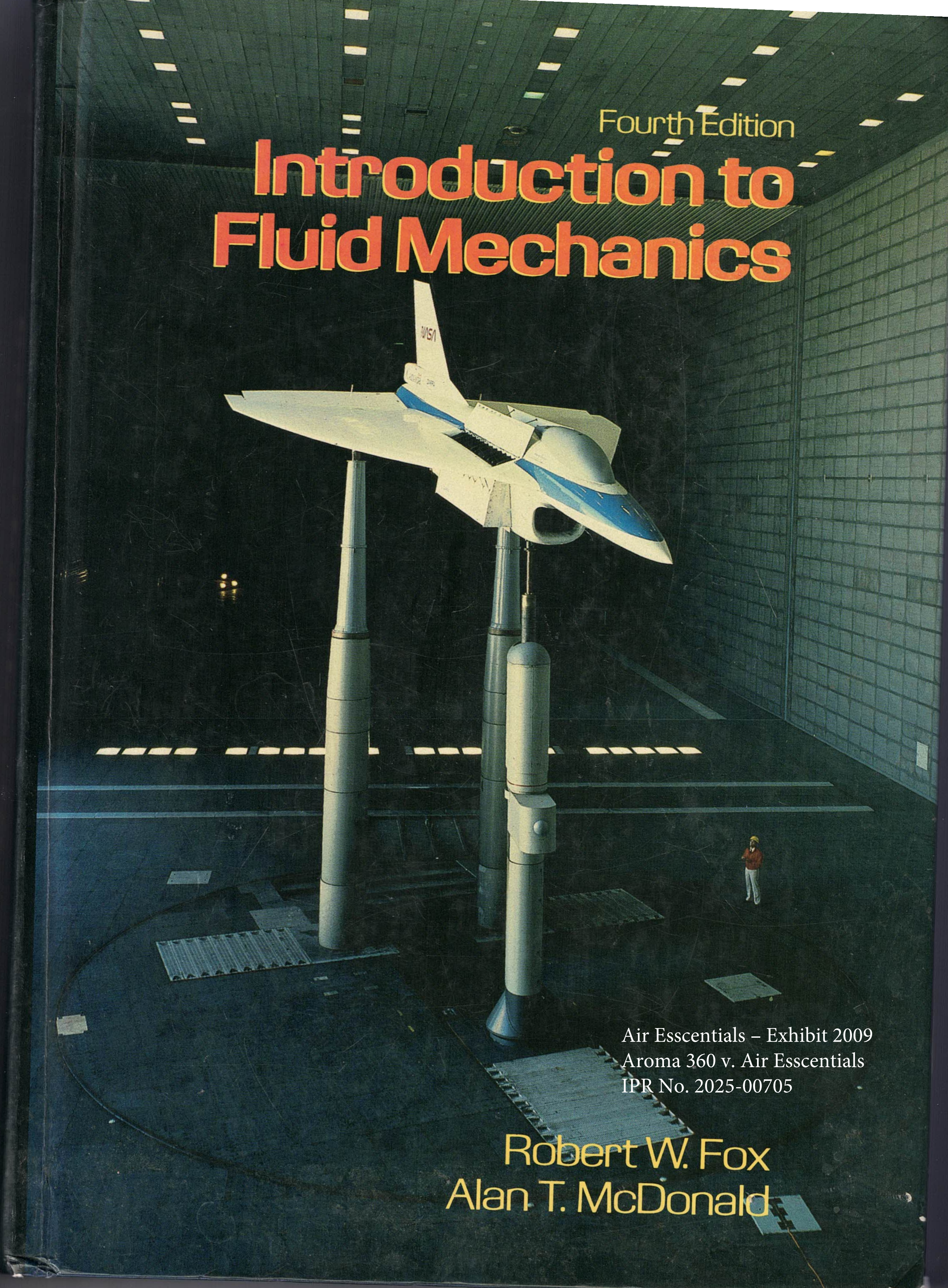

Fourth Edition

Introduction to Fluid Mechanics

Robert W. Fox
Alan T. McDonald

On the Cover: *A Large Wind Tunnel and Model*

The cover photo shows a full-size "model" of a short takeoff, vertical landing, supersonic aircraft. The model is mounted 40 ft above the floor of the world's largest wind tunnel—the new 80×120 ft test section—at NASA Ames Research Center. (The man on the tunnel floor shows the scale.)

This aircraft uses the ejector concept, in which relatively cool fan air is directed to the ejector system—the inlet louvers visible along the aircraft fuselage—to produce lift. This creates a relatively cool flow field on the ground beneath the vehicle. The remaining air is exhausted out the rear nozzle to provide thrust for forward flight.

A key test objective was to measure the thrust augmentation of the ejector system. Full-scale testing allows duplication of flight Reynolds number, which is important in the transition from hover to wing-borne flight following vertical takeoff. Large-scale testing also provides more reliable powerplant and ejector performance data than small-scale tests. The very large test section minimizes extraneous effects on vertical force data.

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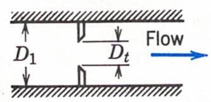
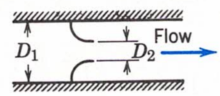
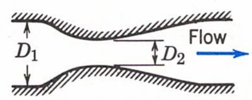
equations and curves of coefficients versus Reynolds number are given for specific metering elements in the next three subsections, following a general comparison of their characteristics.

As we have noted, selection of a flow meter depends on factors such as cost, accuracy, need for calibration, and ease of installation and maintenance. Some of these factors are compared for orifice plate, flow nozzle, and venturi meters in Table 8.5.

Table 8.5 Characteristics of Orifice, Flow Nozzle, and Venturi Flow Meters

Flow Meter Type	Diagram	Head Loss	Cost
Orifice		High	Low
Flow Nozzle		Intermediate	Intermediate
Venturi		Low	High

Flow meter coefficients reported in the literature have been measured with fully developed turbulent velocity distributions at the meter inlet (Section ①). If a flow meter is to be installed downstream from a valve, elbow, or other disturbance, a straight section of pipe must be placed in front of the meter. Approximately 10 diameters of straight pipe are required for venturi meters, and up to 40 diameters for orifice plate or flow nozzle meters. When a meter has been properly installed, the flow rate may be computed from Eqs. 8.49 or 8.51, after choosing an appropriate value for the empirical discharge coefficient, C, or flow coefficient, K, defined in Eqs. 8.47 and 8.50. Some design data for incompressible flow are given in the next few sections. The same basic methods can be extended to compressible flows, but these will not be treated here. For complete details, see [8] or [24].

8-10.1 The Orifice Plate

The orifice plate (Fig. 8.22) is a thin plate that may be clamped between pipe flanges. Since its geometry is simple, it is low in cost and easy to install or replace. The sharp edge of the orifice will not foul with scale or suspended matter. However, suspended matter can build up at the inlet side of a concentric orifice in a horizontal pipe; an eccentric orifice may be placed flush with the bottom of the pipe to avoid this difficulty. The primary disadvantages of the orifice are its limited capacity and the high permanent head loss due to the uncontrolled expansion downstream from the metering element.

Pressure taps for orifices may be placed in several locations, as shown in Fig. 8.22 (see [8] or [24] for additional details). Since the location of the pressure taps

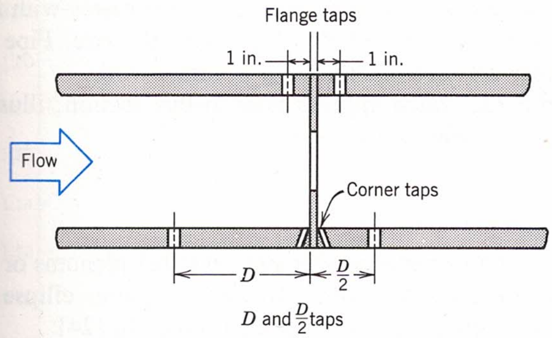


Fig. 8.22 Orifice geometry and pressure tap locations [8].

... the empirically determined flow coefficient, one must select handbook values of C or K consistent with the location of pressure taps.

The correlating equation recommended for a concentric orifice with corner taps is

$$C = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^{8} + \frac{91.71\beta^{2.5}}{Re_{D_1}^{0.75}} \tag{8.54}$$

Equation 8.54 predicts orifice discharge coefficients within ± 0.6 percent for $0.2 < \beta < 0.75$ and for $10^4 < Re_{D_1} < 10^7$. Some flow coefficients calculated from Eqs. 8.54 and 8.53 are presented in Fig. 8.23. Flow coefficients are relatively insensitive to Reynolds number for Re_{D_1} above 10^5 when $\beta > 0.5$.

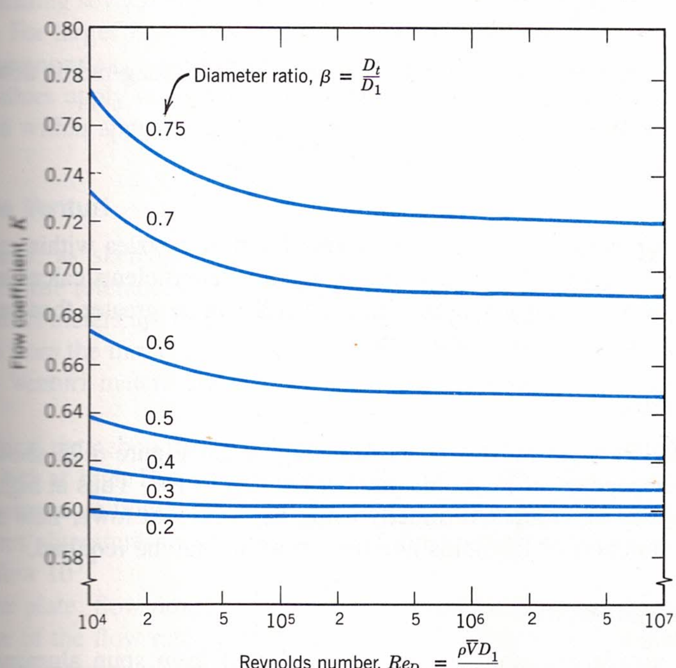


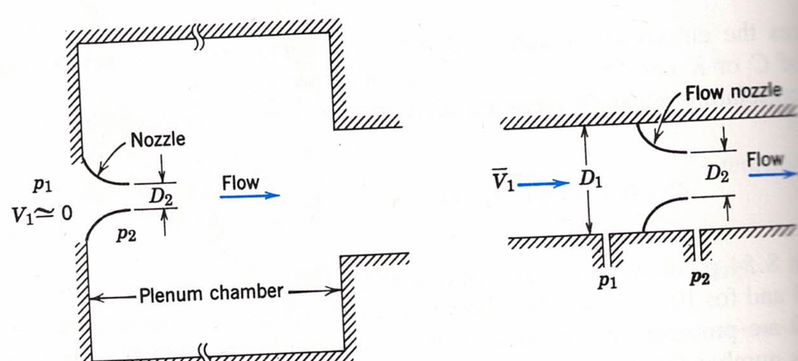
Fig. 8.23 Flow coefficients for concentric orifices with corner taps.

A similar correlating equation is available for orifice plates with D and $D/2$ taps. Flange taps require a different correlation for every line size. Pipe taps, located at $2\frac{1}{2}$ and $8\ D$, no longer are recommended for accurate work.

Example Problem 8.12, which appears later in this section, illustrates the application of flow coefficient data to orifice sizing.

8-10.2 The Flow Nozzle

Flow nozzles may be used as metering elements in either plenums or ducts, as shown in Fig. 8.24; the nozzle section is approximately a quarter ellipse. Design details and recommended locations for pressure taps are given in [24].



(a) In plenum

(b) In duct

Fig. 8.24 Typical installations of nozzle flow meters.

The correlating equation recommended for an ASME long-radius flow nozzle [is

$$C = 0.9975 - \frac{6.53\beta^{0.5}}{Re_{D_1}^{0.5}} \tag{8.55}$$

Equation 8.55 predicts discharge coefficients for flow nozzles within ± 2.0 percent for $0.25 < \beta < 0.75$ for $10^4 < Re_{D_1} < 10^7$. Some flow coefficients calculated from Eq. 8.55 and Eq. 8.50 are presented in Fig. 8.25. (K can be greater than one when the velocity of approach factor exceeds one.)

a. Pipe Installation

For pipe installation, K must be read from Fig. 8.25. Figure 8.25 shows that K is essentially independent of Reynolds number for $Re_{D_1} > 10^6$. Thus at high flow rates the flow rate may be computed directly using Eq. 8.51. At lower flow rates, where K is a weak function of Reynolds number, iteration may be required.

b. Plenum Installation

For plenum installation, nozzles may be fabricated from spun aluminum, molded fiberglass, or other inexpensive materials. Thus they are simple and cheap to make and install. Since the plenum pressure is equal to p_2, the location of the downstream

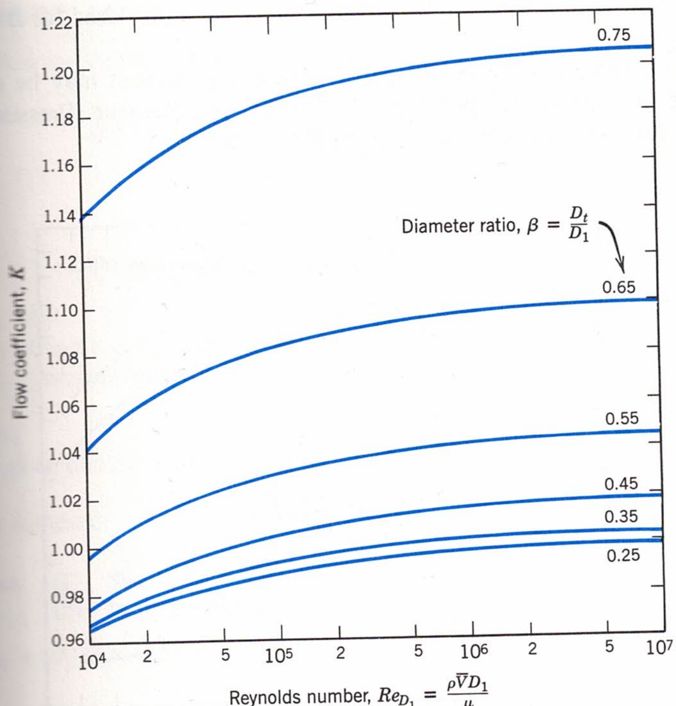


Fig. 8.25 Flow coefficients for ASME long-radius flow nozzles.

pressure tap is not critical. Meters suitable for a wide range of flow rates may be made by installing several nozzles in a plenum. At low flow rates, most of them may be plugged. For larger flow rates, more nozzles may be used.

For plenum nozzles, typical flow coefficients are in the range, $0.95 < K < 0.99$; the larger values apply at high Reynolds numbers. Thus the mass rate of flow can be computed within approximately ± 2 percent using Eq. 8.51 with $K = 0.97$.

The Venturi

Venturi meters, as sketched in Table 8.5, are generally made from castings and machined to close tolerances to duplicate the performance of the standard design. As a result, venturi meters are heavy, bulky, and expensive. The conical diffuser section downstream from the throat gives excellent pressure recovery; therefore, overall head loss is low. Venturi meters are also self-cleaning because of their smooth internal contours.

Experimental data show that discharge coefficients for venturi meters range from 0.980 to 0.995 at high Reynolds numbers ($Re_{D_1} > 2 \times 10^5$). Thus $C = 0.99$ can be used to measure mass flow rate within about ± 1 percent at high Reynolds number [8]. Manufacturers' literature should be consulted for specific information at Reynolds numbers below 10^5.

The orifice plate, flow nozzle, and venturi all produce pressure drops proportional to the square of the flow rate, according to Eq. 8.51. In practice, a meter size must be chosen to accommodate the largest flow rate expected. Because the pressure drop versus flow rate relationship is nonlinear, a limited range of flow rate can be measured accurately. Flow meters with single throats usually are considered for flow rates over a 4:1 range [8].

The unrecoverable loss in head across a metering element may be expressed as a fraction of the differential pressure, Δp, across the element. Pressure losses are displayed as functions of diameter ratio in Fig. 8.26 [8].

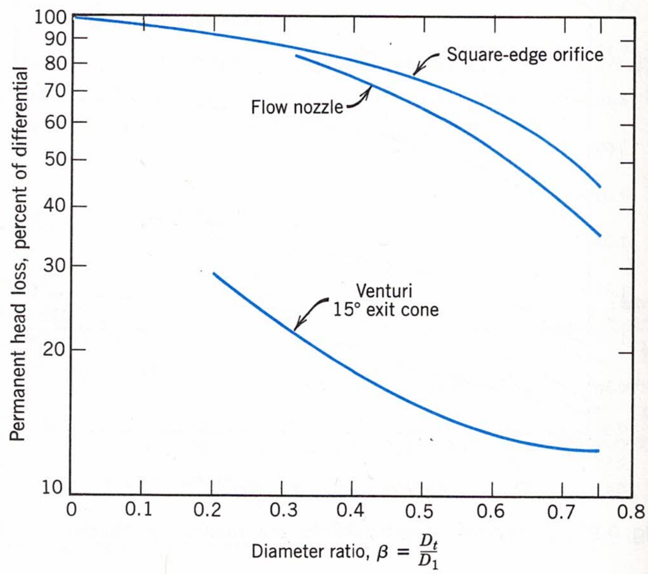


Fig. 8.26 Permanent head loss produced by various flow metering elements [8].

8-10.4 The Laminar Flow Element

The laminar flow element[5] is designed to produce a pressure differential directly proportional to flow rate. The laminar flow element (LFE) contains a metering section subdivided into many passages, each small enough in diameter to assure fully developed laminar flow. As shown in Section 8-3, the pressure drop in laminar duct flow is directly proportional to flow rate. Because the pressure drop versus flow rate relationship is linear, the LFE may be used with reasonable accuracy over a 10:1 range of flow rate. The relationship between the pressure drop and flow rate for laminar flow also depends on viscosity, which is a strong function of temperature. Therefore, the fluid temperature must be known to obtain accurate metering with a LFE.

A laminar flow element costs approximately as much as a venturi, but it is much lighter and smaller. Thus the LFE is becoming widely used in applications where compactness and extended range are important.

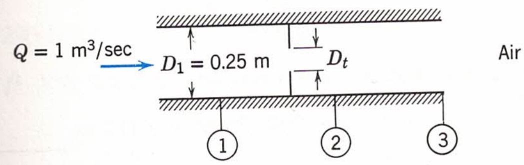
EXAMPLE 8.12—Flow through an Orifice Meter

An air flow rate of 1 m^3/sec at standard conditions is expected in a 0.25 m diameter duct. An orifice meter is used to measure the rate of flow. The manometer available to make the measurement has a maximum range of 300 mm of water. What diameter orifice plate should be used with corner taps? Analyze the head loss if the flow area at the vena contracta is $A_2 = 0.65\ A_t$. Compare with data from Fig. 8.26.

[5] Patented and manufactured by Meriam Instrument Co., 10920 Madison Ave., Cleveland, Ohio.

PROBLEM 8.12

Flow through duct and orifice as shown.



$(p_1 - p_2)_{max} = 300$ mm H_2O

(a) D_t.
(b) Head loss between sections ① and ③.
(c) Compare with data from Fig. 8.26.

plate may be designed using Eq. 8.51 and data from Fig. 8.23.

equation:
$$\dot{m}_{actual} = KA_t\sqrt{2\rho(p_1 - p_2)} \qquad (8.51)$$

(1) Steady flow
(2) Incompressible flow

$A_t/A_1 = (D_t/D_1)^2 = \beta^2$,

$$\dot{m}_{actual} = K\beta^2 A_1\sqrt{2\rho(p_1 - p_2)}$$

$$K\beta^2 = \frac{\dot{m}_{actual}}{A_1\sqrt{2\rho(p_1 - p_2)}} = \frac{\rho Q}{A_1\sqrt{2\rho(p_1 - p_2)}} = \frac{Q}{A_1}\sqrt{\frac{\rho}{2(p_1 - p_2)}}$$

$$= \frac{Q}{A_1}\sqrt{\frac{\rho}{2g\rho_{H_2O}\Delta h}}$$

$$= 1\,\frac{\text{m}^3}{\text{sec}} \times \frac{4}{\pi}\frac{1}{(0.25)^2\ \text{m}^2}\left[\frac{1}{2} \times 1.23\,\frac{\text{kg}}{\text{m}^3} \times \frac{\text{sec}^2}{9.81\ \text{m}} \times \frac{\text{m}^3}{999\ \text{kg}} \times \frac{1}{0.30\ \text{m}}\right]^{1/2}$$

$$K\beta^2 = 0.295 \quad \text{or} \quad K = \frac{0.295}{\beta^2} \qquad (1)$$

function of both β and Re_{D_1}, we must iterate to find β. The duct Reynolds

$$Re_{D_1} = \frac{\rho \bar{V}_1 D_1}{\mu} = \frac{\rho(Q/A_1)D_1}{\mu} = \frac{4Q}{\pi \nu D_1}$$

$$Re_{D_1} = \frac{4}{\pi} \times 1\,\frac{\text{m}^3}{\text{sec}} \times \frac{\text{sec}}{1.45 \times 10^{-5}\ \text{m}^2} \times \frac{1}{0.25\ \text{m}} = 3.51 \times 10^5$$

$\beta = 0.75$. From Fig. 8.23, K should be 0.72. From Eq. 1,

$$K = \frac{0.295}{(0.75)^2} = 0.524$$

for β is too large. Guess $\beta = 0.70$. From Fig. 8.23, K should be 0.69.

$$K = \frac{0.295}{(0.70)^2} = 0.602$$

Thus our guess for β is still too large. Guess $\beta = 0.65$. From Fig. 8.23, K [illegible] 0.67. From Eq. 1,

$$K = \frac{0.295}{(0.65)^2} = 0.698$$

There is satisfactory agreement with $\beta \simeq 0.66$ and

$$D_t = \beta D_1 = 0.66(0.25\text{ m}) = 0.165\text{ m}$$

To evaluate the permanent head loss, apply Eq. 8.28 between sections ① and ③.

Computing equation:

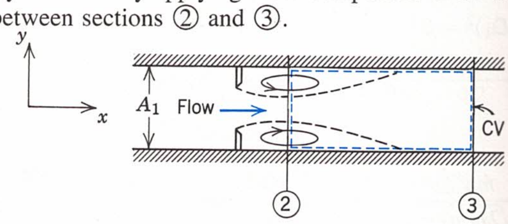
$$\left(\frac{p_1}{\rho} + \alpha_1 \cancel{\frac{\bar{V}_1^2}{2}} + g\cancel{z_1}\right) - \left(\frac{p_3}{\rho} + \alpha_3 \cancel{\frac{\bar{V}_3^2}{2}} + g\cancel{z_3}\right) = h_{l_T}$$

Assumptions: (3) $\alpha_1 \bar{V}_1^2 = \alpha_3 \bar{V}_3^2$
(4) Neglect Δz

Then

$$h_{l_T} = \frac{p_1 - p_3}{\rho} = \frac{p_1 - p_2 - (p_3 - p_2)}{\rho}$$

The pressure at ③ may be found by applying the x component of the momentum equation to a control volume between sections ② and ③.



