 35\}$, according to the rules of roulette.

The probability of winning a simple bet becomes $P(A) = \frac{|A|}{|R|}$,

where $|A|$ means the cardinality of the set *A*. Of course, $|R|$ could be 38 or 37, depending on the roulette type (American or European, respectively).

If a player places a simple bet $B = (A, p_A, S)$, there are two possibilities after the spin:

1) A number from *A* wins and the player makes a positive profit (gain) in the amount of $p_A S$ or

2) A number from $R - A$ wins and the player makes a negative profit, losing the stake *S*.

For a given simple bet *B*, we can define the following function:

$W_B : R \rightarrow \mathbf{R}$, $W_B(e) = 1_A(e)p_A S - 1_{R-A}(e)S$, where **R** is the set of real numbers and 1_A is the characteristic function of a set:

$$1_A(x) = \begin{cases} 1, \textit{if } x \in A \\ 0, \textit{if } x \notin A \end{cases}$$

$W_B(e)$ can be also written as:

$$W_B(e) = 1_A(e)p_A S - [1 - 1_A(e)]S = [1_A(e)(p_A + 1) - 1]S$$

Function W_B is called the *profit* of bet *B*, applying the convention that profit can also be negative (a loss).

The variable e is the outcome of the spin. If $e \in A$ (the player wins bet B), then the player makes the positive profit $p_A S$, and if $e \notin A$ (the player loses the bet B), then the player makes a negative profit of $-S$ (losing an amount equal to S as result of that bet).

Example:
Let us find the probability of winning a split bet on numbers (19, 22) with a stake of \$3, and the profit of that bet at American roulette.

We have $A = \{19, 22\}$, $p_A = 17$, $S = \$3$.
The probability of winning the bet is:
$P(A) = 2/38 = 1/19 = 5.26\%$
The profit is:

$$W_B(e) = [1_A(e) \cdot 18 - 1] \cdot 3 = \begin{cases} 51, \textit{ if } e \in \{19, 22\} \\ -3, \textit{ if } e \in \{19, 22\} \end{cases}$$

So, if 19 or 22 occurs, the player wins \$51. If not, the player loses \$3.

Complex bets

Roulette allows players to spread their chips anywhere on the table by following the rules of placement. In other words, a player can make simultaneously multiple placements of various stakes.

We can call *complex bets* these simultaneous placements. A complex bet consists of several placements of various stakes, so a complex bet is a family of simple bets.

The number of possible multiple placements is huge. If we identify a placement with the set of numbers it covers, this number is in fact the number of all subsets of the set $\mathcal{A}$.

While $\mathcal{A}$ has 154 elements, the number of its subsets is 2^{154}, which is a 47-digit number.

Mathematically, a complex bet can be described by the following: