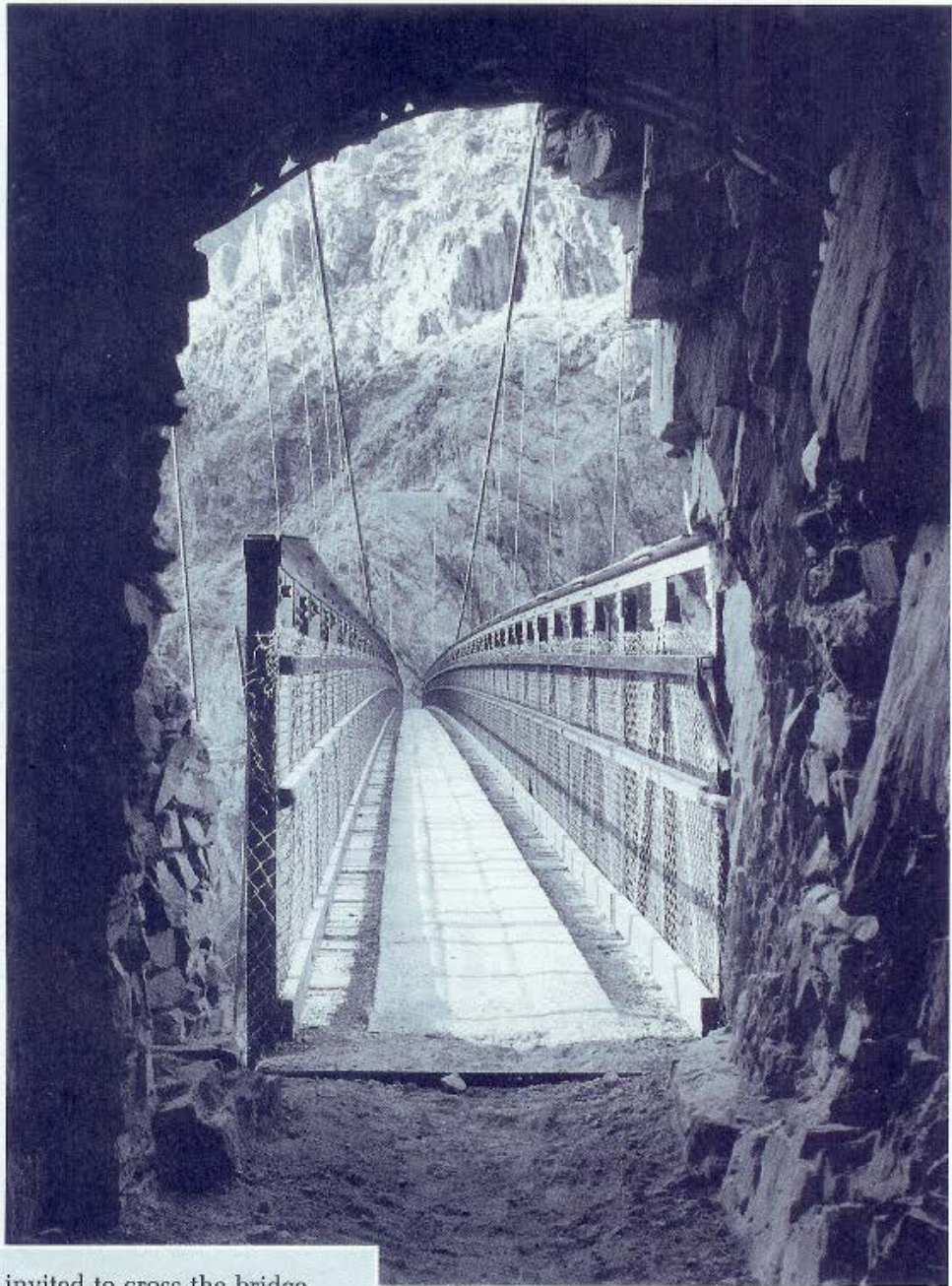


GEOMETRY

for Enjoyment and Challenge

NEW EDITION

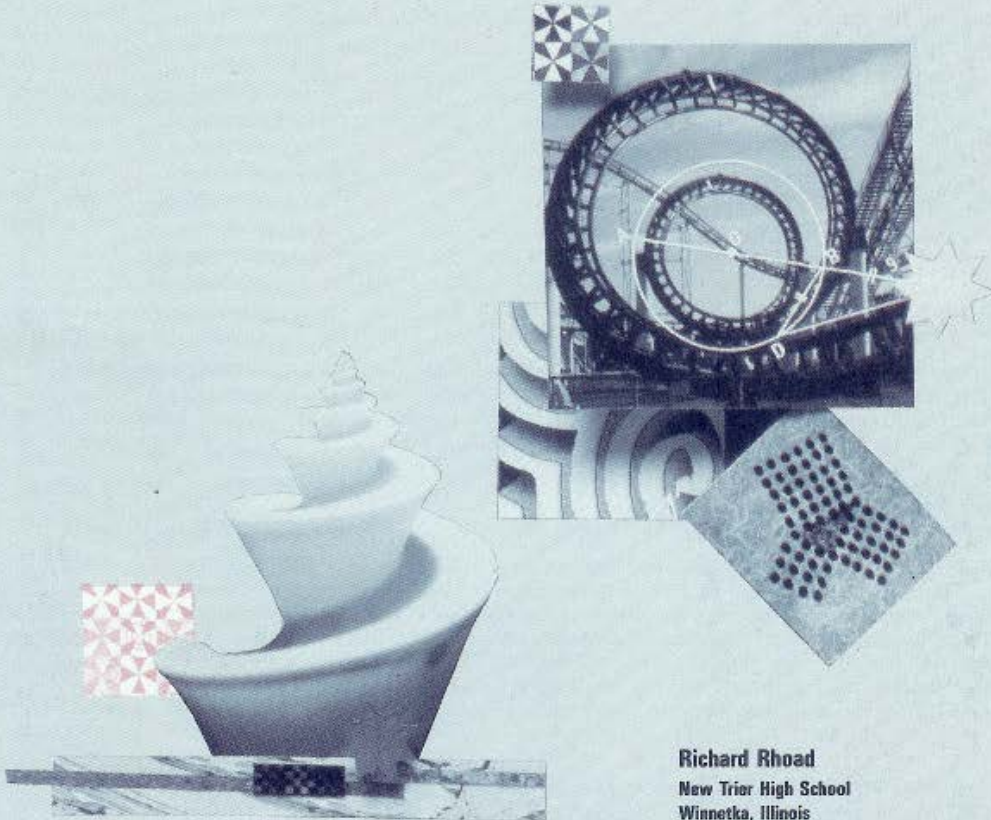


You are invited to cross the bridge into the exciting world of geometry, for enjoyment and challenge.

GEOMETRY

for Enjoyment and Challenge

NEW EDITION



Richard Rhoad
New Trier High School
Winnetka, Illinois

George Milauskas
Illinois Mathematics and
Science Academy
Aurora, Illinois

Robert Whipple
New Trier High School
Winnetka, Illinois

 **McDougal Littell**
A HOUGHTON MIFFLIN COMPANY

Evanston, Illinois ♦ Boston ♦ Dallas

Reviewers

Karen Ardner

Waite High School
Toledo, Ohio

William Duncker

Midland Independent School District
Midland, Texas

Rosanne Hornyak

Wethersfield High School
Wethersfield, Connecticut

Don Nagel

John Marshall High School
Rochester, Minnesota

Irene Norde

Kettering High School
Detroit, Michigan

Sarah Steinmetz

Corona del Sol High School
Tempe, Arizona

Roberta Simonson

Grissom High School
Huntsville, Alabama

Credits

Cover

Cover collage: Carol Tornatore, photograph by Tom Petrillo

1: © Peter Mauss/Esto, Mamaroneck, New York, all rights reserved; 2,5,6,13,15: © Chip Clark, Washington, D.C.; 3,7,8,12: Adrienne McGrath, North Barrington, Illinois; 4,10: (detail) Reprinted by Permission of E.P. Dutton, New York; 9: (detail) Reprinted with Permission of Better Homes and Gardens®, Des Moines, Iowa, American Patchwork and Quilting; 11: JPL, NASA's Regional Planetary Image Facilities, Pasadena, California; 14: Ric Ergenbright, Bend, Oregon.

Fine Art and Photography

Frontispiece: *Suspension Bridge, Kaibab Trail, Grand Canyon, Arizona*, Paul A. Otto, Lake St. Louis, Missouri

2: *Open Book*, 1930, Paul Klée, Guggenheim Museum, New York; 35: Courtesy of Wendell Griffen; 48: © 1985 Sidney Harris, New Haven, Connecticut; 60: Richard Sullivan, Los Angeles; 81: J. Nettis/H. Armstrong Roberts, Chicago; 94: Stember Photography, Lawrenceville, New Jersey; 110: Sheba, 1980, Dorothea Rockburne, National Museum of Women in the Arts, Washington, D.C., Holladay Collection; 130: Norman McGrath, New York; 137: Courtesy of Dorothy Washburne; 168: Algimantas Kezys, Stickney, Illinois; 175: *Ad Parnassum*, 1932, Paul Klée, Kunstmuseum, Bern, Switzerland; 191: (all) E.D. Getzoff/Scripps Clinic and Research Foundation, LaJolla, California; 194: Carmine Fantasia, Chicago; 197: Historical Pictures Service, Chicago; 198: Algimantas Kezys, Stickney, Illinois; Credits continued on p. 769

No part of this book may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, recording, or by any information storage and retrieval system without permission in writing from the Publisher.

ISBN 0-86609-965-4

1997 Impression

Copyright © 1991 by McDougal, Littell & Company

Box 1667, Evanston, Illinois 60204

All rights reserved.

Printed in the United States of America

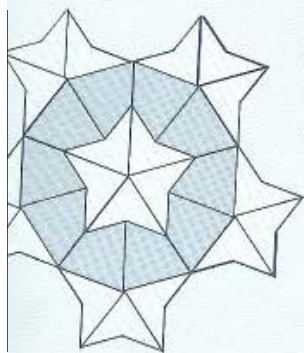
00 00/15 14 13 12 11

CONTENTS

	A LETTER TO STUDENTS	xiv
CHAPTER	1 INTRODUCTION TO GEOMETRY	2
1.1	Getting Started	3
1.2	Measurement of Segments and Angles	9
	<i>Mathematical Excursion: Geometry in Nature</i>	17
1.3	Collinearity, Betweenness, and Assumptions	18
1.4	Beginning Proofs	23
1.5	Division of Segments and Angles	28
	<i>Career Profile: The Science of Deduction</i>	35
1.6	Paragraph Proofs	36
1.7	Deductive Structure	39
1.8	Statements of Logic	44
1.9	Probability	49
	CHAPTER SUMMARY	53
	REVIEW PROBLEMS	54
CHAPTER	2 BASIC CONCEPTS AND PROOFS	60
2.1	Perpendicularity	61
2.2	Complementary and Supplementary Angles	66
2.3	Drawing Conclusions	72
2.4	Congruent Supplements and Complements	76
	<i>Mathematical Excursion: Geometry in Computers</i>	81
2.5	Addition and Subtraction Properties	82
2.6	Multiplication and Division Properties	89
	<i>Career Profile: Studying Bee Communication</i>	94
2.7	Transitive and Substitution Properties	95
2.8	Vertical Angles	100
	CHAPTER SUMMARY	104
	REVIEW PROBLEMS	105

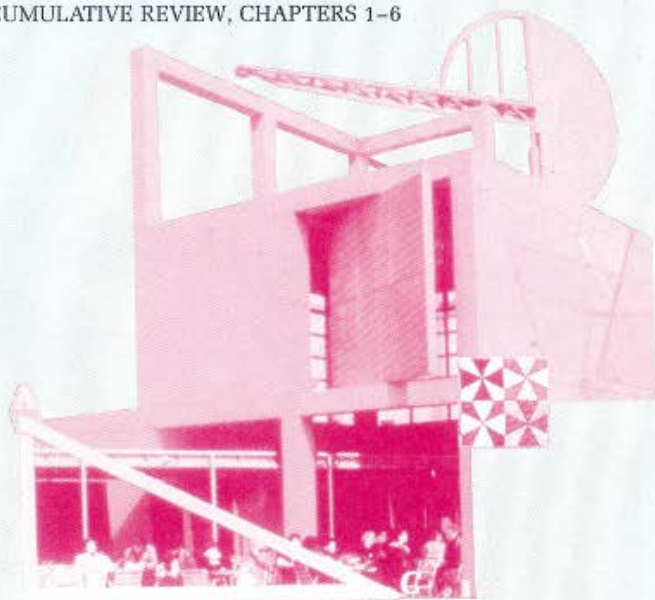
CHAPTER 3	CONGRUENT TRIANGLES	110
3.1	What Are Congruent Figures?	111
3.2	Three Ways to Prove Triangles Congruent	115
3.3	CPCTC and Circles	125
	<i>Mathematical Excursion: Structural Congruent Triangles</i>	130
3.4	Beyond CPCTC	131
	<i>Career Profile: Symmetry Unlocks Culture</i>	137
3.5	Overlapping Triangles	138
3.6	Types of Triangles	142
3.7	Angle-Side Theorems	148
3.8	The HL Postulate	156
	CHAPTER SUMMARY	161
	REVIEW PROBLEMS	162
	CUMULATIVE REVIEW, CHAPTERS 1–3	165

CHAPTER 4	LINES IN THE PLANE	168
4.1	Detours and Midpoints	169
4.2	The Case of the Missing Diagram	176
4.3	A Right-Angle Theorem	180
4.4	The Equidistance Theorems	184
	<i>Career Profile: Plotting the Structure of a Molecule</i>	191
4.5	Introduction to Parallel Lines	192
	<i>Historical Snapshot: From Mud to the Stars</i>	197
4.6	Slope	198
	CHAPTER SUMMARY	205
	REVIEW PROBLEMS	206



CHAPTER 5	PARALLEL LINES AND RELATED FIGURES	210
5.1	Indirect Proof	211
	<i>Career Profile: A Line to the Stars</i>	215
5.2	Proving That Lines Are Parallel	216
5.3	Congruent Angles Associated with Parallel Lines	224
5.4	Four-Sided Polygons	234
	<i>Historical Snapshot: A New Kind of Proof</i>	240
5.5	Properties of Quadrilaterals	241
5.6	Proving That a Quadrilateral Is a Parallelogram	249
5.7	Proving That Figures Are Special Quadrilaterals	255
	CHAPTER SUMMARY	263
	REVIEW PROBLEMS	264

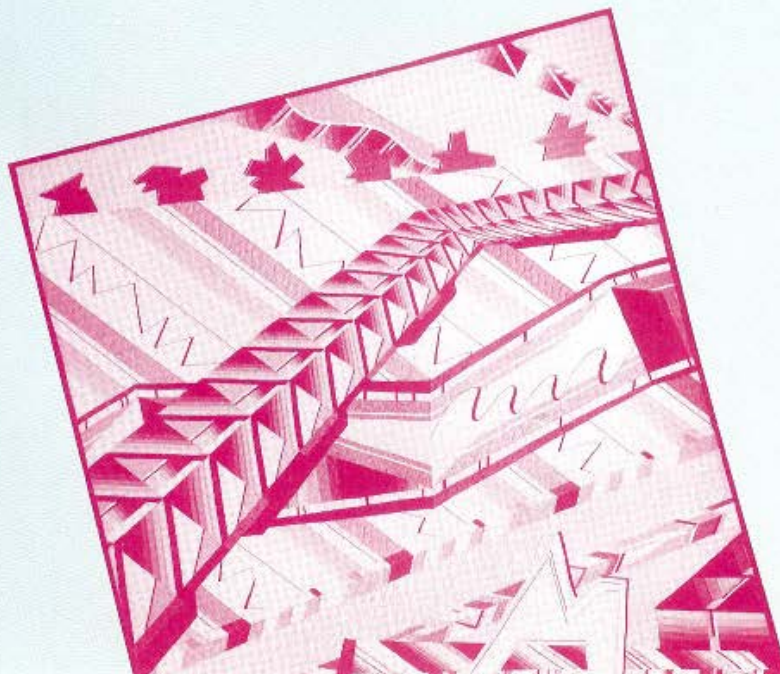
CHAPTER 6	LINES AND PLANES IN SPACE	268
6.1	Relating Lines to Planes	269
	<i>Career Profile: The Geometry of Architecture</i>	275
6.2	Perpendicularity of a Line and a Plane	276
	<i>Historical Snapshot: Probability and Pi</i>	281
6.3	Basic Facts About Parallel Planes	282
	CHAPTER SUMMARY	287
	REVIEW PROBLEMS	288
	CUMULATIVE REVIEW, CHAPTERS 1–6	291



CHAPTER	7	POLYGONS	294
	7.1	Triangle Application Theorems	295
	7.2	Two Proof-Oriented Triangle Theorems	302
	7.3	Formulas Involving Polygons	307
		<i>Career Profile: Precise Angles Pay Off</i>	313
	7.4	Regular Polygons	314
		<i>Mathematical Excursion: Polygons in the North Country</i>	318
		CHAPTER SUMMARY	319
		REVIEW PROBLEMS	320



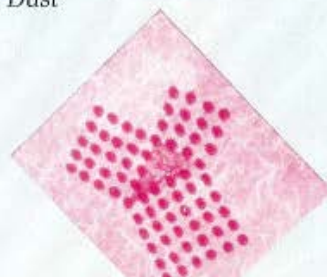
CHAPTER	8	SIMILAR POLYGONS	324
	8.1	Ratio and Proportion	325
	8.2	Similarity	332
	8.3	Methods of Proving Triangles Similar	339
	8.4	Congruence and Proportions in Similar Triangles	345
	8.5	Three Theorems Involving Proportions	351
		<i>Career Profile: Putting Quilts in Perspective</i>	359
		CHAPTER SUMMARY	360
		REVIEW PROBLEMS	361
		<i>Historical Snapshot: A Master Technologist</i>	365



CHAPTER 9	THE PYTHAGOREAN THEOREM	366
9.1	Review of Radicals and Quadratic Equations	367
9.2	Introduction to Circles	370
9.3	Altitude-on-Hypotenuse Theorems	377
	<i>Mathematical Excursion: The Pythagorean Theorem and Trigonometric Ratios</i>	383
9.4	Geometry's Most Elegant Theorem	384
9.5	The Distance Formula	392
	<i>Career Profile: Finding Distances with Lasers</i>	397
9.6	Families of Right Triangles	398
9.7	Special Right Triangles	405
9.8	The Pythagorean Theorem and Space Figures	413
9.9	Introduction to Trigonometry	418
9.10	Trigonometric Ratios	423
	CHAPTER SUMMARY	428
	REVIEW PROBLEMS	429
	CUMULATIVE REVIEW, CHAPTERS 1-9	434



CHAPTER 10	CIRCLES	438
10.1	The Circle	439
10.2	Congruent Chords	446
10.3	Arcs of a Circle	450
10.4	Secants and Tangents	459
10.5	Angles Related to a Circle	468
10.6	More Angle-Arc Theorems	479
10.7	Inscribed and Circumscribed Polygons	486
	<i>Mathematical Excursion: Tangent, Slope, and Loops</i>	492
10.8	The Power Theorems	493
	<i>Career Profile: From Asteroids to Dust</i>	498
10.9	Circumference and Arc Length	499
	CHAPTER SUMMARY	504
	REVIEW PROBLEMS	505



CHAPTER 11 AREA	510
11.1 Understanding Area	511
11.2 Areas of Parallelograms and Triangles	516
11.3 The Area of a Trapezoid	523
11.4 Areas of Kites and Related Figures	528
11.5 Areas of Regular Polygons	531
<i>Mathematical Excursion: Tiling and Area</i>	536
11.6 Areas of Circles, Sectors, and Segments	537
<i>Career Profile: Geometry in Visual Communication</i>	542
11.7 Ratios of Areas	543
11.8 Hero's and Brahmagupta's Formulas	550
CHAPTER SUMMARY	553
REVIEW PROBLEMS	554

CHAPTER 12 SURFACE AREA AND VOLUME	560
12.1 Surface Areas of Prisms	561
12.2 Surface Areas of Pyramids	565
<i>Career Profile: Packaging Ideas</i>	569
12.3 Surface Areas of Circular Solids	570
12.4 Volumes of Prisms and Cylinders	575
12.5 Volumes of Pyramids and Cones	583
12.6 Volumes of Spheres	589
CHAPTER SUMMARY	593
REVIEW PROBLEMS	594
CUMULATIVE REVIEW, CHAPTERS 1-12	598
<i>Historical Snapshot: The Shape of the Universe</i>	603



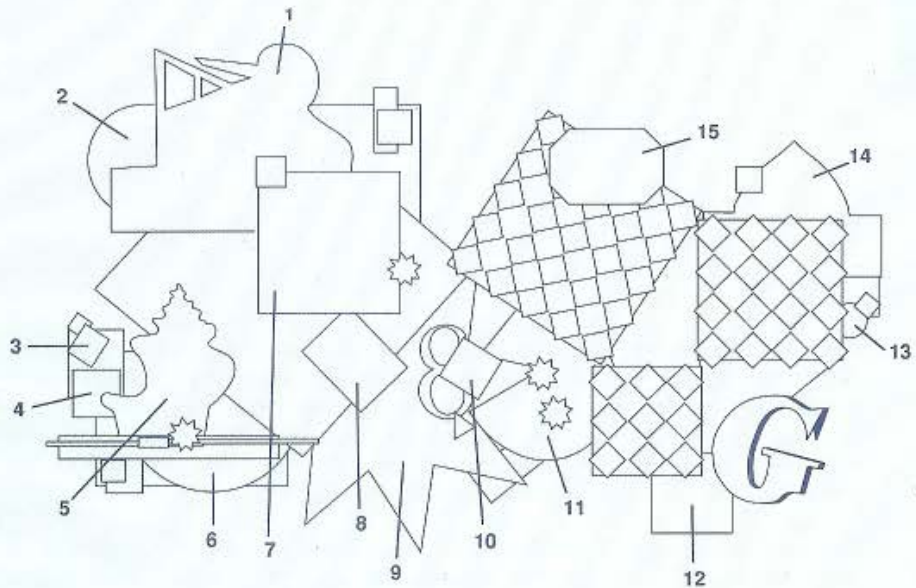
CHAPTER 13	COORDINATE GEOMETRY EXTENDED	604
13.1	Graphing Equations	605
13.2	Equations of Lines	610
13.3	Systems of Equations	618
13.4	Graphing Inequalities	622
13.5	Three-Dimensional Graphing and Reflections	626
	<i>Career Profile: Image-Producing Waves</i>	632
13.6	Circles	633
13.7	Coordinate-Geometry Practice	638
	<i>Historical Snapshot: The Serpent and the Peacock</i>	642
	CHAPTER SUMMARY	643
	REVIEW PROBLEMS	644



CHAPTER 14	LOCUS AND CONSTRUCTIONS	648
14.1	Locus	649
	<i>Historical Snapshot: The Geometry of Music</i>	655
14.2	Compound Locus	656
14.3	The Concurrence Theorems	660
14.4	Basic Constructions	666
14.5	Applications of the Basic Constructions	673
14.6	Triangle Constructions	678
	CHAPTER SUMMARY	682
	REVIEW PROBLEMS	683
	<i>Career Profile: Darkness Visible</i>	685

CHAPTER 15	INEQUALITIES	686
15.1	Number Properties	687
	<i>Career Profile: Deductions from Seismic Waves</i>	690
15.2	Inequalities in a Triangle	691
15.3	The Hinge Theorems	697
	<i>Mathematical Excursion: Inequalities</i>	701
	CHAPTER SUMMARY	702
	REVIEW PROBLEMS	703
	CUMULATIVE REVIEW, CHAPTERS 1–15	706
CHAPTER 16	ENRICHMENT TOPICS	712
16.1	The Point-Line Distance Formula	713
	<i>Career Profile: Planetary Portraits</i>	716
16.2	Two Other Useful Formulas	717
16.3	Stewart's Theorem	721
16.4	Ptolemy's Theorem	724
	<i>Historical Snapshot: Dynamic Geometry</i>	727
16.5	Mass Points	728
16.6	Inradius and Circumradius Formulas	731
16.7	Formulas for You to Develop	734
	CHAPTER SUMMARY	737
	REVIEW PROBLEMS	738
	LIST OF POSTULATES AND THEOREMS	740
	SELECTED ANSWERS	750
	GLOSSARY	758
	INDEX	763
	SYMBOLS USED IN GEOMETRY	770

Cover



1. Café at Parc de la Villette, Paris
- 2, 13. Cuban tree snails
- 3, 8, 12. Pythagorean Theorem proofs
- 4, 10. Pieced quilt, Broken Dishes variation, silk, made by Susan Simpson, New Jersey, c. 1870
5. Miraculous Thatcheria, a marine snail from Taiwan
6. Round blue topaz
7. Amusement-park ride
9. Amish quilt, Star of Bethlehem pattern
11. Saturn's B- and C-rings
14. Hazart Ali Shrine, Mazari-i-Sharif, Afghanistan
15. Malaya, a variety of garnet, from East Africa

FOUR-SIDED POLYGONS

Objectives

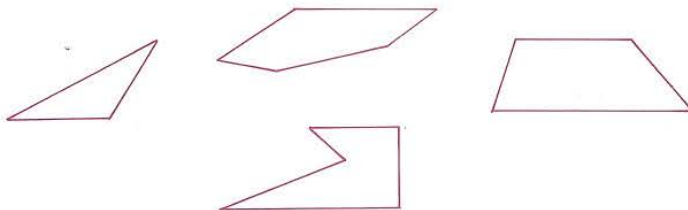
After studying this section, you will be able to

- Recognize polygons
- Understand how polygons are named
- Recognize convex polygons
- Recognize diagonals of polygons
- Identify special types of quadrilaterals

Part One: Introduction

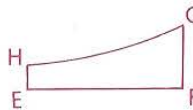
Polygons

Polygons are plane figures. The following are examples of polygons.

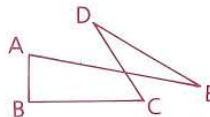


The following are examples of figures that are not polygons.

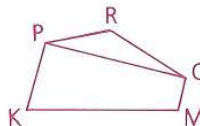
EFGH is not a polygon, because a polygon consists entirely of segments.



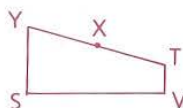
ABCDE is not a polygon. In a polygon, consecutive sides intersect only at endpoints. Nonconsecutive sides do not intersect.



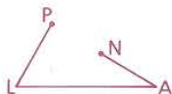
PKMO, PKMOR, and POR are polygons, but PKMOPRO is not, because each vertex must belong to exactly two sides. (Vertex P belongs to three sides in PKMOPRO.)



SVTY is a polygon, but SVTXY is not, because consecutive sides must be noncollinear.

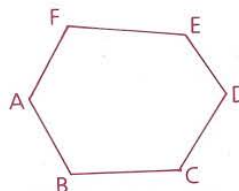


Why is PLAN not a polygon?



Naming Polygons

We name a polygon by starting at any vertex and then proceeding either clockwise or counterclockwise. If we start at A, we can call this polygon ABCDEF or AFEDCB. Can you start at B and name the polygon in two different ways?



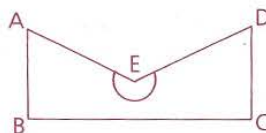
Convex Polygons

Many of the polygons you encounter in your geometry studies will be *convex*.

Definition A *convex polygon* is a polygon in which each interior angle has a measure less than 180.

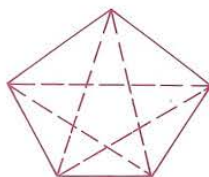
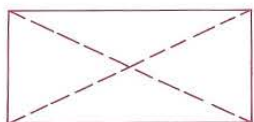
Polygon ABCDE is not convex because the angle that lies in the interior of the polygon at E has a measure greater than 180.

In the rest of this book, unless it is expressly stated otherwise, assume that all polygons are convex.



Diagonals of Polygons

In the two following figures, the dashed segments are *diagonals* of the polygons.



Definition A *diagonal* of a polygon is any segment that connects two nonconsecutive (nonadjacent) vertices of the polygon.

Quadrilaterals

A **quadrilateral** is a four-sided polygon.



The following are special quadrilaterals.

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.



A **rectangle** is a parallelogram in which at least one angle is a right angle.



A **rhombus** is a parallelogram in which at least two consecutive sides are congruent.



A **kite** is a quadrilateral in which two disjoint pairs of consecutive sides are congruent.



A **square** is a parallelogram that is both a rectangle and a rhombus.



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called *bases* of the trapezoid.



An **isosceles trapezoid** is a trapezoid in which the nonparallel sides (*legs*) are congruent. In the figure, $\angle A$ and $\angle B$ are called the **lower base angles**, and $\angle C$ and $\angle D$ are called the **upper base angles**.



We have given the meaning (definition) of each of the previous figures in as simple a manner as possible. Each special quadrilateral will have further properties associated with it. Those properties are discussed in the next section.

Part Two: Sample Problem

Solve the Quadrilateral Mystery!

No solution is provided for the following problem. It is intended to help you understand how mathematicians go about testing ideas that they *think* are true but which they have not yet *proved*. As you work through the problem, think carefully about the ideas you formulate and the ways you test them. (A computer with exploratory geometry software—such as *The Geometric Supposer*, by Sunburst—is an excellent tool for testing ideas. If you do not have access to such resources, try making careful drawings and using a ruler and a protractor to test your ideas.)

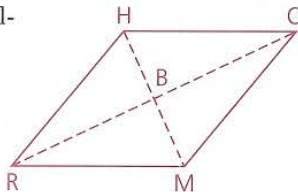
- Problem** What truths can you discover about a parallelogram and a rectangle?
- a Draw a parallelogram ABCD.
 - i What true statements do you think you might be able to make about the parallelogram? Test your ideas and discuss your results in class.
 - ii Draw diagonals \overline{AC} and \overline{BD} . What true statements can be made about the diagonals? Again, test your ideas and discuss your results in class.
 - b Draw a rectangle PQRS.
 - i What true statements can be made about the rectangle? Test your ideas and discuss your results in class.
 - ii Draw diagonals \overline{PR} and \overline{QS} . What true statements can be made about the diagonals? Again, test your ideas and discuss your results in class.

Part Three: Problem Sets

Problem Set A

A computer and exploratory geometry software may be used for problems 1–5.

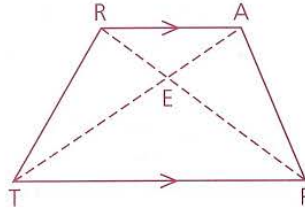
- 1 Examine the rhombus. Which of the following statements appear to be true?
 - a All four sides are congruent.
 - b The diagonals are perpendicular.
 - c The diagonals bisect the angles.
 - d The diagonals bisect each other.
 - e The diagonals are congruent.



Problem Set A, continued

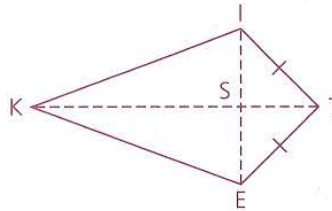
2 Examine the isosceles trapezoid. Which of the following statements appear to be true?

- a The opposite sides are congruent.
- b Opposite sides are parallel.
- c The diagonals bisect the angles.
- d The diagonals bisect each other.
- e The diagonals are congruent.



3 Examine the kite. Which of the following statements appear to be true?

- a The opposite sides are congruent.
- b Opposite sides are parallel.
- c The diagonals bisect the angles.
- d The diagonals bisect each other.
- e The diagonals are congruent.
- f The diagonals are perpendicular.



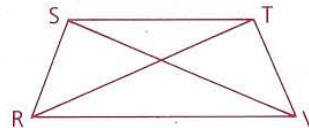
4 List all the properties that a nonisosceles trapezoid appears to have.

5 List all the properties that a square appears to have.

- 6 a Draw an equilateral quadrilateral that is not equiangular.
- b Draw an equiangular quadrilateral that is not equilateral.

7 In the isosceles trapezoid shown, $\overline{ST} \parallel \overline{RV}$.

- Name:
- a The bases
 - b The diagonals
 - c The legs
 - d The lower base angles
 - e The upper base angles
 - f All pairs of congruent alternate interior angles



8 Examine each statement below. If the statement is always true, write A; if sometimes true, write S; if never true, write N.

- a A square is a rhombus.
- b A rhombus is a square.
- c A kite is a parallelogram.
- d A rectangle is a polygon.
- e A polygon has the same number of vertices as sides.
- f A parallelogram has three diagonals.
- g A trapezoid has three bases.

9 Why is a circle not a polygon?

- 10 Using the diagram, explain how the formula for the area of a parallelogram can be the same as that for the area of a rectangle.



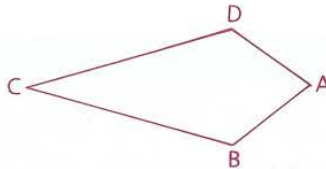
- 11 If the sum of the measures of the angles of a triangle is 180, what is the sum of the measures of the angles in
- A quadrilateral?
 - A pentagon (five-sided polygon)?
- 12 Find the area of a square whose perimeter is 65 feet.

