COMMUNICATION SYSTEMS ENGINEERING

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To Felia, George, and Elena.
—John G. Proakis

To Fariba, Omid, Sina, and my parents.
—Masoud Salehi

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Fourier series). In the Fourier transform for periodic signals, we started with a (time) periodic signal and showed that its Fourier transform consists of a sequence of impulses. Therefore, to define the signal, it was enough to give the weights of these impulses (Fourier series coefficients). In the sampling theorem, we started with an impulse-sampled signal, or a sequence of impulses in the time domain, and showed that the Fourier transform is a periodic function in the frequency domain. Here again, the values of the samples are enough to define the signal completely. This similarity is a consequence of the duality between the time and frequency domains and the fact that both the Fourier series expansion and reconstruction from samples are orthogonal expansions, one in terms of the exponential signals and the other in terms of the sinc functions. This fact will be further explored in the problems.

2.5 BANDPASS SIGNALS

In this section, we examine time domain and frequency domain characteristics of a class of signals frequently encountered in communication system analysis. This class of signals is the class of *bandpass* or *narrowband* signals. The concept of bandpass signals is a generalization of the concept of monochromatic signals, and our analysis of the properties of these signals follows that used in analyzing monochromatic signals.

Definition 2.5.1. A *bandpass* or *narrowband* signal is a signal $x(t)$ whose frequency domain representation $X(f)$ is nonzero for frequencies in a usually small neighborhood of some high frequency f_0; i.e., $X(f) \equiv 0$ for $|f - f_0| \geq W$, where $W < f_0$. A *bandpass system* is a system which passes signals with frequency components in the neighborhood of some high frequency f_0; i.e., $H(f) = 1$ for $|f - f_0| \leq W$ and highly attenuates frequency components outside of this frequency band. Alternatively, we may say that a bandpass system is one whose impulse response is a bandpass signal.

Note that in the above definition, f_0 need not be the center of the signal bandwidth, or be located in the signal bandwidth at all. In fact, all the spectra shown in Figure 2.11 satisfy the definition of a bandpass signal.

With the above precautions, the frequency f_0 is usually referred to as the *central frequency* of the bandpass signal. A monochromatic signal is a bandpass signal for which $W = 0$. A large class of signals used for information transmission, the *modulated signals,* are examples of bandpass signals or at least closely represented by bandpass signals. Throughout this section, we assume that the bandpass signal $x(t)$ is real valued.

To begin our development of bandpass signals, let us start with the tools used in the analysis of systems or circuits driven by monochromatic (or sinusoidal) signals. Let $x(t) = A\cos(2\pi f_0 t + \theta)$ denote a monochromatic signal. To analyze a circuit driven by this signal we first introduce the *phasor* corresponding to this signal as $X = Ae^{j\theta}$, which contains the information about the amplitude and phase of the signal but does not have any information concerning the frequency of it. To find the output of a linear time invariant circuit driven by this sinusoidal signal, it is enough to multiply the phasor of the excitation signal by the value of the frequency response of the system computed at

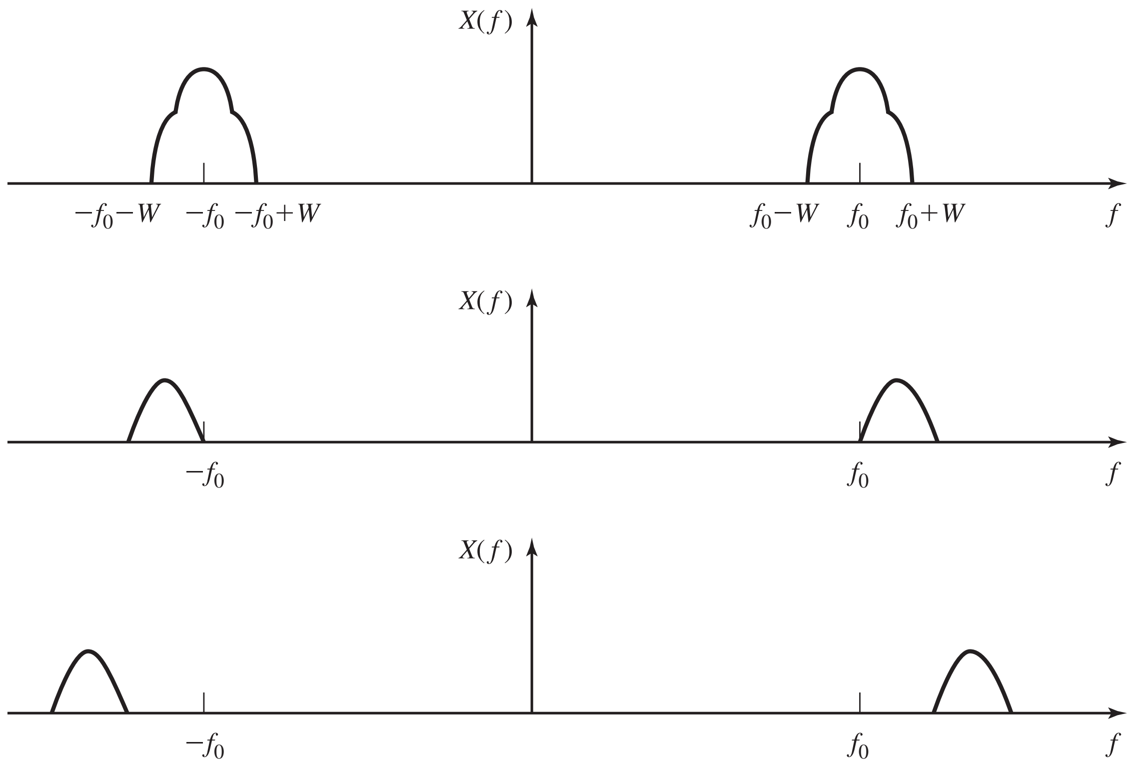

Figure 2.11 Examples of narrowband signals.

the input frequency to obtain the phasor corresponding to the output. From the output phasor, we can find the output signal by noting that the input and output frequencies are the same. To obtain the phasor corresponding to the input, we first introduce the signal $z(t)$ as

$$\begin{aligned} z(t) &= Ae^{j(2\pi f_0 t+\theta)} \\ &= A\cos(2\pi f_0 t+\theta) + jA\sin(2\pi f_0 t+\theta) \\ &= x(t) + jx_q(t) \end{aligned}$$

where $x_q(t) = A\sin(2\pi f_0 t+\theta)$ is a 90° phase shift version of the original signal and the subscript stands for *quadrature*. Note that $z(t)$ represents a vector rotating at an angular frequency equal to $2\pi f_0$ as shown in Figure 2.12, and X, the phasor is obtained from $z(t)$ by deleting the rotation at the angular frequency of $2\pi f_0$, or equivalently by

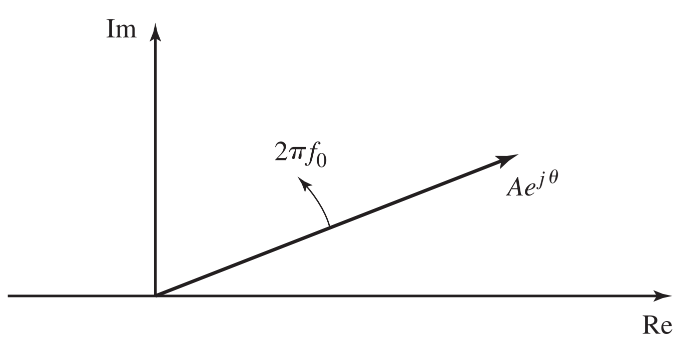

Figure 2.12 Phasor of a monochromatic signal.

rotating the vector corresponding to $z(t)$ at an angular frequency equal to $2\pi f_0$ in the opposite direction, which is equivalent to multiplying by $e^{-j2\pi f_0 t}$, or

$$X = z(t)e^{-j2\pi f_0 t}$$

In the frequency domain, this is equivalent to shifting $Z(f)$ to the left by f_0. Also note that the frequency domain representation of $Z(f)$ is obtained by deleting the negative frequencies from $X(f)$ and multiplying the positive frequencies by 2.

To obtain a parallel development for narrowband signals, we start with a signal corresponding to $z(t)$. We define $z(t)$ for narrowband signals in the same way that $z(t)$ was defined for monochromatic signals; i.e., by multiplying the positive frequencies in $X(f)$ by 2 and deleting the negative frequencies. By doing this we have

$$Z(f) = 2u_{-1}(f)X(f) \tag{2.5.1}$$

The signal $z(t)$ defined by the above relation is called the *analytic signal* corresponding to $x(t)$, or *pre-envelope* of $x(t)$. To obtain the time domain representation of $z(t)$, we first start with finding a signal whose Fourier transform is $u_{-1}(f)$. From Table 2.1 we know that

$$\mathscr{F}[u_{-1}(t)] = \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$$

Applying the duality theorem, we obtain

$$\mathscr{F}\left[\frac{1}{2}\delta(t) + \frac{j}{2\pi t}\right] = u_{-1}(f) \tag{2.5.2}$$

Now, using the convolution theorem, we have

$$\begin{aligned} z(t) &= \left(\delta(t) + \frac{j}{\pi t}\right) \star x(t) \\ &= x(t) + j\frac{1}{\pi t} \star x(t) \\ &= x(t) + j\hat{x}(t) \end{aligned} \tag{2.5.3}$$

where

$$\hat{x}(t) = \frac{1}{\pi t} \star x(t) \tag{2.5.4}$$

Comparing this result with the corresponding monochromatic result

$$z(t) = \underbrace{A\cos(2\pi f_0 t + \theta)}_{x(t)} + j\,\underbrace{A\sin(2\pi f_0 t + \theta)}_{\hat{x}(t)} \tag{2.5.5}$$

we see that $\hat{x}(t)$ plays the same role as $A\sin(2\pi f_0 t + \theta)$. $\hat{x}(t)$ is called the *Hilbert transform* of $x(t)$. The name "transform" is somewhat misleading because there is no change of domain involved, (as is the case for Fourier, Laplace, and Z transforms, for example). In fact, Hilbert transform is a simple filter (seen from the fact that it can be expressed in terms of a convolution integral). To see what the Hilbert transform does in the frequency domain we note that

$$\begin{aligned}
\mathscr{F}\left[\frac{1}{\pi t}\right] &= -j\,\mathrm{sgn}(f) \\
&= \begin{cases} -j & f > 0 \\ 0 & f = 0 \\ +j & f < 0 \end{cases} \\
&= \begin{cases} e^{-j\frac{\pi}{2}} & f > 0 \\ 0 & f = 0 \\ e^{j\frac{\pi}{2}} & f < 0 \end{cases} \\
&= e^{-j\frac{\pi}{2}\mathrm{sgn}(f)}
\end{aligned} \tag{2.5.6}$$

This means that the Hilbert transform is equivalent to a $-\frac{\pi}{2}$ phase shift for positive frequencies and $+\frac{\pi}{2}$ phase shift for negative frequencies and can be represented by a filter with transfer function $H(f) = -j\,\mathrm{sgn}(f)$. This filter is called a *quadrature filter,* emphasizing its role in providing a 90° phase shift. In the problems, we will investigate some of the most important properties of the Hilbert transform.

To obtain the equivalent of a "phasor" for the bandpass signal we have to shift the spectrum of $z(t)$; i.e., $Z(f)$, to the left by f_0 to obtain a signal denoted by $x_l(t)$, which is the *lowpass representation of the bandpass signal* $x(t)$. Hence,

$$X_l(f) = Z(f + f_0) = 2u_{-1}(f + f_0)X(f + f_0) \tag{2.5.7}$$

and

$$x_l(t) = z(t)e^{-j2\pi f_0 t} \tag{2.5.8}$$

Figure 2.13 shows $Z(f)$ and $X_l(f)$ corresponding to a bandpass signal $x(t)$.

As seen $x_l(t)$ is a lowpass signal, meaning that its frequency components are located around the zero frequency, or $X_l(f) \equiv 0$ for $|f| \geq W$ where $W < f_0$. $x_l(t)$ plays the role of the phasor for bandpass signals. In general, $x_l(t)$ is a complex signal having $x_c(t)$ and $x_s(t)$ as its real and imaginary parts respectively; i.e.,

$$x_l(t) = x_c(t) + jx_s(t) \tag{2.5.9}$$

$x_c(t)$ and $x_s(t)$ are lowpass signals, called *in-phase* and *quadrature* components of the bandpass signal $x(t)$. Substituting for $x_l(t)$ and rewriting $z(t)$, we obtain

$$\begin{aligned}
z(t) &= x(t) + j\hat{x}(t) \\
&= x_l(t)e^{j2\pi f_0 t} \\
&= (x_c(t) + jx_s(t))e^{j2\pi f_0 t} \\
&= (x_c(t)\cos(2\pi f_0 t) - x_s(t)\sin(2\pi f_0 t)) \\
&\quad + j(x_c(t)\sin(2\pi f_0 t) + x_s(t)\cos(2\pi f_0 t))
\end{aligned} \tag{2.5.10}$$

Equating the real and imaginary parts, we have

$$x(t) = x_c(t)\cos(2\pi f_0 t) - x_s(t)\sin(2\pi f_0 t) \tag{2.5.11}$$

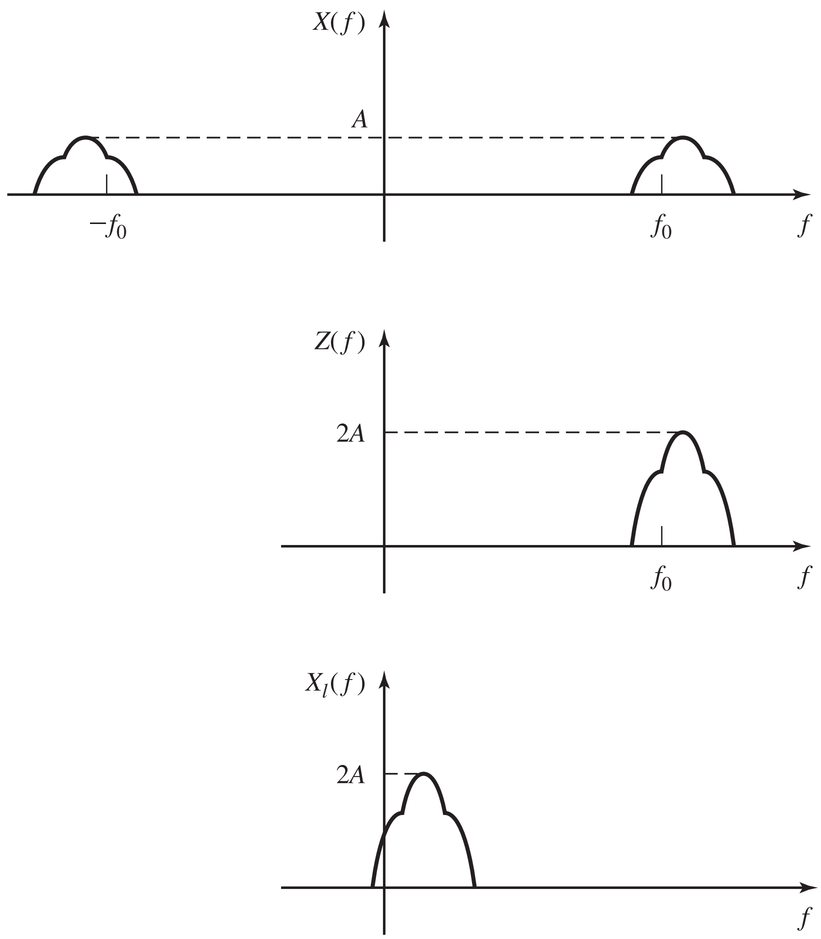

Figure 2.13 $Z(f)$ and $X_l(f)$ corresponding to $x(t)$.

and

$$\hat{x}(t) = x_c(t)\sin(2\pi f_0 t) + x_s(t)\cos(2\pi f_0 t) \tag{2.5.12}$$

These relations give $x(t)$ and $\hat{x}(t)$ in terms of two lowpass quadrature component signals $x_c(t)$ and $x_s(t)$ and are known as *bandpass to lowpass transformation relations*.

If we define $V(t)$, the *envelope* of $x(t)$ as

$$V(t) = \sqrt{x_c^2(t) + x_s^2(t)} \tag{2.5.13}$$

and $\Theta(t)$, the *phase* of $x(t)$, as

$$\Theta(t) = \arctan\frac{x_s(t)}{x_c(t)} \tag{2.5.14}$$

we can write

$$x_l(t) = V(t)e^{j\Theta(t)} \tag{2.5.15}$$

which looks more like the familiar phasor relation $X = Ae^{j\theta}$. The only difference is that in this case the envelope $(V(t))$ and phase $(\Theta(t))$ are both (slowly) time-varying functions. Therefore, in contrast to the monochromatic phasor which has constant

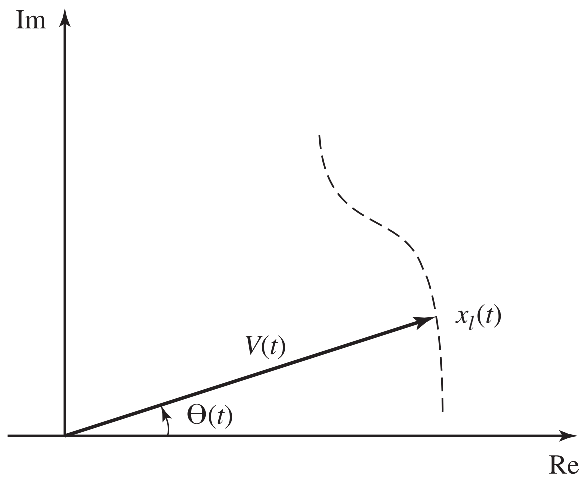

Figure 2.14 The phasor of a bandpass signal.

amplitude and phase, the envelope and phase of a bandpass signal vary slowly with time, and therefore the vector representation of it moves on a curve in the complex plane (see Figure 2.14).

