



# Distortion

by

B. Hönlinger and H. H. Nasse

## Preface

***“I found somebody say ‘terrible moustache distortion’, but I don’t see any distortion at all”.***

As you can see in above statement in a forum discussion one can have very different opinions about distortion of lenses. For some photographers straight lines are of low importance or their subjects don't have them. Others strive for perfection and spend a lot of efforts, to heal geometric aberrations at least in the image post-processing.

Those who try to avoid these efforts search for lenses which deliver perfect quality right from the beginning without later improvements. At this point the problem with numbers arises, because absolute perfection exists only in a few types. Most lenses exhibit at least small distortion errors, and to decide whether they are acceptable, one has to understand the numbers in data sheets and in lens test publications. This issue of Camera Lens News tries to support you a little bit in understanding this matter.

And since distortion is of major importance in wide-angle lenses, we will in addition deal with some other strange effects typical for this class of lenses, so that you understand why image composition with the short ones is at the same time so difficult and attractive.

## What is distortion?

The majority of camera lenses produce images in line with the laws of **central perspective**. This kind of projection of three-dimensional space onto a two-dimensional image surface is also called **gnomonic projection**. This Greek term (γνομον = gnomon = shadow-producing rod) denotes a type of sundial because, as with a sundial, an image point is produced by connecting a point in the object space to the center of the

projection using a straight line; the image point is where this straight line intersects the flat projection surface.

In painting, central perspective was invented during the time of Renaissance. The methods used by the artist to strictly apply this perspective are shown to us by Albrecht Dürer:



*Instructions in the application of perspective, woodcarving by Albrecht Dürer, approx. 1527*

The center of projection of the image is the tip of the rod used by Dürer's artist to draw attention to the charms of the young lady in this picture. The projection surface is the frame fitted with a grid placed between the two, and it is used by the artist to transfer the image points onto his drawing paper with the proper perspective.

The center of projection of camera lenses is their **entrance pupil**, i.e. the image of the aperture stop viewed from the front. When taking panorama photos, it is necessary to swivel around the entrance pupil to ensure that objects in the foreground and background are not shifted with respect to each other.

This special point is often also called the **nodal point**, but this term has a very different meaning in optics. There is nothing particularly mysterious about the entrance pupil either, since anyone can see it without any special aids and estimate its approximate position.

However, the entrance pupil is not the physical aperture stop, but rather its virtual image – and as such may even be situated outside the lens altogether. This is often the case with short telephoto lenses.

The gnomonic (central perspective) projection has the special feature where all straight lines of the object space are reproduced in the image as a straight line again regardless of where they are situated and to where they are projected. Lenses with this property are called '**rectilinear**'. During the history of photographic lenses they appeared first in the sixties of the 19<sup>th</sup> century.

Distortion is defined as a lens aberration in which this property is no longer exactly fulfilled. A lens that exhibits distortion produces slightly curved images of all those lines that do not pass through the center of the image. Thus this effect is also called '**curvilinear distortion**'.

## What can we learn from the data sheet of the lens?

The reason for the curved images of straight lines is that the image scale is not constant throughout the entire image field. In other words: the focal length of a lens showing distortion changes with the distance of an image point from the optical axis.

If the reader allows me to use the language of mathematics, it will be much easier to comprehend what the numbers we use to describe distortion actually mean.

Let us first look at the image equation of a rectilinear lens with ideal gnomonic projection:

$$u' = f' \cdot \tan W$$

$W$  is the object-side field angle, i.e. the angle between the optical axis and the line from the object point to the entrance pupil,  $u'$  is the off-axis distance, i.e. the distance of the image point from the optical axis. To express this equation in words: the off-axis distance is proportional to the tangent of the field angle, and the focal length  $f'$  is the proportionality constant.

I would now like to mention that this is exactly how the focal length of lenses is defined and measured: one measures the angle to the optical axis and the off-axis distance of the corresponding image point created by a ray of light arriving from a source at an infinite distance. You may also encounter measurements, in which the focal length is calculated in the near range from the distance and image scale factor. However, this result often deviates substantially from the specifications of the manufacturer.

However, this does not necessarily mean that the manufacturer has cheated vastly. Many lenses change their focal length when focused on short distances, and calculations of this kind fail to take into account other lens parameters such as principal plane distances. For this reason, a long zoom lens, for example, often has smaller object fields at short distances than a comparable prime lens.

\* In optical tradition the off-axis distance of an image point is also called 'image height'. This should not be confused with the vertical frame length.

Another factor must be included in the gnomonic image equation if the focal length is not constant throughout the field of view:

$$u' = f' \cdot \left( 1 + \frac{D\%}{100} \right) \cdot \tan W$$

$D$  is the measure of the distortion error. If  $D=0$ , the term in parentheses is equal to 1, and the situation is the same as with the ideal lens.

If  $D$  is a **positive** number, the term in parentheses is larger than 1. The off-axis distance  $u'$  is then larger than with the distortion-free lens of the same focal length. Since the distortion increases with increasing distance in most cases, a rectangle is distorted to a **pincushion** shape. We shall see later though that pincushion-shaped images can also occur with negative values of  $D$ .

If  $D$  is a **negative** number, the term in parentheses is smaller than 1. The off-axis distance  $u'$  is then smaller than with the distortion-free lens of the same focal length. The lines of a rectangle are bulging and we call this a **barrel shape**.

In both cases considered here, the image point is shifted in a radial direction, i.e. on the radius of the field of view. Accordingly, parameter  $D$  is also called **radial distortion**. Our data sheets specify the radial distortion by means of a curve, i.e. as a percentage value as a function of the off-axis distance \*.

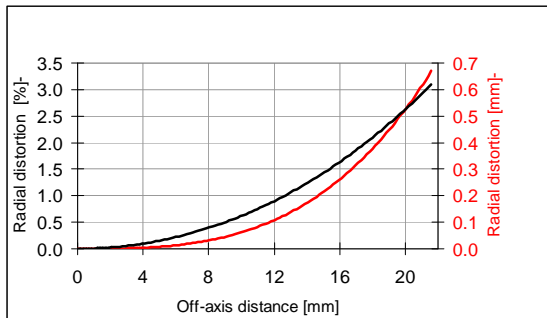
This curve contains all information about the geometric properties of the lens. For proper interpretation, though, we need to understand it even better. What makes our eyes so sensitive to distortion? Taking a photo of concentric circles, for example a target for shooting practice, we can barely recognize distortions as long as they remain moderate since we have little feeling for the correct absolute size of a circle. In contrast, we can see pretty well that straight lines are reproduced with some curvature, i.e. we need to understand the relationship between radial distortion and this kind of **bending of straight lines**.

# Radial distortion and TV distortion

## Types of distortion

### 1. Pincushion distortion

Pincushion distortion at an already detectable level in a lens for the 35mm format is described by the following curve:



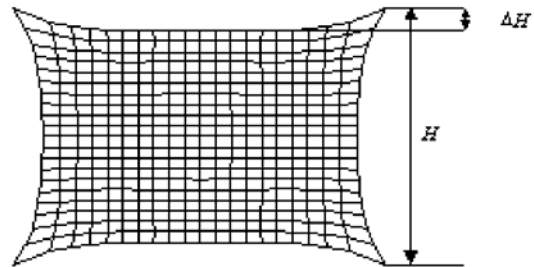
The black curve shows that the radial distortion error increases gradually from zero in the middle of the image to 3% in the corner of the image where the focal length is 3% larger than in the middle.

The red curve shows the absolute magnitude of the radial shift of the image points in units of millimeters. These values tell us that distortion shift is more than ten times larger than the usual circle of confusion. Thus perceivable distortion is nearly never caused by poor build quality or shock damage of the lens. Distortion is not an issue for repair service, it is determined by the lens design. And distortion is not changed by stopping down.

Looking at this curve, one keeps in mind: "The lens exhibits a 3% distortion", but might then be surprised if a test report claims: "The lens shows 1.1% pincushion distortion". How can we reconcile this apparent discrepancy?

Well, the test report does not refer to radial distortion, but rather to **TV distortion**. This is a measure of how strongly the image of a straight line that is situated at the edge of the image (in particular on the long edge of the image in a rectangular format) is curved. This amplitude of curvature is then related to the total frame height and expressed as a percentage value.

It is therefore very important to note which percentage value is actually meant. The values of TV distortion are always smaller than the radial distortion specified by us.

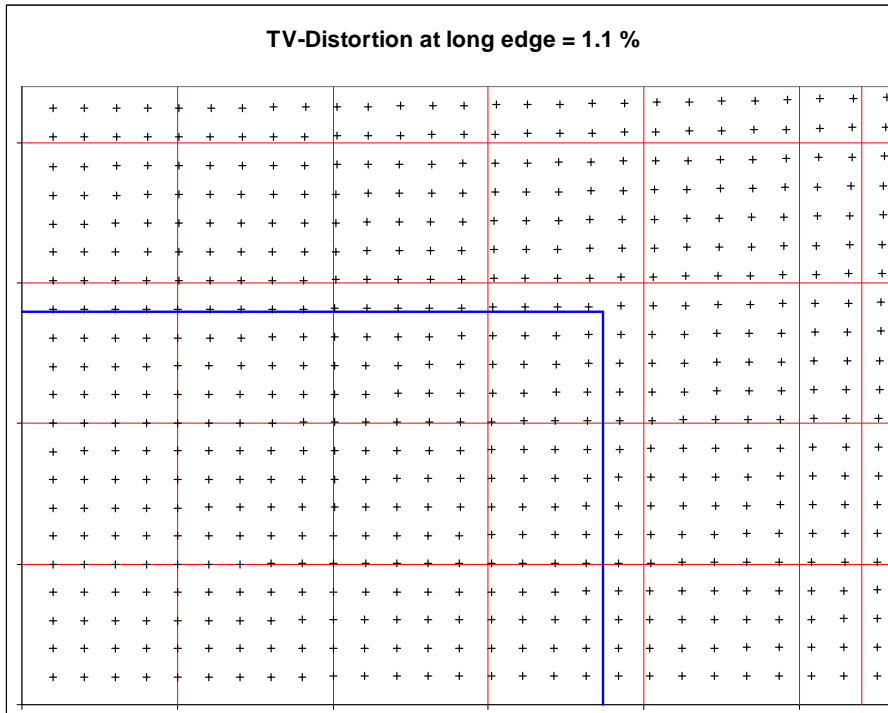


$$D_{TV} = \frac{\Delta H}{H} \cdot 100$$

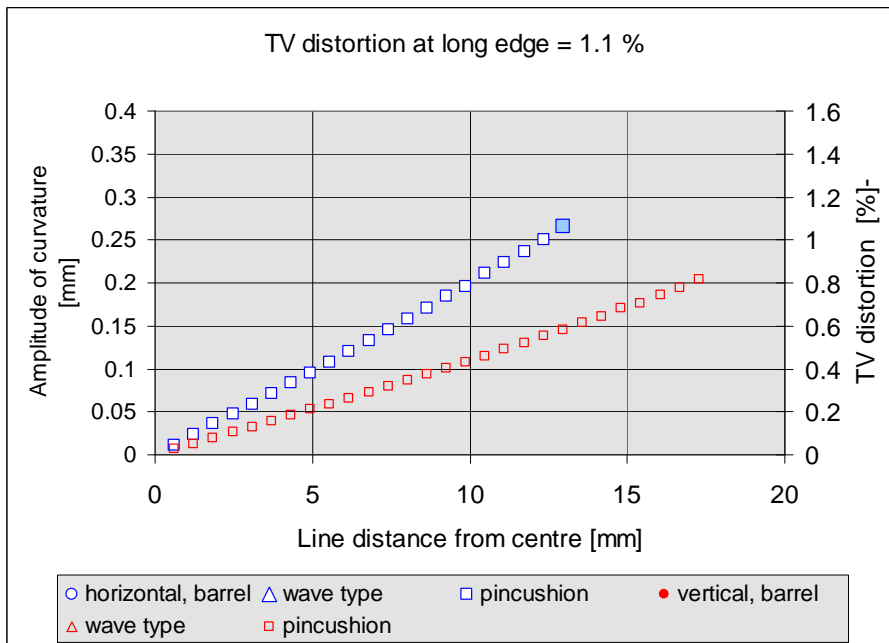
Sometimes the total height difference of the distorted rectangle is related to the frame height. This is then twice the above figure.