Problem Set B

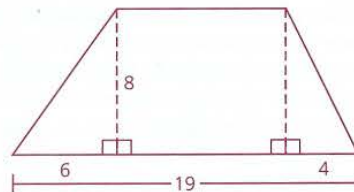
- 13 Prove that in a parallelogram each pair of consecutive angles are supplementary.
- 14 Prove that in a parallelogram each pair of opposite sides are congruent.
- 15 Prove that the diagonals of a rectangle are congruent.

- 16 Given: ABCD is a kite.

$$\begin{aligned} AB &= x + 3, \\ BC &= x + 4, \\ CD &= 2x - 1, \\ AD &= 3x - y \end{aligned}$$

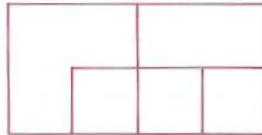


- Solve for x and y .
 - What is the perimeter of the kite?
 - Is it possible for \overline{AC} to be 19 units long? Why or why not?
- 17 a PQRS is a kite and also a rectangle. What else do we know about PQRS?
- Draw a quadrilateral that is not convex and still satisfies the definition of a kite.
- 18 What is the area of a triangle whose vertices are $(-4, -3)$, $(8, 7)$, and $(8, -3)$?
- 19 The trapezoidal region is actually the union of two triangles and a rectangle. Find the area of the trapezoid.



Problem Set B, continued

- 20 How many rectangles are shown in the figure at the right, in which all of the angles are right angles?



Problem Set C

- 21 a How many diagonals does a triangle have?
b How many diagonals does a quadrilateral have?
c How many diagonals does a five-sided polygon have?
d How many diagonals does a six-sided polygon have?
e How many diagonals meet at one vertex of a polygon with n sides?
f How many vertices does an n -sided polygon have?
g How many diagonals does an n -sided polygon have?
- 22 Refer to the seven special quadrilaterals on page 236. What is the probability that if two are picked at random, each will have a pair of congruent opposite sides?

HISTORICAL SNAPSHOT

A NEW KIND OF PROOF

The computer and the four-color conjecture

How many colors does it take to color any map so that no two adjacent regions will be the same color? (Regions that touch only at a single point are not considered to be adjacent.) In 1852 it was suggested that four colors are enough for any possible map. Although no one ever succeeded in constructing a map that needed more than four colors, for over 100 years no one was able to furnish a satisfactory proof that such a map could not exist.

Then, in 1976, it was announced that a group of mathematicians led by Kenneth Appel and Wolfgang Haken at the University of Illinois had proved the four-color conjecture. Having determined that all possible maps could be represented by a set of 1936 particular configurations of regions, they programmed a computer to test each of these cases for four-colorability.



The computer found no instance in which more than four colors were required.

Traditionally, however, a proof has been considered a way of presenting mathematical reasoning that can be understood and verified by other people. The four-color proof is so complex that it would take lifetimes to verify it by hand. It is one of the first examples of a proof that can be produced and checked only using a computer.

PROPERTIES OF QUADRILATERALS

Objectives

After studying this section, you will be able to

- Identify some properties of parallelograms
- Identify some properties of rectangles
- Identify some properties of kites
- Identify some properties of rhombuses
- Identify some properties of squares
- Identify some properties of isosceles trapezoids

Part One: Introduction

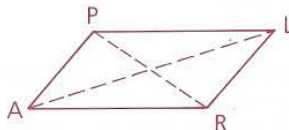
Properties of Parallelograms

In this section, we will list some of the properties of special quadrilaterals, beginning with parallelograms. (You should be able to prove many of these properties.) Read the properties carefully and learn them. They will be used often in the sections to follow.

Learning so many properties may seem overwhelming at first, but most are concepts that you already know or that you discovered in Section 5.4. With some effort you will soon learn them all.

In a parallelogram,

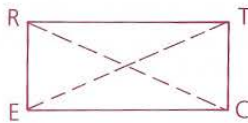
- 1 The opposite sides are parallel by definition ($\overline{PL} \parallel \overline{AR}$, $\overline{AP} \parallel \overline{RL}$)
- 2 The opposite sides are congruent ($\overline{PL} \cong \overline{AR}$, $\overline{AP} \cong \overline{RL}$)
- 3 The opposite angles are congruent ($\angle PAR \cong \angle PLR$, $\angle ARL \cong \angle APL$)
- 4 The diagonals bisect each other (\overline{AL} bis. \overline{PR} , \overline{PR} bis. \overline{AL})
- 5 Any pair of consecutive angles are supplementary ($\angle PAR$ supp. $\angle ARL$, etc.)



Properties of Rectangles

In a rectangle,

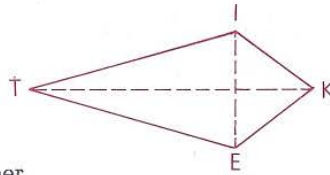
- 1 All the properties of a parallelogram apply by definition
- 2 All angles are right angles ($\angle REC$ is a right angle, etc.)
- 3 The diagonals are congruent ($\overline{ET} \cong \overline{RC}$)



Properties of Kites

In a kite,

- Two disjoint pairs of consecutive sides are congruent by definition ($\overline{IT} \cong \overline{ET}$, $\overline{IK} \cong \overline{EK}$)
- The diagonals are perpendicular ($\overline{TK} \perp \overline{IE}$)
- One diagonal is the perpendicular bisector of the other ($\overline{TK} \perp \text{bis. } \overline{IE}$)
- One of the diagonals bisects a pair of opposite angles (\overrightarrow{TK} bis. $\angle ITE$, \overrightarrow{TK} bis. $\angle IKE$)
- One pair of opposite angles are congruent ($\angle TIK \cong \angle TEK$)

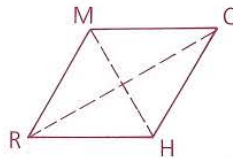


Properties 3–5 are sometimes called the *half properties* of kites.

Properties of Rhombuses

In a rhombus,

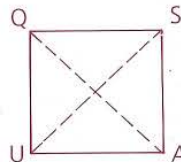
- All the properties of a parallelogram apply by definition
- All the properties of a kite apply (In fact, the half properties become full properties)
- All sides are congruent—that is, a rhombus is equilateral ($\overline{RH} \cong \overline{HO} \cong \overline{OM} \cong \overline{MR}$)
- The diagonals bisect the angles (\overrightarrow{RO} bis. $\angle MRH$, \overrightarrow{RO} bis. $\angle MOH$, etc.)
- The diagonals are perpendicular bisectors of each other ($\overline{RO} \perp \text{bis. } \overline{MH}$, $\overline{MH} \perp \text{bis. } \overline{RO}$)
- The diagonals divide the rhombus into four congruent right triangles



Properties of Squares

In a square,

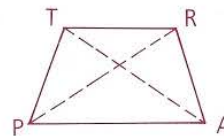
- All the properties of a rectangle apply by definition
- All the properties of a rhombus apply by definition
- The diagonals form four isosceles right triangles (45° - 45° - 90° triangles)



Properties of Isosceles Trapezoids

In an isosceles trapezoid,

- The legs are congruent by definition ($\overline{TP} \cong \overline{RA}$)
- The bases are parallel (by definition of trapezoid) ($\overline{TR} \parallel \overline{PA}$)
- The lower base angles are congruent ($\angle RAP \cong \angle TPA$)
- The upper base angles are congruent ($\angle PTR \cong \angle ART$)
- The diagonals are congruent ($\overline{PR} \cong \overline{AT}$)
- Any lower base angle is supplementary to any upper base angle ($\angle PAR$ supp. $\angle PTR$, etc.)