Substituting $x_l(t) = V(t)e^{j\Theta(t)}$ in $z(t)$ in Equation (2.5.10), we obtain

$$\begin{aligned} z(t) &= x(t) + j\hat{x}(t) \\ &= x_l(t)e^{j2\pi f_0 t} \\ &= V(t)e^{j\Theta(t)}e^{j2\pi f_0 t} \\ &= V(t)\cos(2\pi f_0 t + \Theta(t)) + jV(t)\sin(2\pi f_0 t + \Theta(t)) \end{aligned} \tag{2.5.16}$$

from which we have

$$x(t) = V(t)\cos(2\pi f_0 t + \Theta(t)) \tag{2.5.17}$$

and

$$\hat{x}(t) = V(t)\sin(2\pi f_0 t + \Theta(t)) \tag{2.5.18}$$

These relations show why $V(t)$ and $\Theta(t)$ are called the envelope and phase of the signal $x(t)$. Figure 2.15 shows the relation between $x(t)$, $V(t)$, and $\Theta(t)$.

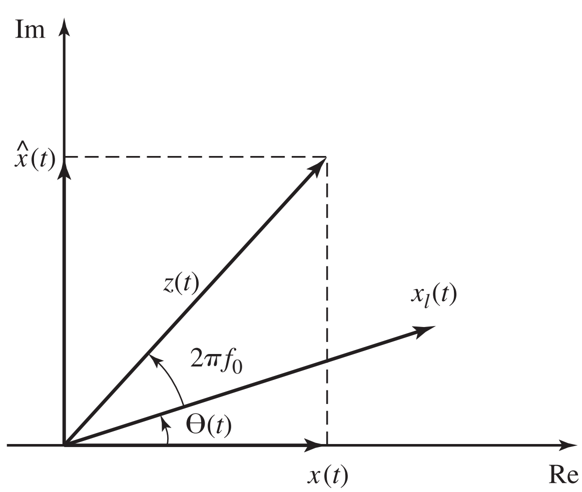

Figure 2.15 The envelope and phase of a bandpass signal.

Example 2.5.1

Show that $X(f)$ can be written in terms of $X_l(f)$ as

$$X(f) = \tfrac{1}{2}[X_l(f - f_0) + X_l^*(-f - f_0)] \tag{2.5.19}$$

Solution To obtain $X(f)$ from $X_l(f)$, we do exactly the inverse of what we did to get $X_l(f)$ from $X(f)$. First we shift $X_l(f)$ to the right by f_0 to get $Z(f)$. We have

$$Z(f) = X_l(f - f_0)$$

To get $X(f)$ from $Z(f)$, we have to multiply the positive frequencies by a factor of $\frac{1}{2}$ and reconstruct the negative frequencies. Since $x(t)$ is assumed to be real valued, its Fourier transform has Hermitian symmetry (see Section 2.2.1). Therefore, if we write

$$X(f) = X_+(f) + X_-(f)$$

where $X_+(f)$ denotes the positive frequency part of $X(f)$ and $X_-(f)$ denotes the negative frequency part, we have

$$X_-(f) = X_+^*(-f)$$

Since

$$X_+(f) = \tfrac{1}{2}X_l(f - f_0)$$

we obtain

$$X(f) = \tfrac{1}{2}[X_l(f - f_0) + X_l^*(-f - f_0)]$$

The relation between the various signals discussed in this section are summarized in Table 2.2.

Transmission of Bandpass Signals through Bandpass Systems. In the same way that phasors make analysis of systems driven by monochromatic signals easier, lowpass equivalents of bandpass signals can be employed to find the outputs of bandpass systems driven by bandpass signals. Let $x(t)$ be a bandpass signal with center frequency f_0, and let $h(t)$ be the impulse response of an LTI system. Let us assume that $h(t)$ is narrowband with the same center frequency as $x(t)$. To find $y(t)$, the output of the system when driven by $x(t)$, we use frequency domain analysis. In the frequency domain, we have $Y(f) = X(f)H(f)$. The signal $y(t)$ is obviously a bandpass signal, and therefore it has a lowpass equivalent $y_l(t)$. To obtain $Y_l(f)$ we have

$$\begin{aligned} Y_l(f) &= 2u_{-1}(f + f_0)Y(f + f_0) \\ &= 2u_{-1}(f + f_0)X(f + f_0)H(f + f_0) \end{aligned} \tag{2.5.20}$$

By writing $H(f)$ and $X(f)$ in terms of their lowpass equivalents, we obtain

$$X_l(f) = 2u_{-1}(f + f_0)X(f + f_0)$$

the most likely errors caused by noise involve the erroneous selection of an adjacent phase to the transmitted phase, only a single bit error occurs in the k-bit sequence with Gray encoding.