In order to understand how these fairly different numbers arise, let us look at a line at the upper edge of the 35mm format. A point in the middle of this line has a distance of 12 mm from the middle of the image. Here, the lens shows approx. one percent radial distortion. Consequently, the image of this line is not only curved, but also shifted by 0.12 mm. However, we do not notice this.

If we travel along the line until we get to the corner, the off-axis distance increases from 12 to 21.6 mm and, according to the curve presented above, the radial distortion increases to 3.1% in the process. The endpoint of the line is therefore shifted in a radial direction by  $0.031 \times 21.6 = 0.67$  mm. However, in the 24x36 rectangle of the 35 mm format, the diagonal is inclined by  $56^\circ$  with respect to the vertical direction. Splitting the radial distortion into horizontal and vertical components, the distortion shift in the vertical direction is  $0.67 \times \cos(56^\circ) = 0.37$  mm. Accordingly, the vertical shift of the image points in the middle of the line and at its end is 0.12 mm and 0.37 mm, respectively. The amplitude  $\Delta H$  is the difference between these two values, i.e. 0.25 mm. This corresponds to approx. 1.1% for a frame height of 24 mm.



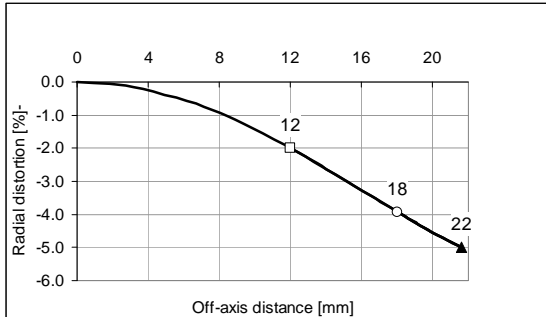
This diagram shows a simulated image characterized by the pincushion distortion described above. The crosses are the image points that are imaged with distortion error. The thin red lines are reference lines without curvature. For reasons of symmetry, it is sufficient to show just one quarter of the image, meaning that the middle of the image is situated in the lower left corner of the graph. The blue lines show the crop factor of 1.5x.



Each point in this graph represents the **amplitude of line curvature** plotted over the distance of the line from the image centre. The amplitude of curvature increases uniformly from the middle towards the edge in this distortion profile. TV distortion values usually refer to a line on the long edge of the image (symbol filled in blue). Curvature is less pronounced on the short edge, since this line proceeds only through off-axis distances between 18 and 21.6 mm.

## 2. Barrel distortion

A lens with substantial barrel distortion of straight lines shows negative radial distortion whose absolute amount increases towards the corner:



The lens reaches -5% radial distortion in the corner of the image, which is a typical value of many wide-angle to tele zoom lenses at shortest focal length.

The curvature of a peripheral line at the long edge of the format is easy to calculate. We need to bear in mind that the radial distortion in the corner needs to be multiplied by the cosine of  $56^\circ = 0.55$  in order to take into account only its contribution to the shift in the vertical direction.

$$D_{TV} = (21.6 \times 0.05 \times 0.55 - 12 \times 0.02)/24$$

Result: 1.5 % TV distortion.

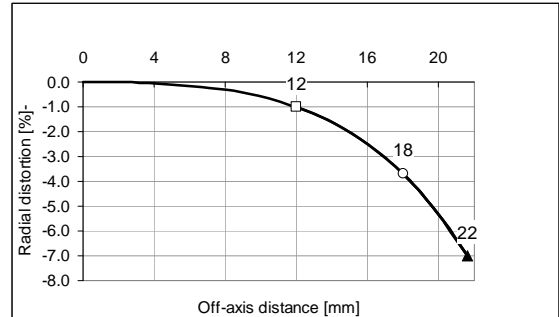
Similarly, we can calculate the curvature of a peripheral line at the short edge of the format, but now have to take into account the inclination of the diagonal with respect to the horizontal direction. This angle is only  $34^\circ$  and the cosine of  $34^\circ$  is 0.83.

$$D_{TV} = (21.6 \times 0.05 \times 0.83 - 18 \times 0.04)/24$$

The result obtained (0.73%) is smaller again, as expected.

This shows how easy it is to get lost in the dense forest of percentage values! Let me demonstrate this even better by making the lens just a little bit worse.

Let us have the negative radial distortion drop just a little bit more, let's say to -7% - i.e. it is 1.4 times larger than before:



Values like this do indeed occur with real-life lenses when these are used outside their normal field of application, e.g. a zoom lens in an extreme macro-photography setting.

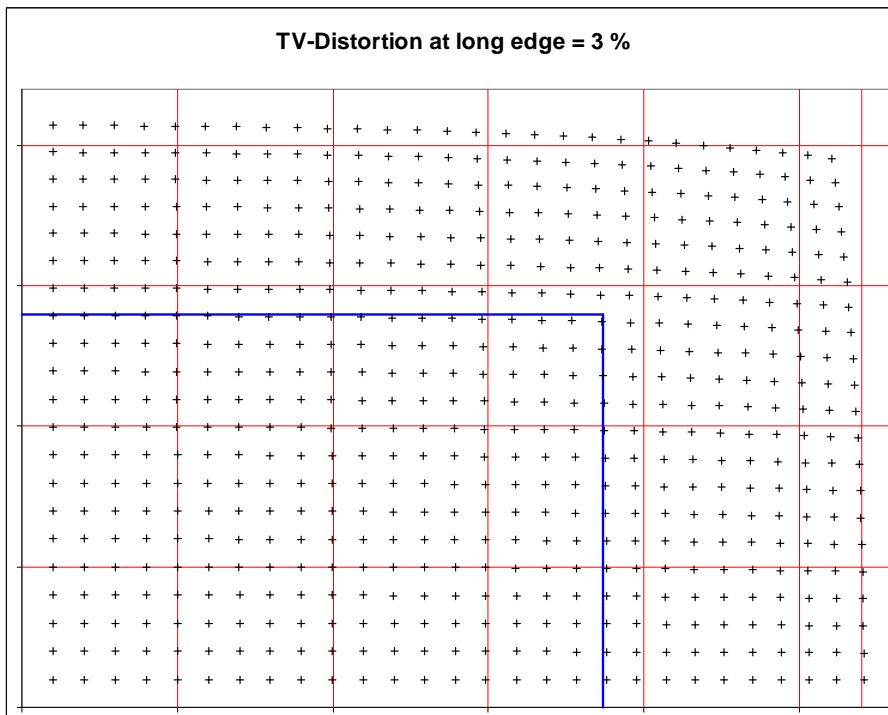
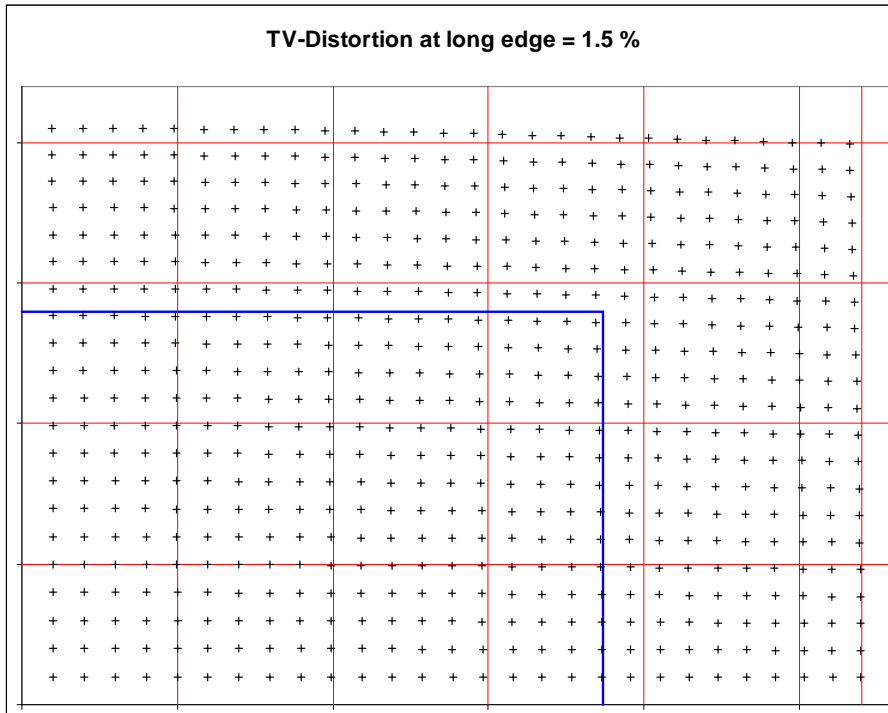
The same calculation for the long edge of the format as before yields:

$$D_{TV} = (21.6 \times 0.07 \times 0.55 - 12 \times 0.01)/24$$

This means approx. 3% TV distortion for this line - quite a bit. Although we increased the maximal radial distortion by only 40%, we managed to double the TV distortion.