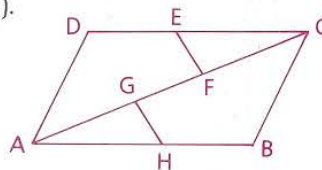
In the problems that follow, you will be asked to prove some of these properties. You may use any prior property to help in the proof of a later property. For example, if you are asked to prove property 5 of parallelograms, you may use properties 1–4 to help you in the proof.

Part Two: Sample Problems

Problem 1 Given: ABCD is a \square (parallelogram).

$$\begin{aligned} \angle GHA &\cong \angle FEC, \\ \overline{HB} &\cong \overline{DE} \end{aligned}$$

Conclusion: $\overline{GH} \cong \overline{EF}$



Proof

1 ABCD is a \square .	1 Given
2 $DC \parallel AB$	2 Opposite sides of a \square are \parallel .
3 $\angle ECF \cong \angle HAG$	3 \parallel lines \Rightarrow alt. int. \angle s \cong
4 $\overline{AB} \cong \overline{DC}$	4 Opposite sides of a \square are \cong .
5 $\overline{HB} \cong \overline{DE}$	5 Given
6 $\overline{HA} \cong \overline{EC}$	6 Subtraction Property
7 $\angle GHA \cong \angle FEC$	7 Given
8 $\triangle GAH \cong \triangle FCE$	8 ASA (3, 6, 7)
9 $\overline{GH} \cong \overline{EF}$	9 CPCTC

Problem 2 Given: VRZA is a \square .

$$\begin{aligned} AV &= 2x - 4, \\ VR &= 3y + 5, \\ RZ &= \frac{1}{2}x + 8 \\ ZA &= y + 12 \end{aligned}$$



Find: The perimeter of VRZA

Solution

The opposite sides of a \square are congruent, so we can write two equations.

$$\begin{aligned} 2x - 4 &= \frac{1}{2}x + 8 \\ \frac{1}{2}x - 4 &= 8 \\ \frac{1}{2}x &= 12 \\ x &= 8 \end{aligned}$$

$$AV = 12 \text{ and } RZ = 12$$

$$3y + 5 = y + 12$$

$$2y + 5 = 12$$

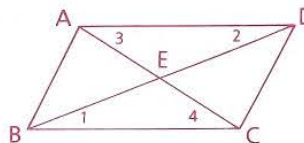
$$2y = 7$$

$$y = 3\frac{1}{2}$$

$$VR = 15\frac{1}{2} \text{ and } ZA = 15\frac{1}{2}$$

Adding the measures of the four sides, we find that the perimeter is 55.

Problem 3 Prove property 4 of parallelograms.
 Given: $\square ABCD$
 Prove: \overline{AC} and \overline{BD} bisect each other.



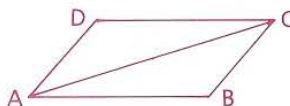
Proof

1 $\square ABCD$	1 Given
2 $\overline{AD} \parallel \overline{BC}$	2 Opposite sides of a \square are \parallel .
3 $\angle 1 \cong \angle 2$	3 \parallel lines \Rightarrow alt. int. \angle s \cong
4 $\angle 3 \cong \angle 4$	4 \parallel lines \Rightarrow alt. int. \angle s \cong
5 $\overline{AD} \cong \overline{BC}$	5 Opposite sides of a \square are \cong .
6 $\triangle BEC \cong \triangle DEA$	6 ASA (3, 5, 4)
7 $\overline{BE} \cong \overline{DE}$	7 CPCTC
8 $\overline{AE} \cong \overline{EC}$	8 CPCTC
9 \overline{AC} and \overline{BD} bisect each other.	9 If two segments divide each other into \cong segments, they bisect each other.

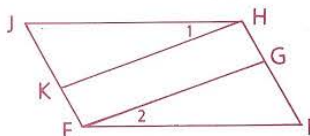
Part Three: Problem Sets

Problem Set A

1 Given: $\square ABCD$ (ABCD is a \square .)
 Conclusion: $\triangle ABC \cong \triangle CDA$



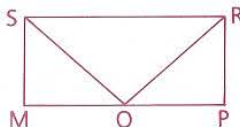
2 Given: $\square EFHJ$,
 $\angle 1 \cong \angle 2$
 Conclusion: $\overline{KH} \cong \overline{EG}$



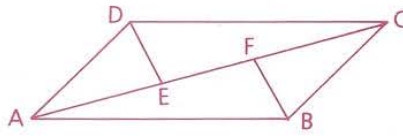
Supply each missing reason.

1 $\square EFHJ$	1 _____
2 $\angle J \cong \angle F$	2 _____
3 $\overline{JH} \cong \overline{EF}$	3 _____
4 $\angle 1 \cong \angle 2$	4 _____
5 $\triangle KJH \cong \triangle GFE$	5 _____
6 $\overline{KH} \cong \overline{EG}$	6 _____

3 Given: Rectangle MPRS,
 $\overline{MO} \cong \overline{PO}$
 Prove: $\triangle ROS$ is isosceles.



- 4 Given: $\square ABCD$,
 $\overline{AE} \cong \overline{CF}$
 Conclusion: $\overline{DE} \cong \overline{BF}$



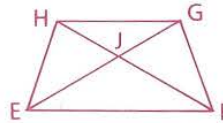
- 5 Given: $\square WSTV$,
 $WS = x + 5$,
 $WV = x + 9$,
 $VT = 2x + 1$
 Find the perimeter of $WSTV$.



- 6 Given: $\square ABCD$,
 $\angle A = (x)^\circ$,
 $\angle D = (3x - 4)^\circ$
 Find: $m\angle D$ and $m\angle C$



- 7 Given: $EFGH$ is an isosceles trapezoid,
 with legs \overline{HE} and \overline{GF} .
 $EJ = x + 5$,
 $JG = 2x - 1$,
 $HF = 13$
 Find: EJ , JG , and HJ

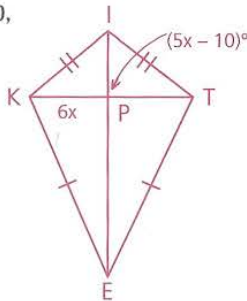


- 8 Prove property 3 of parallelograms.