The Euclidean distance between any two signal points in the constellation is

$$d_{mn} = \sqrt{\|\mathbf{s}_m - \mathbf{s}_n\|^2} = \sqrt{2\mathcal{E}_s\left(1 - \cos\frac{2\pi(m-n)}{M}\right)} \tag{7.3.18}$$

and the minimum Euclidean distance (distance between two adjacent signal points) is simply

$$d_{\min} = \sqrt{2\mathcal{E}_s\left(1 - \cos\frac{2\pi}{M}\right)} \tag{7.3.19}$$

As we shall demonstrate in Equation (7.6.10), the minimum Euclidean distance $d_{\min}$ plays an important role in determining the error-rate performance of the receiver that demodulates and detects the information in the presence of additive Gaussian noise.

7.3.3 Two-dimensional Bandpass Signals—Quadrature Amplitude Modulation

In our discussion of carrier-phase modulation, we observed that the bandpass signal waveforms may be represented as given by Equation (7.3.15), in which the signal waveforms are viewed as two orthogonal carrier signals, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$, modulated by the information bits. However, the carrier-phase modulation signal waveforms are constrained to have equal energy $\mathcal{E}_s$, which implies that the signal points in the geometric representation of the signal waveforms lie on a circle of radius $\sqrt{\mathcal{E}_s}$. If we remove the constant energy restriction, we can construct signal waveforms that are not constrained to fall on a circle.

The simplest way to construct such signals is to impress separate information bits on each of the quadrature carriers, $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$. This type of digital modulation is called *quadrature amplitude modulation (QAM)*. We may view this method of information transmission as a form of quadrature-carrier multiplexing, previously described in Section 3.2.6.

The transmitted signal waveforms have the form

$$u_m(t) = A_{mc} g_T(t) \cos 2\pi f_c t + A_{ms} g_T(t) \sin 2\pi f_c t, \quad m = 1, 2, \ldots, M \tag{7.3.20}$$

where $\{A_{mc}\}$ and $\{A_{ms}\}$ are the sets of amplitude levels that are obtained by mapping k-bit sequences into signal amplitudes. For example, Figure 7.21 illustrates a 16-QAM signal constellation that is obtained by amplitude modulating each quadrature carrier by $M = 4$ PAM. In general, rectangular signal constellations result when two quadrature carriers are each modulated by PAM.

More generally, QAM may be viewed as a form of combined digital amplitude and digital-phase modulation. Thus, the transmitted QAM signal waveforms may be

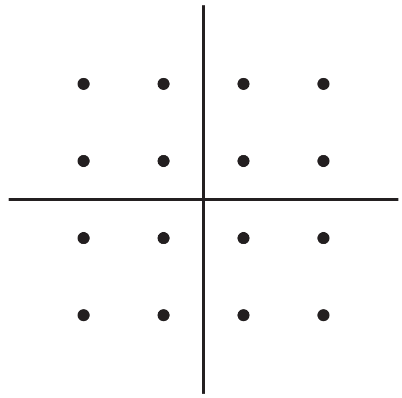

Figure 7.21 $M = 16$-QAM signal constellation.

expressed as

$$u_{mn}(t) = A_m g_T(t)\cos(2\pi f_c t + \theta_n), \quad \begin{array}{l} m = 1, 2, \ldots, M_1 \\ n = 1, 2, \ldots, M_2 \end{array} \tag{7.3.21}$$

If $M_1 = 2^{k_1}$ and $M_2 = 2^{k_2}$, the combined amplitude- and phase-modulation method results in the simultaneous transmission of $k_1 + k_2 = \log_2 M_1 M_2$ binary digits occurring at a symbol rate $R_b/(k_1 + k_2)$. Figure 7.22 illustrates the functional block diagram of a QAM modulator.