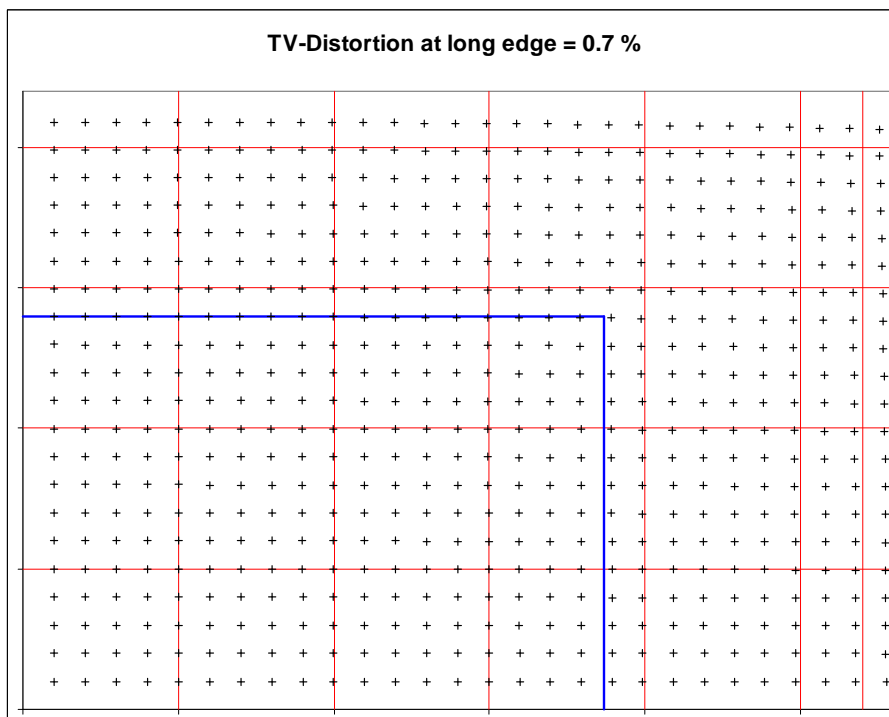
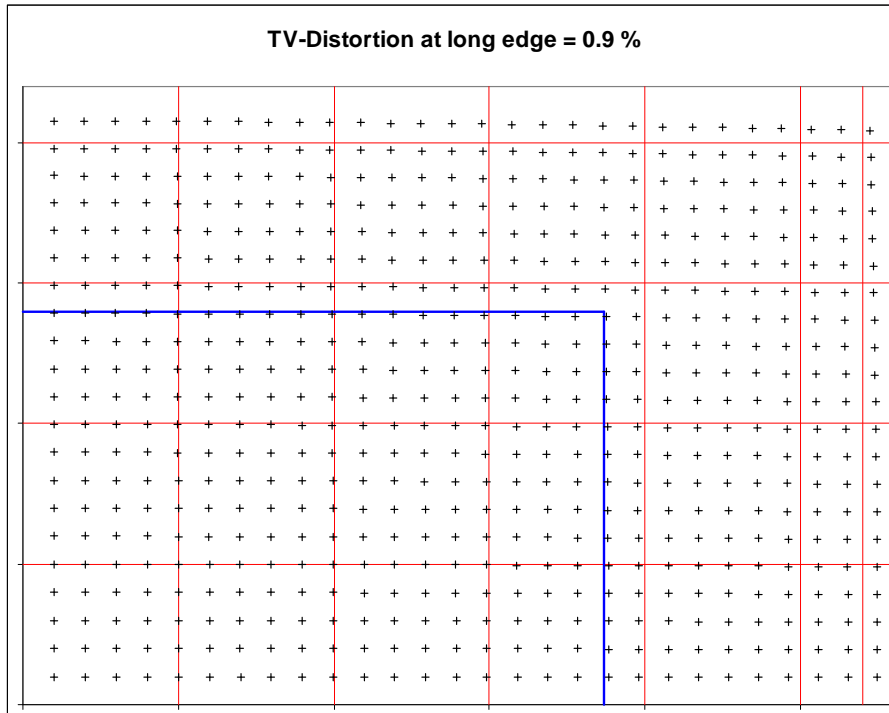
The cause of this phenomenon is that we are considering a difference (subtraction) in our calculation, to be precise, the difference between the smallest and the largest shift due to distortion. In the second lens considered above, not only did the distortion increase in the corner but it became smaller at the off-axis distance of 12 mm at the same time.

Thus, the perception of distortion, i.e. the curving of what are actually straight lines, by our eyes, appears to depend on the **change** of the radial distortion along a line. And, at this point, TV distortion appears to be the better measure. But I will show you some examples demonstrating the opposite in the next chapter - but first let us have a look at the image simulations of the two lenses:

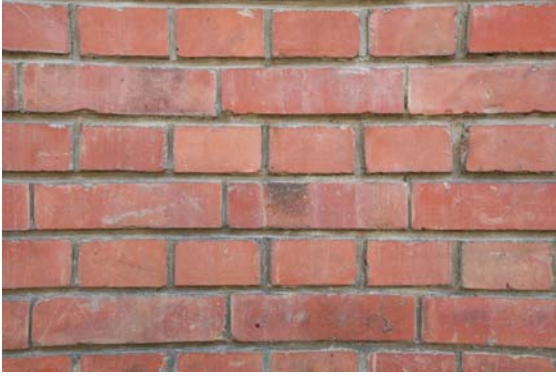


*Simulated image with the negative radial distortion described on the previous page. Both lenses exhibit strong barrel distortion in the entire field of view. The doubling of TV distortion in the lower example results from the higher gradient of radial distortion towards the edge of the image.*

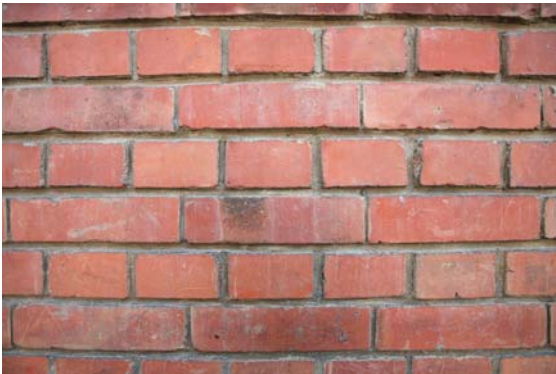




*However, using the two lenses in the **APS-C** format, the second one shows only 0.7% TV distortion and thus is even slightly better. This is because its radial distortion curve is initially flatter at the smaller image heights.*



*Not all motifs reveal the pincushion distortion of a zoom lens in longest focal length setting as clearly as this one. Horizontal and vertical lines close to the frame edges enable an easy comparison.*



*The same zoom lens as above shows a pronounced barrel type of distortion when it is used at the shortest focal length. This causes the illusion that the brick surface forms a kind of dome towards the observer.*



*Even if this door is already quite old all its elements are straight, parallel and rectangular to each other. A wide angle zoom makes us believe that the door is not really closed.*

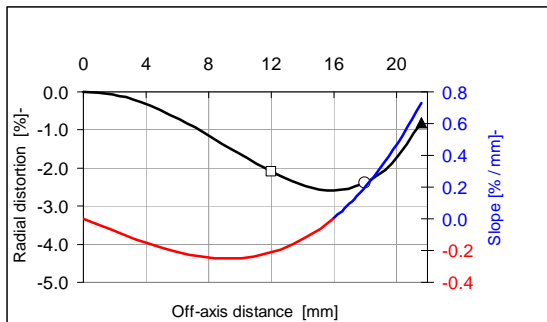


*The image of wooden floor covering boards on the left exhibits a kind of distortion which seems to be a mixture of both types mentioned above. Please read on the following pages how this can happen.*

*The image actually was a 2164x256 crop from a 4288x2848 file taken from the top right edge of the frame. The height was then exaggerated by a factor of 8, much like we will do it in some of the next graphs.*

### 3. Moustache distortion

Unfortunately, it is not always this easy to calculate the TV distortion from the radial data since there are many lenses whose radial distortion does not increase or decrease uniformly between the middle and the corner but rather reaches an extreme value somewhere in between. This is caused by countermeasures in the design to limit negative distortion. Many retro-focus wide-angle lenses have a radial distortion function like the one shown in the following example:



The radial distortion of this lens is negative throughout; it reaches its highest amount of -2.6% at 16mm off-axis distance and then decreases to -0.8% towards the corner.

While we may have remembered until now the simple rule: "positive D → pincushion, negative D → barrel", this lens teaches us that this simple rule is an oversimplification (unfortunately it was also said on earlier ZEISS data sheets).

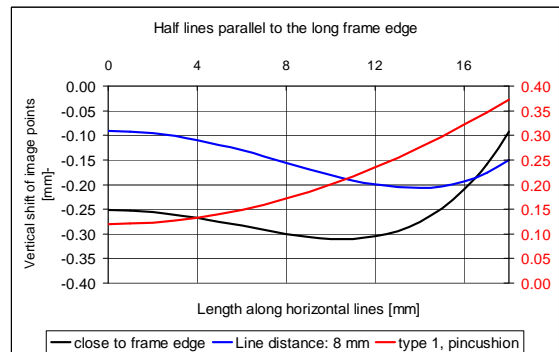
Looking at the image of lines that are parallel to the edge and whose points have off-axis distances of between 0 and 16 mm, the distortion curve looks just like that of type 2 - barrel distortion is evident.

Looking at lines near the short edge of this format and whose shortest off-axis distances exceed 16 mm, the distortion curve of this section looks just like that of type 1 - it is negative, but the values increase towards the corner. Accordingly, we see pincushion distortion.

This shows that the attributes "pincushion" and "barrel" actually do not depend on whether the radial distortion is positive or negative, but rather on the slope of the distortion function. This change, in % per mm, is drawn in color in the diagram above: red where it is negative and blue where it is positive.

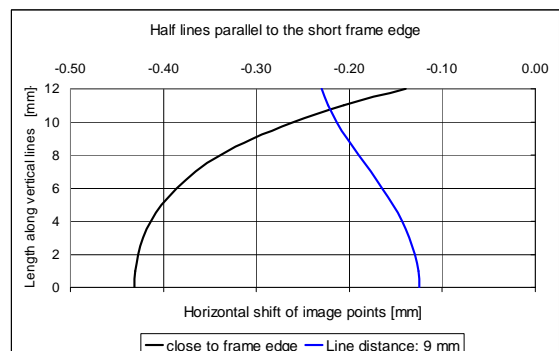
Many lines that are parallel to the long edge of the format and extend across the entire width of the image have some points in the red and some points in the blue section. Accordingly, there must be a switch between barrel and pincushion distortion on these lines. We say that they exhibit moustache distortion, also called wave-type or gull-wing distortion.