$A_2 = A_{\text{vena contracta}}$

Basic equation:

$$F_{S_x} + \cancel{F_{B_x}} = \cancel{\frac{\partial}{\partial t}\int_{\text{CV}} u\rho\, d\forall} + \int_{\text{CS}} u\rho\vec{V}\cdot d\vec{A}$$

$= 0(5) \quad = 0(1)$

Assumptions: (5) $F_{B_x} = 0$
(6) Uniform flow at sections ② and ③
(7) Pressure uniform across duct at sections ② and ③
(8) Neglect friction force on CV

Then

$$(p_2 - p_3)A_1 = u_2\{-|\rho\bar{V}_2A_2|\} + u_3\{|\rho\bar{V}_3A_3|\} = (u_3 - u_2)\rho Q = (\bar{V}_3 - \bar{V}_2)\rho Q$$

or

$$p_3 - p_2 = (\bar{V}_2 - \bar{V}_3)\frac{\rho Q}{A_1}$$

Now $\bar{V}_3 = Q/A_1$, and

$$\bar{V}_2 = \frac{Q}{A_2} = \frac{Q}{0.65A_t} = \frac{Q}{0.65\beta^2 A_1}$$

Thus

$$p_3 - p_2 = \frac{\rho Q^2}{A_1^2}\left[\frac{1}{0.65\beta^2} - 1\right]$$

$$p_1 - p_2 = 1.23\ \frac{\text{kg}}{\text{m}^3} \times (1)^2\ \frac{\text{m}^6}{\text{sec}^2} \times \frac{4^2}{\pi^2}\ \frac{1}{(0.25)^4\ \text{m}^4}\left[\frac{1}{0.65(0.66)^2} - 1\right]\frac{\text{N}\cdot\text{sec}^2}{\text{kg}\cdot\text{m}}$$

$$p_1 - p_2 = 1290\ \text{N/m}^2$$

ratio, β, was selected to give maximum manometer deflection at maximum . Thus

$$p_1 - p_2 = \rho_{\text{H}_2\text{O}}\, g\, \Delta h = 999\ \frac{\text{kg}}{\text{m}^3} \times 9.81\ \frac{\text{m}}{\text{sec}^2} \times 0.30\ \text{m} \times \frac{\text{N}\cdot\text{sec}^2}{\text{kg}\cdot\text{m}} = 2940\ \text{N/m}^2$$

into Eq. 2 gives

$$h_{l_T} = \frac{p_1 - p_3}{\rho} = \frac{p_1 - p_2 - (p_3 - p_2)}{\rho}$$

$$h_{l_T} = (2940 - 1290)\ \frac{\text{N}}{\text{m}^2} \times \frac{\text{m}^3}{1.23\ \text{kg}} = 1340\ \text{N}\cdot\text{m/kg}$$

h_{l_T}

with Fig. 8.26, express the permanent pressure loss as a fraction of the meter

$$\frac{p_1 - p_3}{p_1 - p_2} = \frac{(2940 - 1290)\ \text{N/m}^2}{2940\ \text{N/m}^2} = 0.561$$

from Fig. 8.26 is about 0.57. This is satisfactory agreement!

illustrates flow meter calculations and shows use of the momentum compute the pressure rise in a sudden expansion.

FLOW METERS

meter types produce outputs that are directly proportional to flow rate. produce signals without the need to measure differential pressure. The linear flow meters are discussed briefly in the following paragraphs. may be used to indicate flow rate directly for liquids or gases. An shown in Fig. 8.27. In operation, the ball or float is carried upward clear tube by the flowing fluid until the drag force and float weight

Fig. 8.27 Float-type variable-area flow meter. (Courtesy of Dwyer Instrument Co., Michigan City, Indiana.)