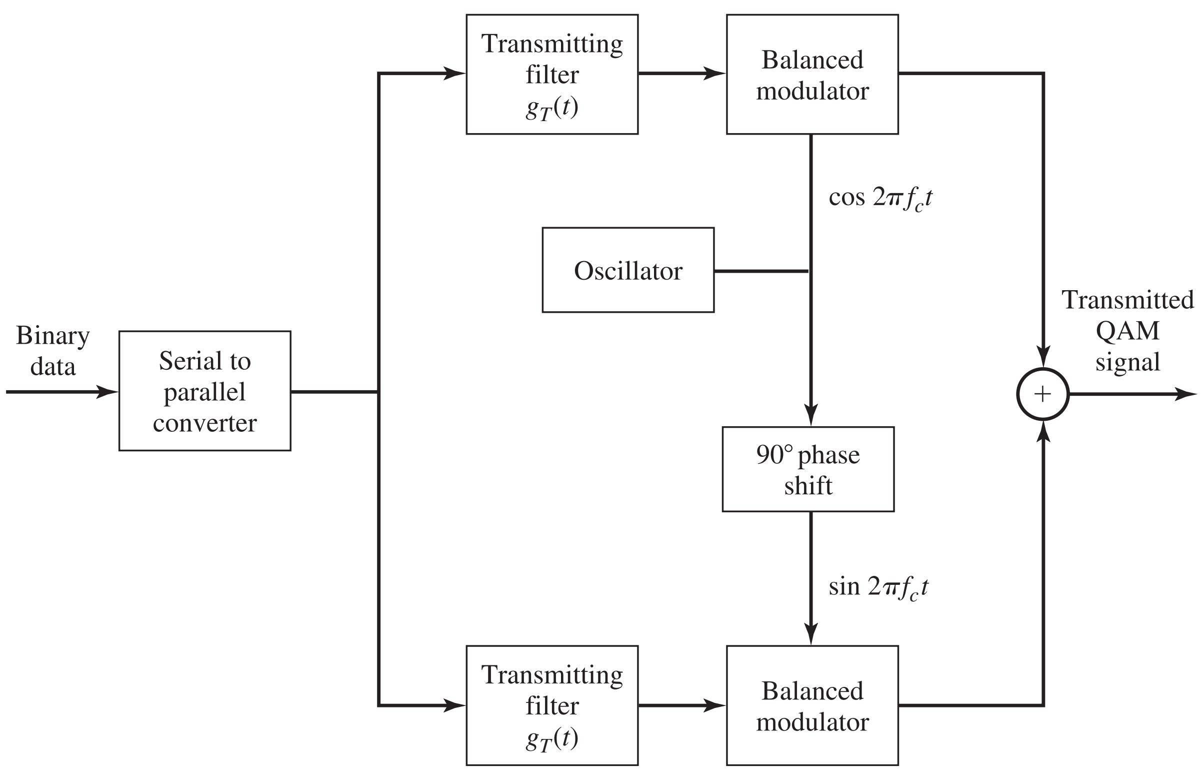

Figure 7.22 Functional block diagram of modulator for QAM.

It is clear that the geometric signal representation of the signals given by Equations (7.3.20) and (7.3.21) is in terms of two-dimensional signal vectors of the form

$$s_m = \left(\sqrt{\mathcal{E}_s}\, A_{mc}, \sqrt{\mathcal{E}_s}\, A_{ms}\right), \quad m = 1, 2, \ldots, M \tag{7.3.22}$$

Examples of signal space constellations for QAM are shown in Figure 7.23.