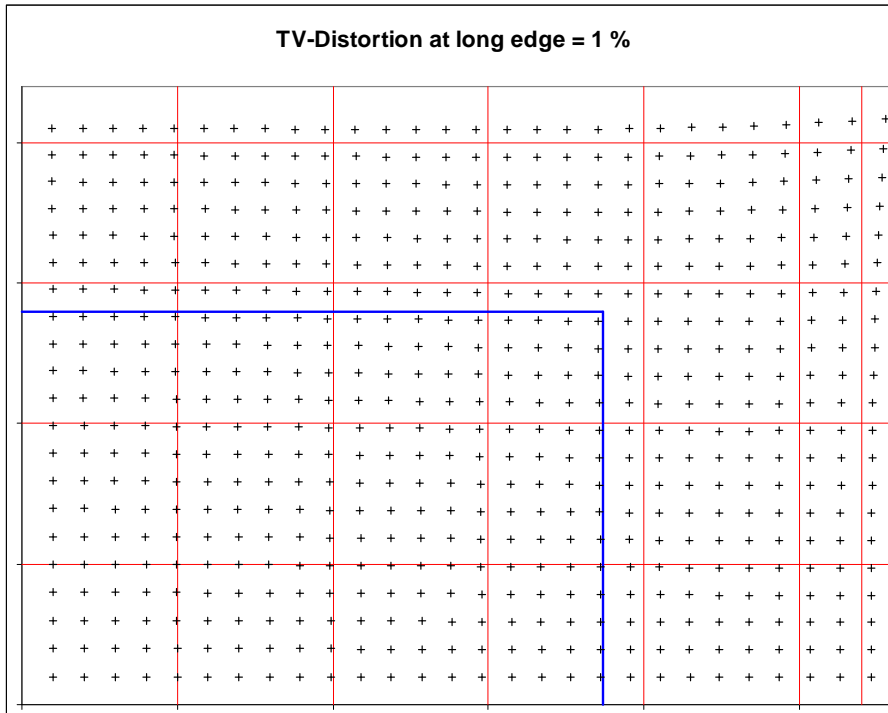
Looking at the entire image and at lines that differ in length, all three types of line curvature are evident. This is shown in an exaggerated manner in the following diagrams:



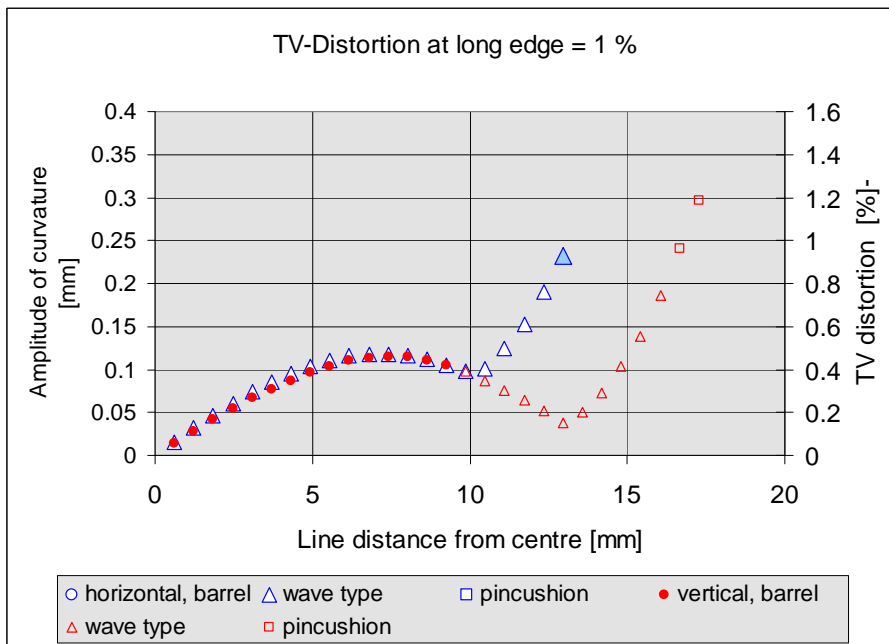
The pure pincushion type 1 is also plotted as a red line for purposes of comparison. Although it has even slightly higher amplitude than the black line, this may often be less evident since it is not as steep near the edge.

The following two lines are parallel to the short edge of the format and show purely barrel or pincushion features:





Simulated image with the negative radial distortion with local extreme value described on the previous page. Pincushion-type dominates on the edges of the 35 mm format, whereas the distortion is mainly of barrel-type in APS-C format (blue lines).

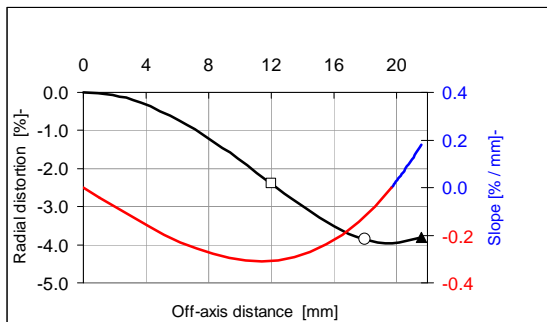


The distortion profile of lines with a different distance from the middle of the image is quite complex: after increasing gradually at first, it decreases for a while until there is a dramatic increase towards the edge. The shape changes in the process. The standard TV distortion (blue triangle) does not represent the line which exhibits the most severe curvature.

The example shown above clearly demonstrates that distortion errors, much like many other properties of lenses, are difficult to describe in detail by means of a single number. The maximum value of radial distortion can be misleading; the numerical value of TV distortion is sometimes somewhat closer to subjective perception - but it still does not tell us enough about the distortion throughout the image and it does not tell us the profile of distortion along a line.

The lens discussed above can be used to take images without annoying distortion if one ensures that prominent lines of the object that are near the long edge of the format do not extend into the extreme corner and by avoiding the placement of critical lines too close to the short edge of the format.

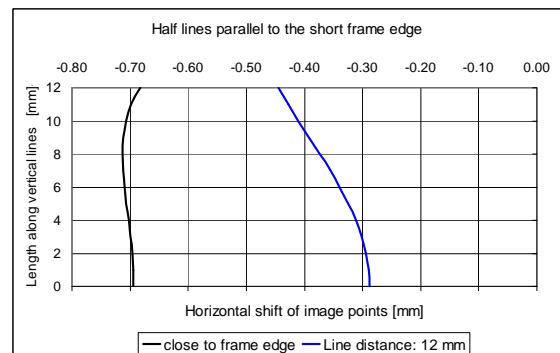
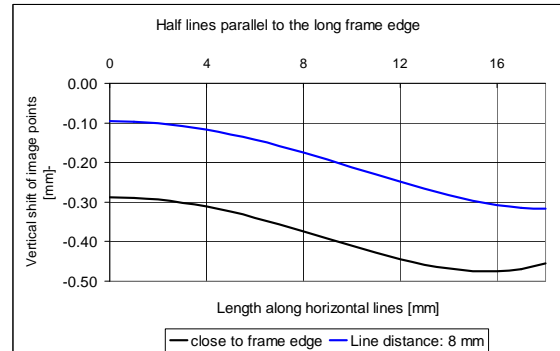
The limited value of having only a single number is even more evident in the following example in which the amount of negative radial distortion is increased to - 4% and the curvature is nevertheless lower for the most part:



Since the extreme value of radial distortion is now situated near the corner of the image and since the values increase only over a very short distance and much less steeply, barrel distortion predominates in much of the image.

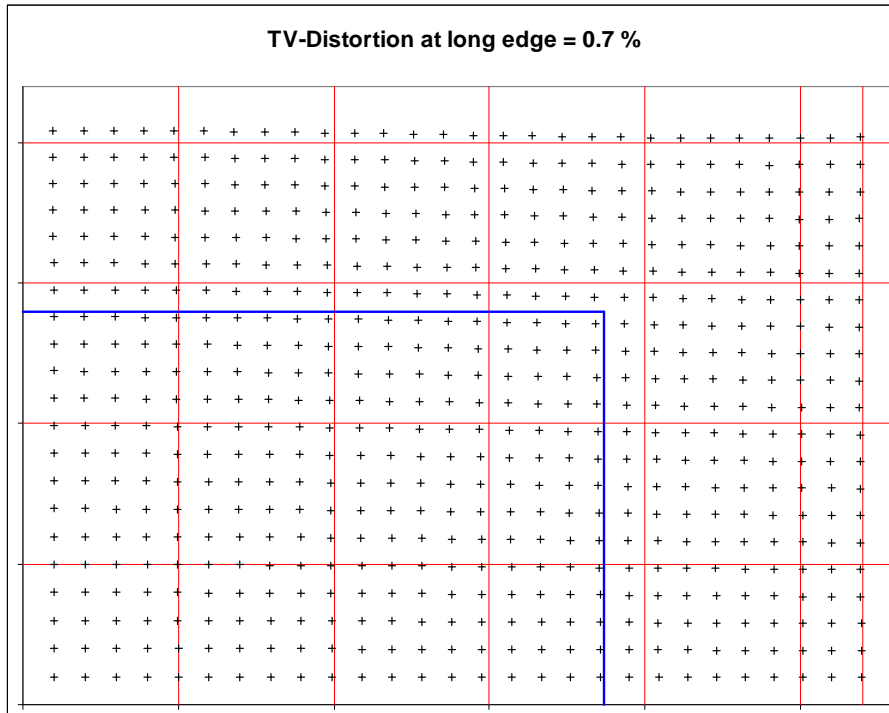
For lines at the short edge of the format with off-axis distances between 18 and 21.6 mm, the radial distortion is quite large. As a result, the entire line is shifted more strongly, but this is not perceived by us. The crucial point is that the **change** of the radial distortion in this range of image heights is very small, since this causes the lines at this edge of the image to be practically straight - despite - 4% radial distortion.

This is clearly evident in the strongly exaggerated plot:

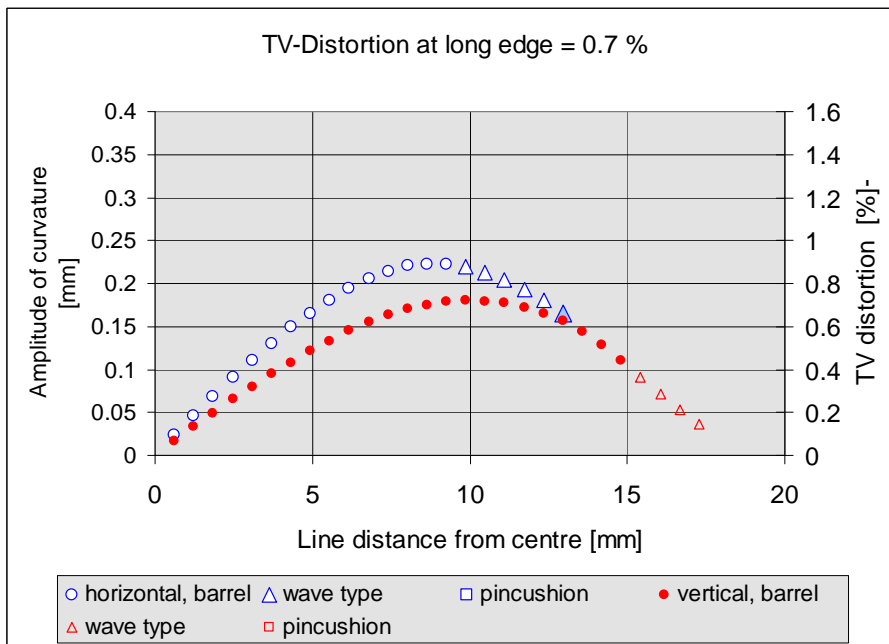


Precisely because the slope of the distortion curve near the edge of the image is small and has a less steep profile than in the previous example, the distortion, which appears to be high in the data sheet, is "masked" fairly well. Our perception is often particularly sensitive at the periphery of the image since the straight edge serves as a reference to the eye, much like a ruler.

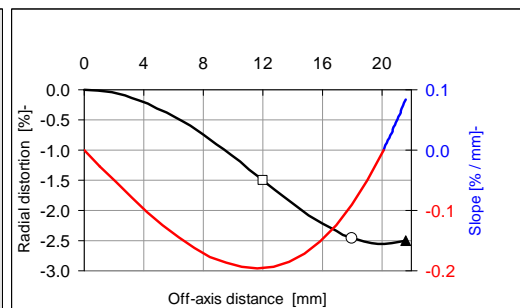
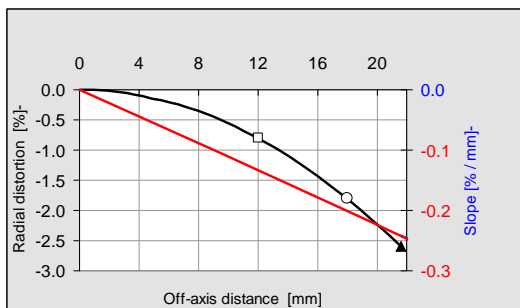
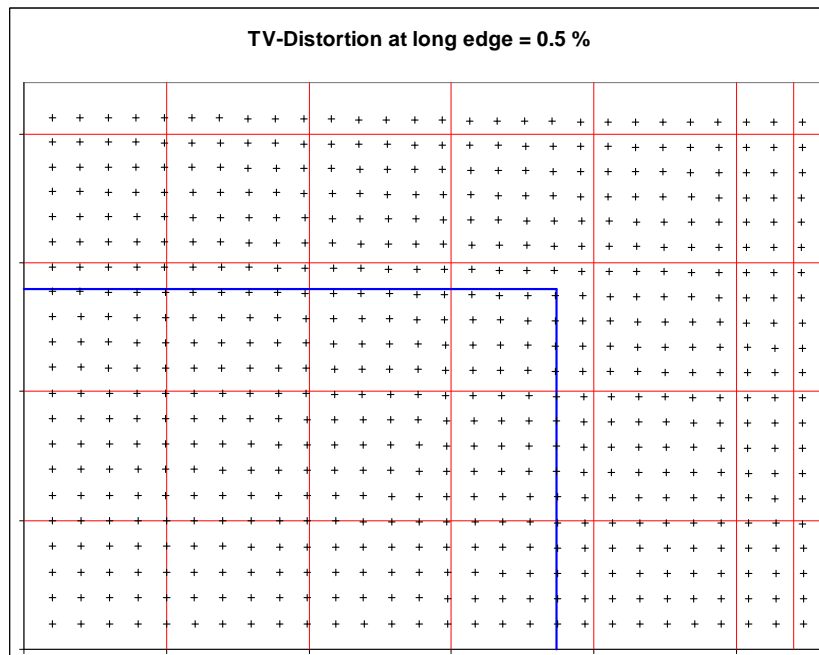
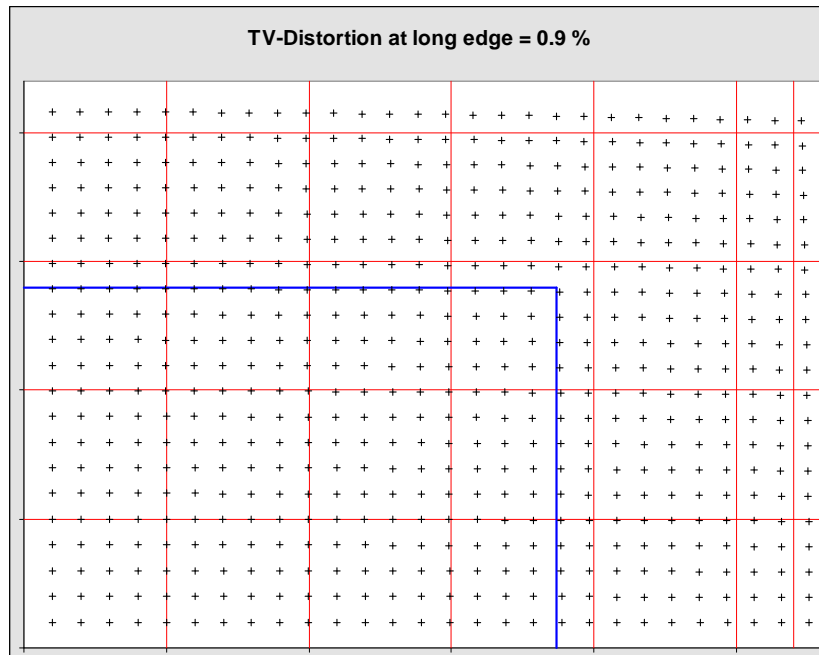
The figure of the TV distortion is moderate but it withholds something from us, namely that the curvature further inwards is somewhat larger than on the periphery of the image. It also does not tell us that the short edge does not show any recognizable distortion at all (see next page).



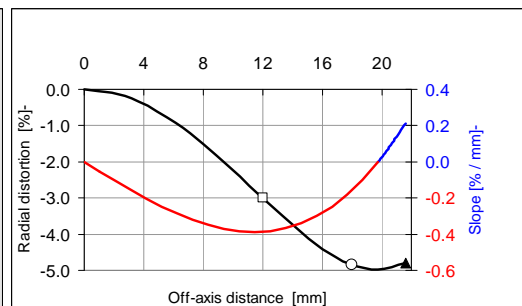
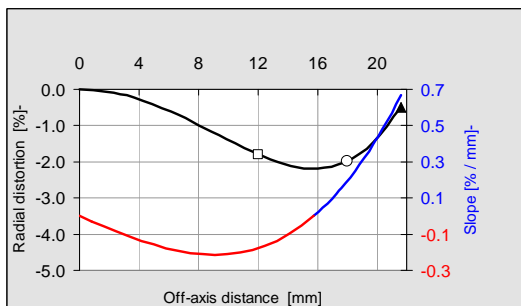
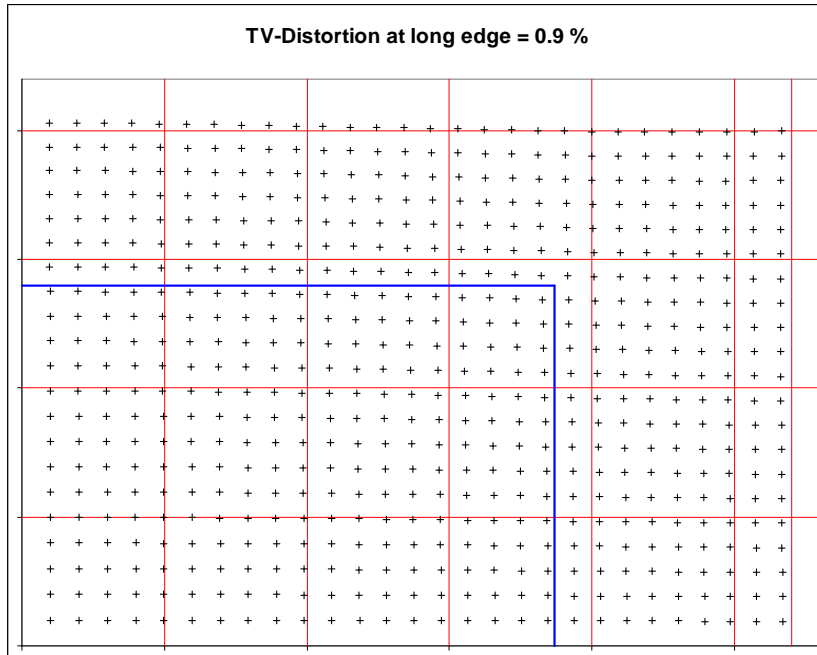
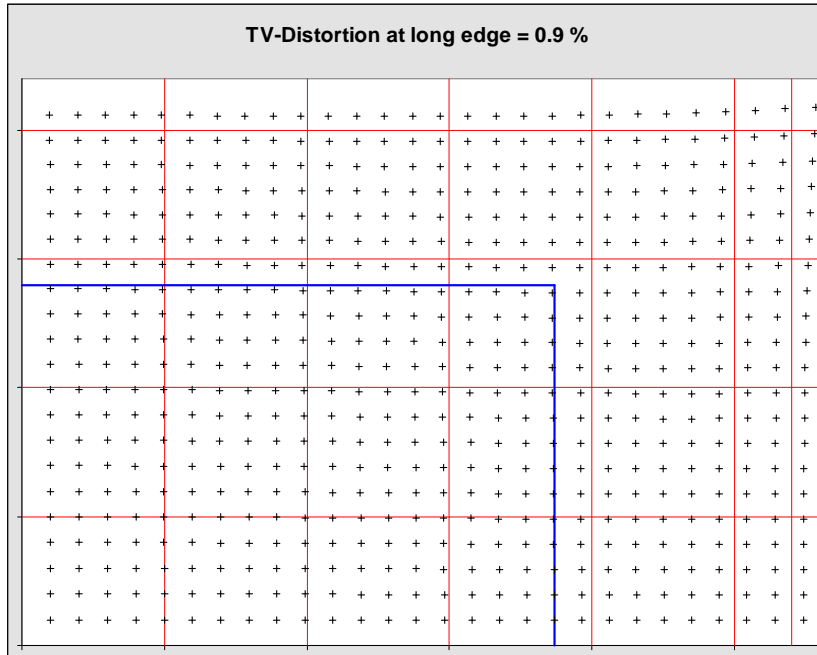
Simulated image of the negative radial distortion described on the previous page. Barrel distortion dominates on the upper edge of the image whereas the vertical edge of the image on the right is basically free of curvature – in spite of a - 4% radial distortion figure.



For lines with different distances from the middle of the image, the profile of curvature is simpler than in the previous example: it increases gradually, reaches a maximum value at a distance of between 9 and 10 mm from the middle and then decreases again and disappears almost completely at the short edge. The standard TV distortion (blue triangle) does not indicate the largest curvature.



Two more examples with the **same maximum** value of negative **radial** distortion: they clearly show us that the higher slope of the radial distortion curve causes more curvilinear distortion at the frame edges (top and left graph).



These two examples and one on the previous page have all the **same** amount of **TV distortion**. When you however examine the lines close to the frame edges you will notice quite obvious differences.

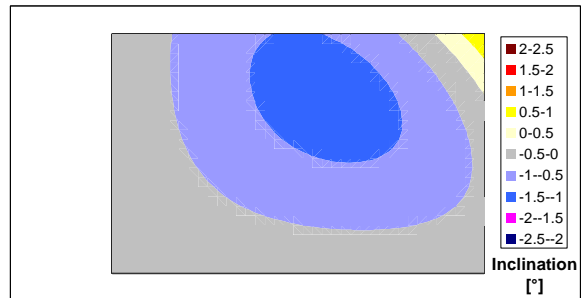
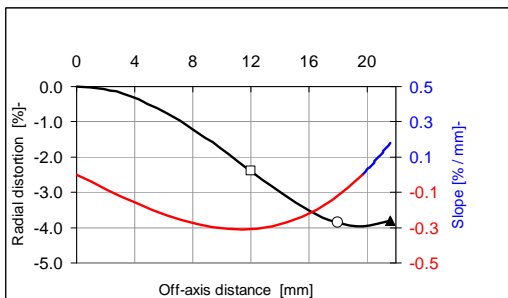
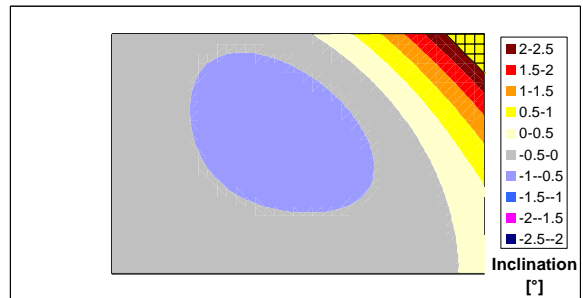
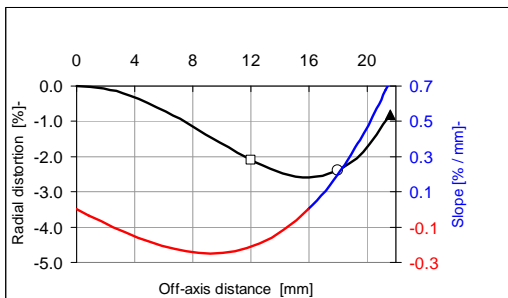
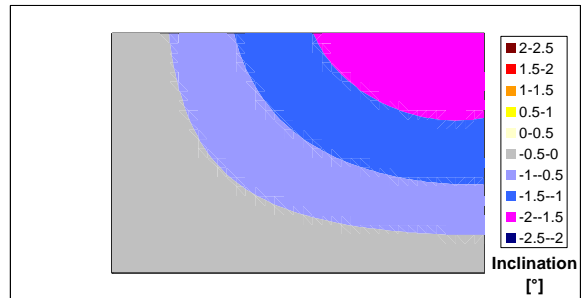
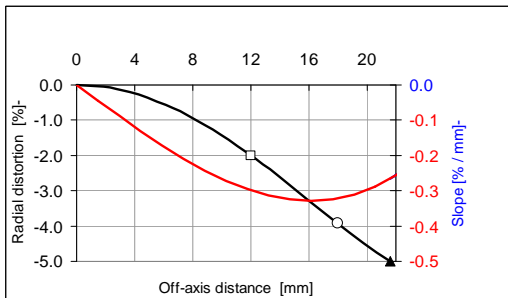
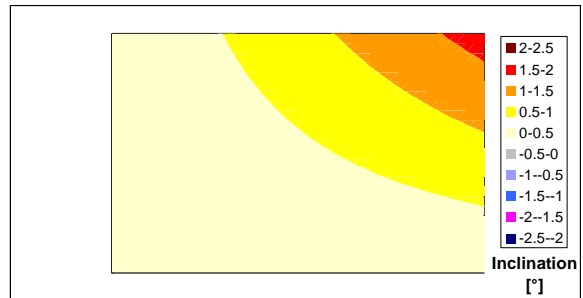
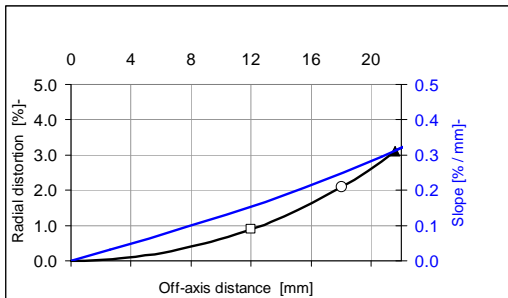


# Angle of Inclination

The examples we have discussed above teach us that distortion can hardly be described by one single number. And the distortion charts of the data sheet need further 'translation' to get what we see.

A much better and more informative presentation of distortion features would be a kind of map which we show on this page to explain the four most important types of distortion.

This map represents the inclination angle of horizontal lines in a quarter of the image. The middle of the image is again the lower left corner of the map. Without distortion this angle would of course be 0° within the complete image area. A positive inclination means that a line in the image rises from left to right in the map (yellow and red colors). A line with negative inclination angle drops from left to right. Each single color represents a range of 0.5°. Where the colors change rapidly lines are strongly curved.



## Types of lenses

The optical reasons of lens distortion are related to the distribution of the refractive power in the lens. However, at the present time this should only be used for predicting distortion with great caution since many modern corrective tools are available that render simple rules obsolete. Keeping this in mind, the following types can be distinguished:

Very little distortion is evident in all **symmetrical** lenses in which the distribution of refractive power to the front component ahead of the aperture stop and the rear component behind the aperture stop is very well-balanced. The ideal case is represented by special repro-lenses for a 1:1 scale which are perfectly symmetrical such that the path of rays is also symmetrical. Under these circumstances there is simply no distortion.

Wide-angle lenses designed to be **nearly symmetrical**, such as the **Biogon**, approximate to this ideal as they have a short backfocal distance that corresponds to their short focal length. This allows for favorable design conditions with specialized cameras. The backfocal distance, i.e. the distance of the last lens from the image plane, is a very sensitive parameter. The older types of the Biogon were even slightly better since they did not need to take into consideration the modern TTL exposure measurement and were designed to have even shorter backfocal distances than their modern successors. But still, the TV distortion of the new types is only approx. 0.2%, which is negligible in normal imaging.

If we analyze the images by means of metrology, mathematical corrections are applied even to such small levels of distortion. One goes so far as to no longer assume the aberration resulting from distortion to be rotationally symmetrical. The distortion of our Biogon lenses for the NASA was always measured in several azimuths.

Usually, **short tele lenses, such as the Planar 1.4/85**, also show very little distortion, especially when they are not really tele designs at all, such as the **Tele-Tessar 4/85 ZM**.

True **telephoto lenses**, in which positive refractive powers predominate in the front and negative refractive powers predominate in the back, tend to show pincushion distortion.

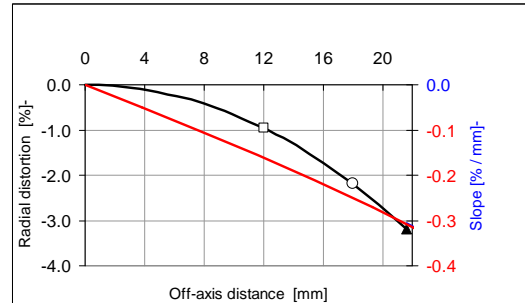
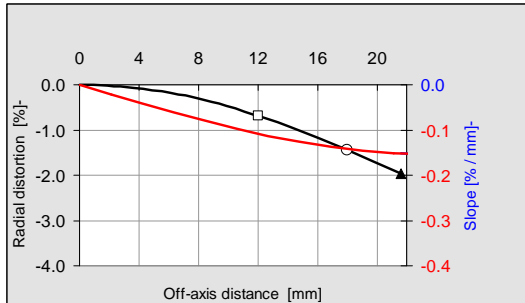
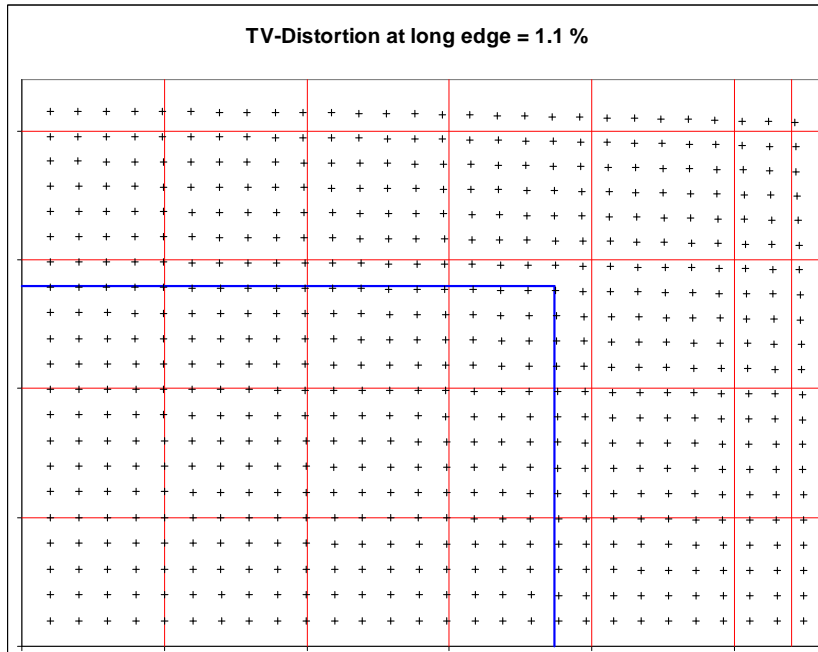
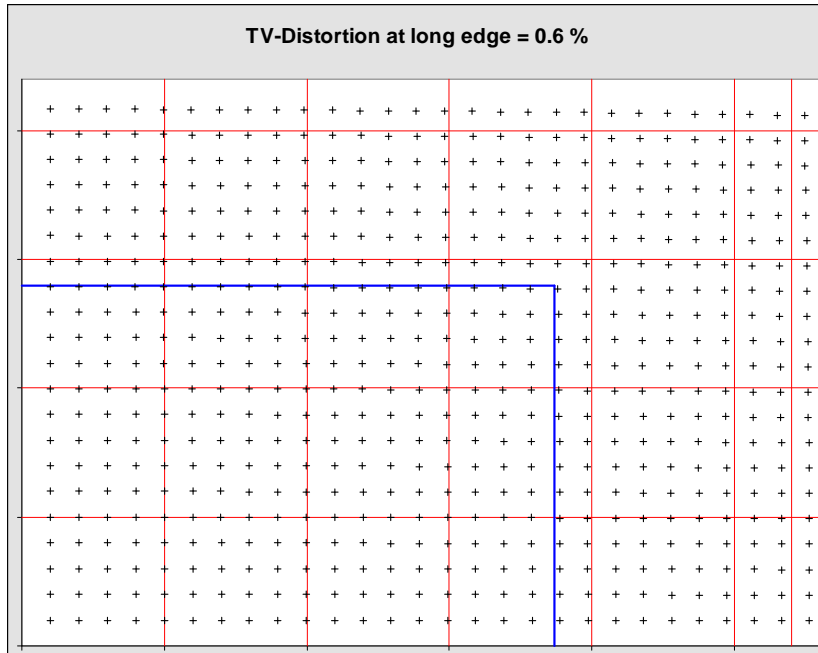
In contrast, all **retro-focus wide-angle designs** whose much extended backfocal distance is needed for single-lens reflex cameras, show barrel or moustache distortion.