- 9 Prove property 4 of rhombuses.

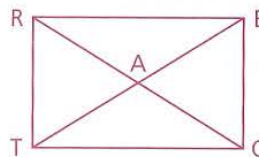
- 10 Prove property 5 of isosceles trapezoids.

- 11 Given: $m\angle IPT = 5x - 10$,
 $KP = 6x$
 Find: KT



- 12 Given: $RECT$ is a rectangle.
 $RA = 43x$,
 $AC = 214x - 742$

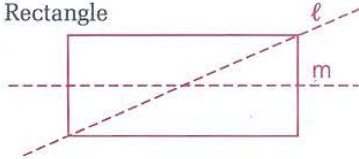
Find: The length of \overline{ET} to the nearest tenth



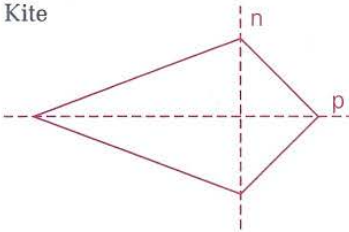
Problem Set A, continued

- 13 Which of the dotted lines represent an axis of symmetry of the figure? (One side of a figure is a reflection of the other side over an axis of symmetry.)

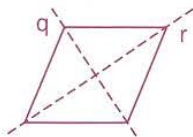
a Rectangle



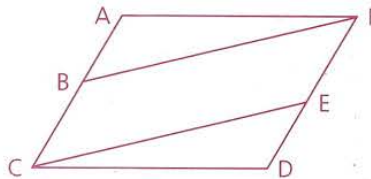
b Kite



c Rhombus

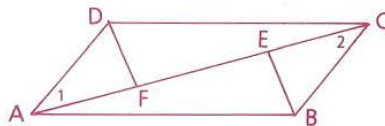


- 14 Given: $\angle AFB \cong \angle DCE$,
 $\triangle AFB \cong \triangle DCE$
 Prove: $ACDF$ is not a parallelogram.

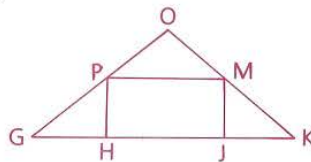


Problem Set B

- 15 Given: $ABCD$ is a \square .
 $\overline{AF} \cong \overline{CE}$
 Prove: $\overline{DF} \parallel \overline{EB}$

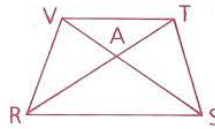


- 16 Given: $PHJM$ is a rectangle.
 $\overline{PG} \cong \overline{MK}$
 Prove: $\triangle O GK$ is isosceles.



- 17 Given: $VRST$ is an isosceles trapezoid,
with legs \overline{VR} and \overline{TS} .

Prove: $\triangle ARS$ is isosceles.



- 18 Prove that the diagonals of a rhombus divide the rhombus into four $\cong \triangle$.

- 19 Given: $\square KMOP$,
 $\angle M = (x + 3y)^\circ$,
 $\angle O = (x - 4)^\circ$,
 $\angle P = (4y - 8)^\circ$

Find: $m\angle K$

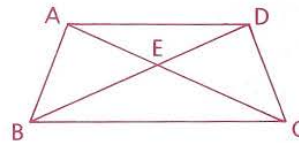


- 20 $ABCD$ is an isosceles trapezoid with
upper base \overline{AD} .

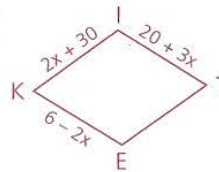
$BE = x + 7$, $CE = y - 3$,

$AE = x + 5$, $BD = y + 4$

Find AC .



- 21 An author wrote a problem involving
kite $KITE$ but forgot to say which pairs
of sides were congruent. Work the prob-
lem twice to see which pairs of sides are
congruent.



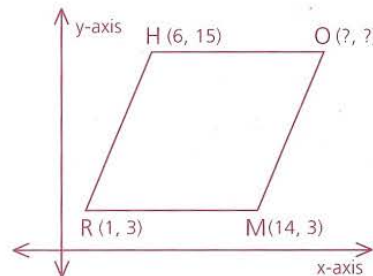
- 22 Prove, in paragraph form, that one diagonal of a kite divides it
into two congruent triangles, while the other diagonal divides it
into two isosceles triangles.

- 23 $RHOM$ is a rhombus.

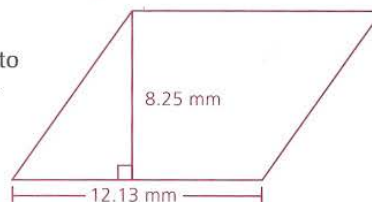
a Find the coordinates of point O .

b Find the slopes of \overleftrightarrow{HM} and \overleftrightarrow{RO} .

c What does the result in part b verify?

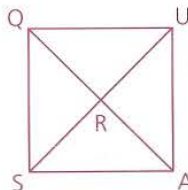


- 24 The area of a parallelogram is equal to
the product of its base and its height.
Find the area of the parallelogram.

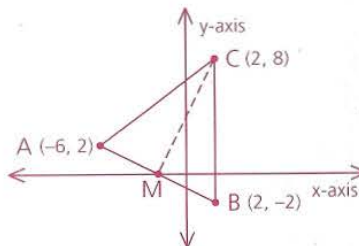


Problem Set B, continued

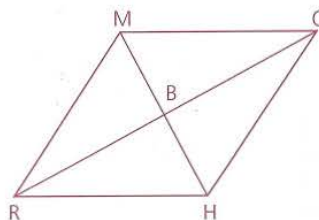
- 25 SQUA is a square. If one of the triangles shown in the figure is chosen at random, what is the probability that it is isosceles?



- 26 \overline{CM} is a median.
 a Find the coordinates of M.
 b Is \overline{CM} an altitude?
 c What type of triangle is $\triangle ABC$?



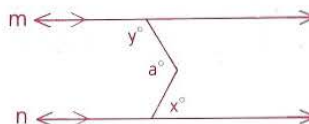
- 27 RHOM is a rhombus.
 $m\angle MBR = 21x + 13$,
 $MR = 6.2x$
 Find the length of \overline{RH} to the nearest tenth.



Problem Set C

- 28 TRAP is an isosceles trapezoid. The measure of one of its angles is 2.43 greater than 5.12 times the measure of another. If $m\angle T$ is less than $m\angle R$, find $\angle A$ to the nearest second.

- 29 Given: $m \parallel n$
 a Solve for a in terms of x and y .
 b If $a > 90$, what must be true of $y - x$?



- 30 In the solid box,
 $\square ABCD \cong \square EFGH$.
 Prove: $\overline{HF} \cong \overline{DB}$