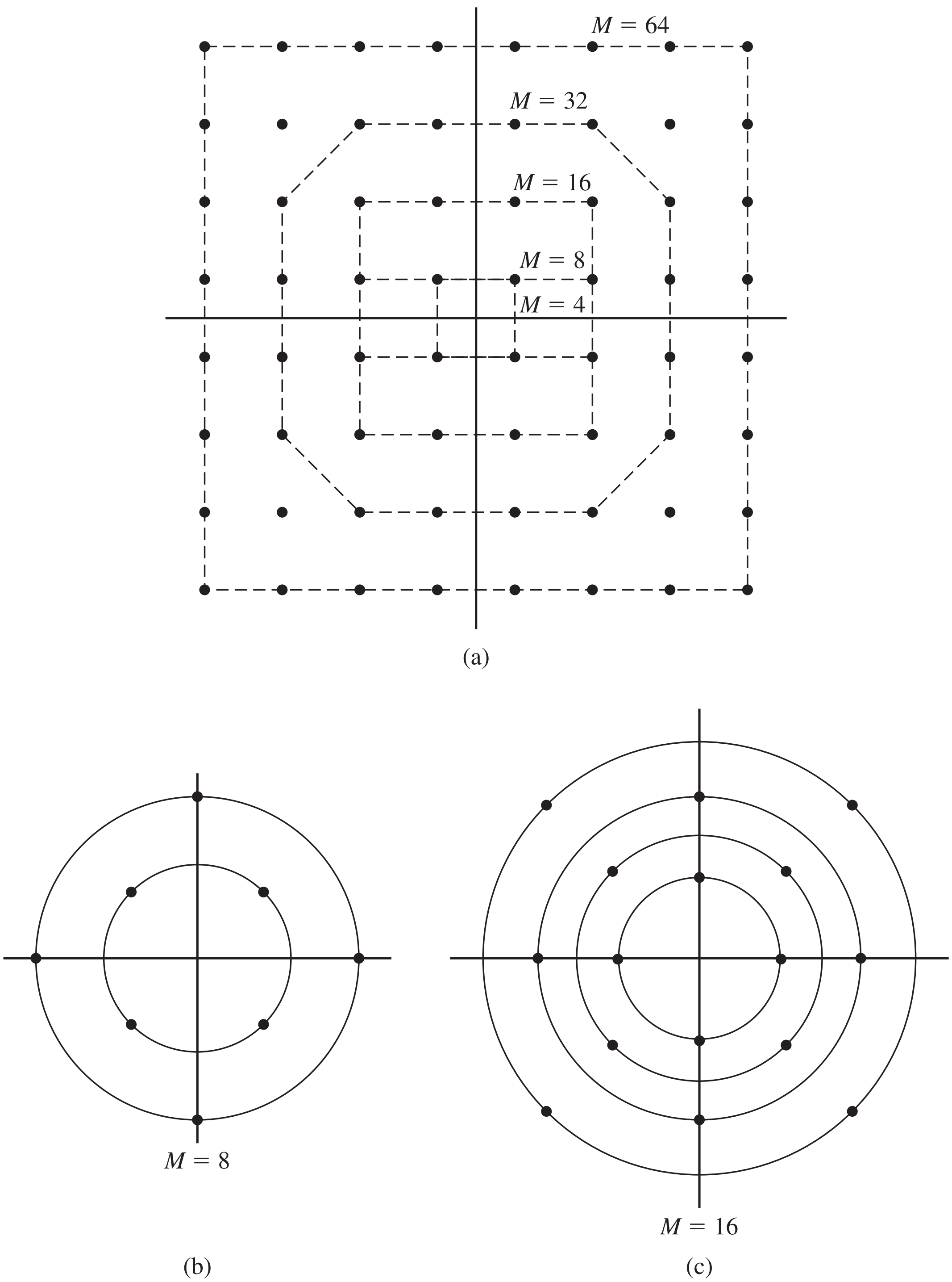

Figure 7.23 (a) Rectangular signal-space constellations for QAM. (b, c) Examples of combined PAM-PSK signal-space constellations.

The average transmitted energy for these signal constellations is simply the sum of the average energies on each of the quadrature carriers. For rectangular signal constellations, as shown in Figure 7.23(a), the average energy/symbol is given by $\mathcal{E}_{av} = \frac{1}{M}\sum_{i=1}^{M} \|\mathbf{s}_i\|^2$. The Euclidean distance between any pair of signal points is

$$d_{mn} = \sqrt{\|\mathbf{s}_m - \mathbf{s}_n\|^2} \tag{7.3.23}$$

7.4 MULTIDIMENSIONAL SIGNAL WAVEFORMS

In the previous section, we observed that a number of signal waveforms, say $M = 2^k$, can be constructed in two dimensions. By transmitting any one of these M signal waveforms in a given interval of time, we convey k bits of information. In this section, we consider the design of a set of $M = 2^k$ signal waveforms having more than two dimensions. We will show in Section 7.6.6 the advantages of using such multidimensional signal waveforms to transmit information.

We begin by constructing M signal waveforms that are mutually orthogonal, where each waveform has dimension $N = M$.

7.4.1 Orthogonal Signal Waveforms

First, we consider the construction of baseband orthogonal signals and, then, discuss the design of bandpass signals.

Baseband Signals. Orthogonal signal waveforms at baseband can be constructed in a variety of ways. Figure 7.24 illustrates two sets of $M = 4$ orthogonal signal waveforms. We observe that the signal waveforms $s_i(t)$, $i = 1, 2, 3, 4$ completely overlap over the interval $(0, T)$, while the signal waveforms $s_i'(t)$, $i = 1, 2, 3, 4$ are nonoverlapping in time. These are just two examples of a set of $M = 4$ orthogonal signal waveforms. In general, if we begin with a set of K baseband signal waveforms, we can use the Gram-Schmidt procedure to construct $M \leq K$ mutually orthogonal signal waveforms. The M signal waveforms are simply the orthonormal signal waveforms $\psi_i(t)$, $i = 1, 2, \ldots, M$, obtained from the Gram-Schmidt procedure. For example, a set of $M = 2^k$ overlapping orthogonal signal waveforms can be constructed from Hadamard sequences, also called Walsh-Hadamard sequences (see Problem 7.31).

When the M orthogonal signal waveforms are nonoverlapping in time, the digital information that is transmitted is conveyed by the time interval in which the signal pulse occupies. This type of signaling is usually called pulse position modulation (PPM). In this case, the M baseband signal waveforms may be expressed as

$$\begin{aligned} s_m(t) = A g_T(t - (m-1)T/M), \quad & m = 1, 2, \ldots, M \\ & (m-1)T/M \leq t \leq mT/M \end{aligned} \tag{7.4.1}$$

where $g_T(t)$ is a signal pulse of duration T/M and of arbitrary shape.

Although each signal waveform in a set of M orthogonal signal waveforms may be designed to have different energy, it is desirable for practical reasons that all M signal waveforms have the same energy. For example, in the case of M PPM signals