Due to their nature, the type of distortion of **zoom lenses** varies with the focal length since the distribution of refractive powers changes in the process of zooming. While barrel or moustache distortion tends to predominate at the short end of the range of focal lengths, this tendency decreases towards longer focal lengths or changes completely to pincushion distortion. When the radial distortion changes its sign in the process of zooming, it must become very small in certain favorable focal length ranges. Within these ranges, the zoom lens is often better than a comparable prime lens. This may seem paradoxical to some of us since it seems that one could simply design a prime lens the same way as the zoom lens and just dispense with the movement of the zoom components. However, this is undesirable since the zoom lens is much larger than the comparable prime lens.

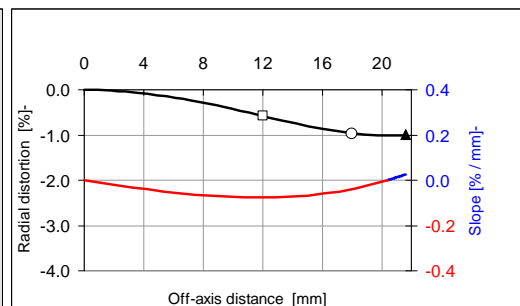
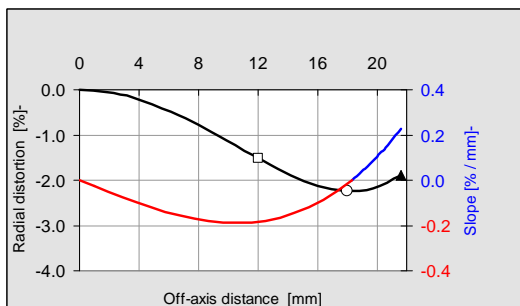
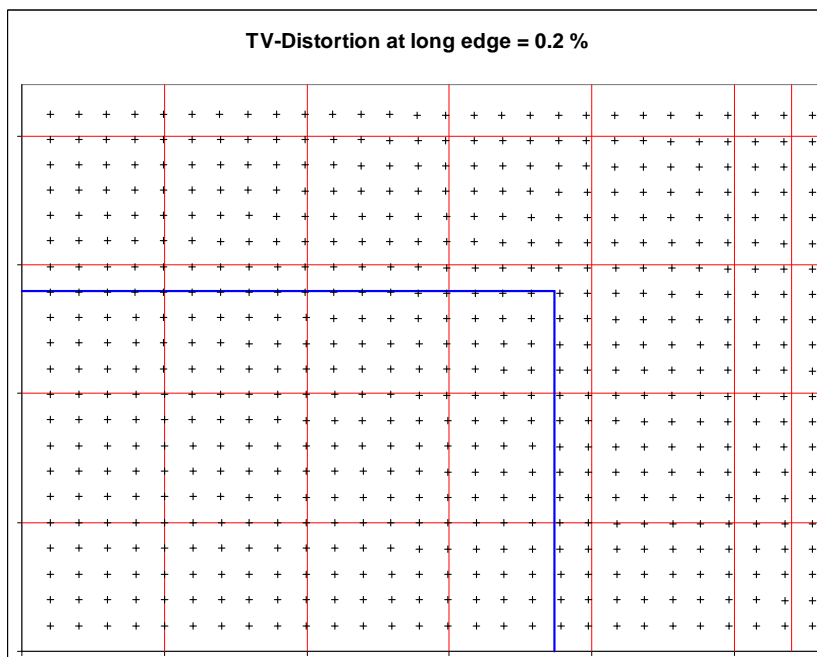
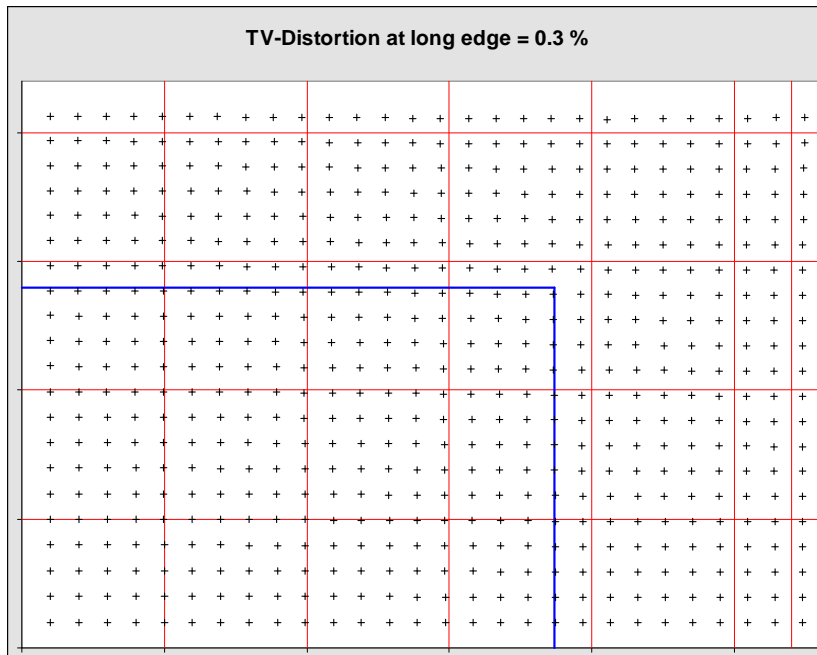
This shows that compromises with regard to distortion always need to take into consideration the size and weight of a lens. Often, a basically favorable lens design has been rejected in order to stick with the standard filter thread.

Lenses with a very **large aperture** are usually associated with somewhat higher distortion as compared to a more moderate initial aperture if all other parameters are identical. For this reason, macro-photography lenses offer better image geometry than high speed lenses with the same focal length.

One should keep in mind that distortion is a function of scale and can be changed by any kind of optical attachments.



*Distortion of the fast standard lens **Planar 1.4/50 ZF/ZE/ZK** at taking distance **infinity** (top and left graph) and at the minimum focus distance of **0.45 m***



Comparison of **Distagon 2.8/25 ZF** (top and left graphs) and **Biogon 2.8/25 ZM** (bottom and right): please note how small perceived distortion is even in the retrofocus lens. But the Biogon is still better: the peak TV distortion of the Distagon is about 0.5% at 8 or 9mm distance from the centre, while the Biogon has a peak value of just over 0.2%.

## Not all strange geometry is caused by distortion

Whether or not distortion bothers us in taking photos depends mainly on the actual object being photographed. In nature and landscape photography straight lines are rare with the result that we can tolerate larger errors since we do not perceive them at all. This also applies to portrait photography.

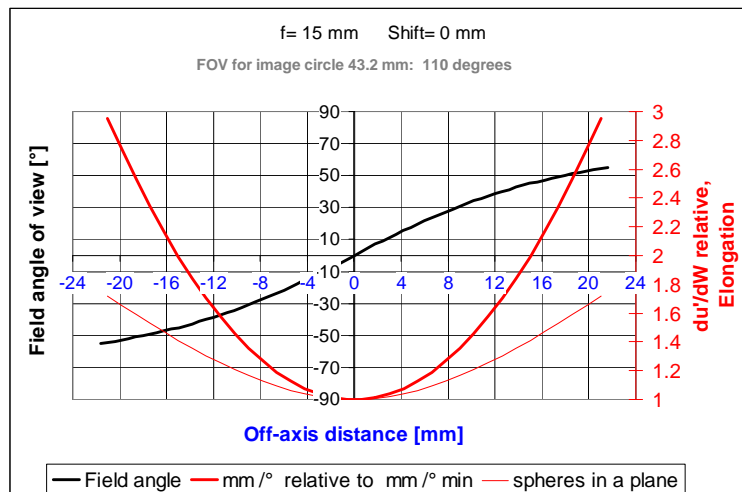
However, the more the world created by man is shown in the image, the greater is the need for distortion correction because man-made objects are full of geometric lines. In architecture and reproduction we tend to be very critical of distortion. Even if the straight lines are only background or of minor significance, e.g. in photos of people in rooms, next to furniture, etc., we find major distortion to be bothersome.

On the other hand, a lens that is completely free of all distortion does not solve all problems of photography with wide-angle lenses. For example in photos of groups of people, those poor souls standing close to the periphery of the picture appear weirdly warped and skewed and are bedecked with an 'egg-shaped' head. Is this due to yet another lens aberration?

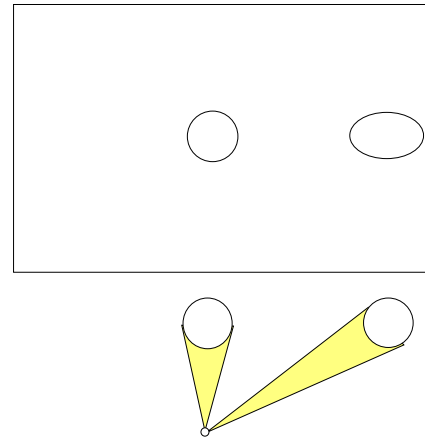
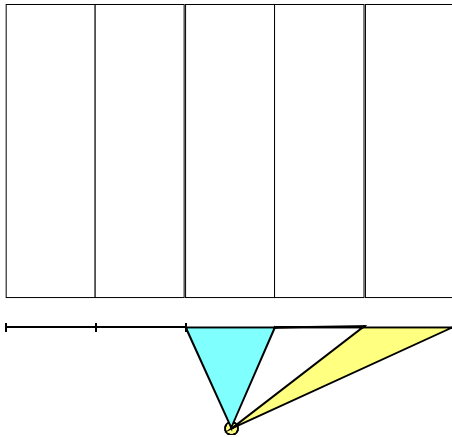


*The picture of this couple exhibits skewed "eggheads" while the balloon in the middle looks quite normal. The shot was taken with a 16mm-lens on 35mm format.*

No, rather it is a mistake made by the photographer who took the picture setting the focal length too short. It is an inevitable feature of lenses based on gnomonic, rectilinear projection that equal angle intervals need to be imaged onto a larger image distance towards the periphery. This is demonstrated in the following diagram using the example of a 15mm-lens for the 35 mm format:



*Plot of the image equation for a rectilinear 15mm-lens: the black curve indicates the field angle as a function of off-axis distance; this curve gets flatter towards the corners, which indicates to us that distortion-free lenses can never be used to obtain a diagonal field angle of 180°. The bold red curve is related to the slope of the black curve; its values are obtained by calculating the reciprocal value of the slope of the black curve and relating it to its value in the middle of the image. This means that the red curve tells us the factor by which the image of the same field angle interval is larger at the edge as compared to the middle: in the 15mm-lens, spherical objects at constant distance from the camera are elongated by a factor of 3 in the radial direction, if they are imaged in the corner. This is what causes the people to have "egg-shaped heads" as mentioned above. The thin red line is for spheres in a plane which don't have constant distance from the camera.*



*This effect does not occur in a flat two-dimensional object (left image) containing stripes of equal width or circles because the lens sees a stripe at the periphery at a smaller angle than in the middle. This decrease in angle fits exactly in the image equation, meaning that the distortion-free lens reproduces the geometry of the flat object in a true- to-nature manner.*

*However, when dealing with 3-dimensional objects, e.g. spheres or heads, the situation is different. The spheres are seen at the same angle regardless of where they are and therefore the increase of the angle scale (red curve on previous page) in fact generates the clear radial stretching of the image.*

If one wishes to avoid this effect, one either needs to use a longer focal length or, if this is not possible because of cramped conditions, to use a fisheye lens. Then however, once again one has to accept the strong curvature of straight lines. To optimize both at the same time is ultimately impossible since there is no way to image three-dimensional space onto two-dimensional image in a true-to-nature fashion - something has to give here.



*The image of a brick wall looks very strange when taken with a fisheye lens.*

These projection problems are some of the challenges of creating the image when using wide-angle lenses. Depending on the motif chosen and individual taste they might render the images either interesting or totally unacceptable.



*The geometry of some motifs nicely matches the imaging character of a fisheye lens and the strong curvilinear distortion of lines at the periphery is of minor importance.*

Aside from these problems which are particularly evident with round objects, there also is the well-known phenomenon of "convergent lines", i.e. lines converging when the image plane and the object plane are not parallel to each other. This is not a lens aberration either.

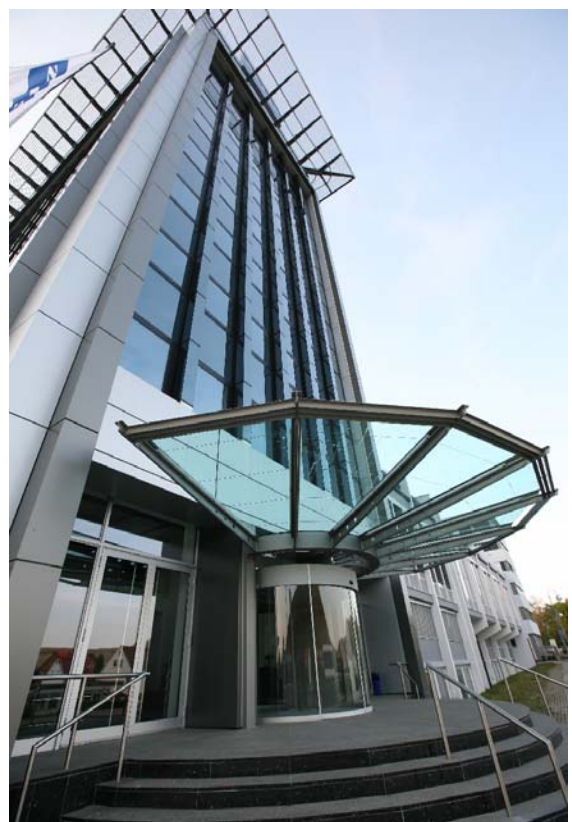


*To frame the complete building the camera had to be tilted upwards. Converging vertical lines are then a natural consequence and no lens aberration.*

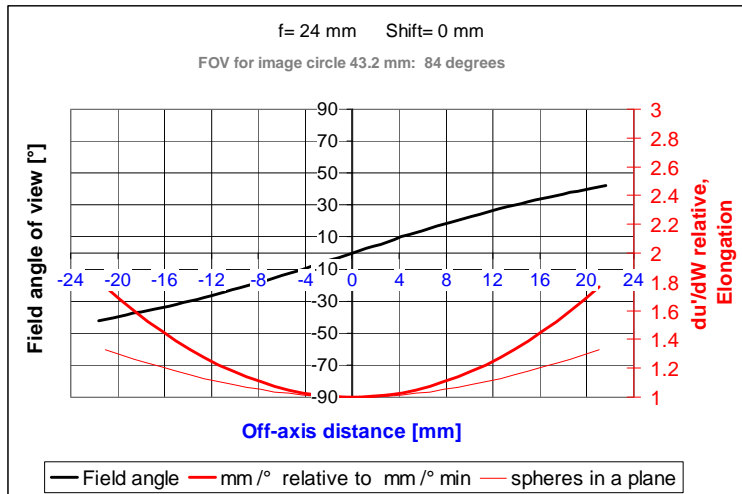
We perceive convergent parallel lines as a mistake mainly when they extend in the vertical direction. This creates a feeling that a building is toppling over. In contrast, very strongly convergent lines in the vertical direction suggest height to us, while convergent parallel lines in the horizontal direction are perceived as perfectly normal since they match our visual experience.



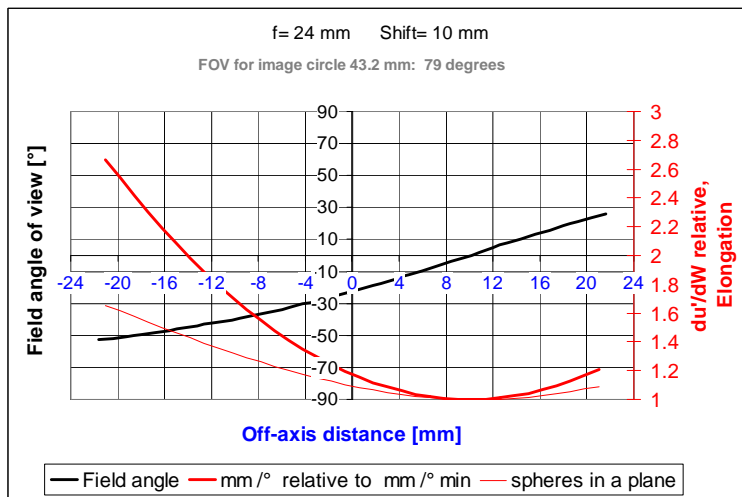
*Nobody would assume here that the building gets lower towards the right. We are used to the experience that parallel lines of the building converge to a common distant vanishing point.*



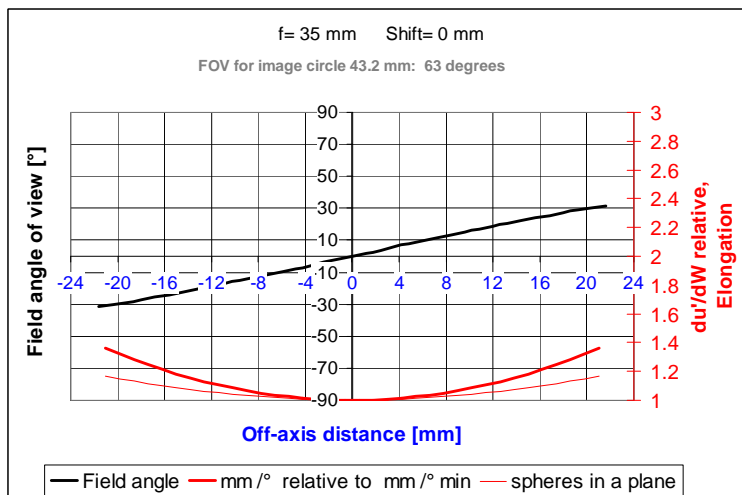
*In this dynamic image composition the strong vertical and horizontal convergence makes some amount of distortion invisible.*



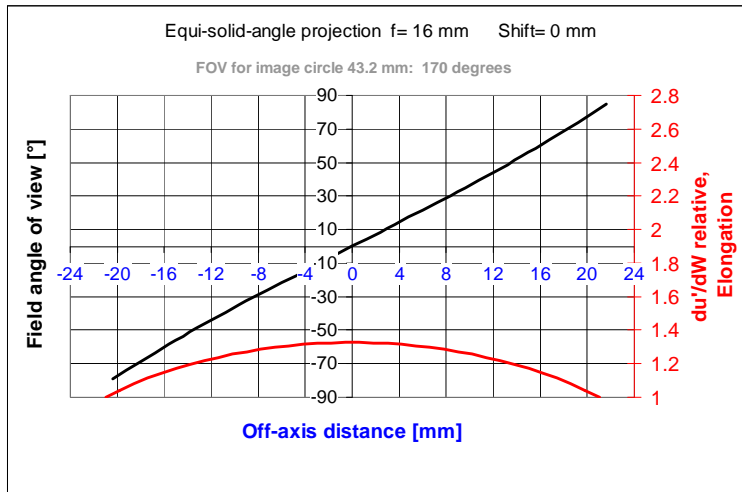
Imaging function and angle scale factor for a 24mm-lens. The radial elongation is already much less pronounced than with the 15mm focal length.



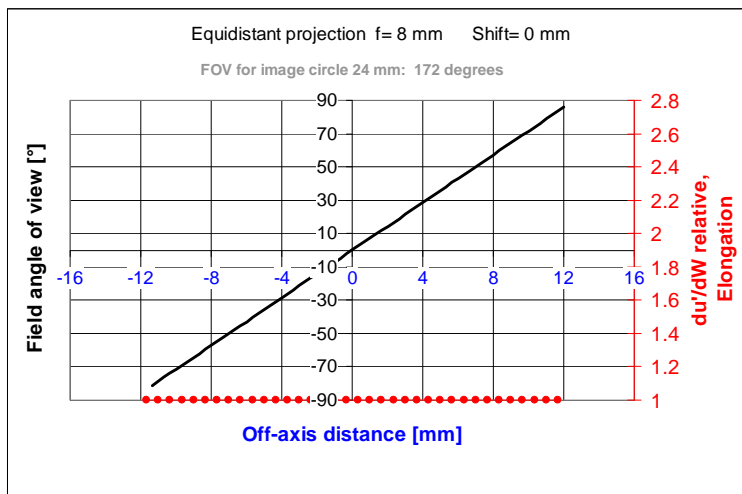
If the 24mm-lens has a larger image circle diameter and its optical axis can be **shifted** laterally by 10 mm, the angular scale factor in the lateral direction becomes unsymmetrical and we have approx. the same situation as with the 15 mm lens on one side, while on the opposite side the elongation is as low as with a 35mm-lens (below).







Imaging function and angular scale factor for a format-filling fisheye lens with a so-called “equi-solid-angle” projection: here, the angular scale factor decreases towards the periphery to the same degree in which it increases in a rectilinear 35mm-lens. Thus the distortion of spheres by the projection is minor - but the radial distortion value in the corner of the image is approx. - 90 %. All lines that do not run through the middle of the image exhibit a very strong barrel distortion.

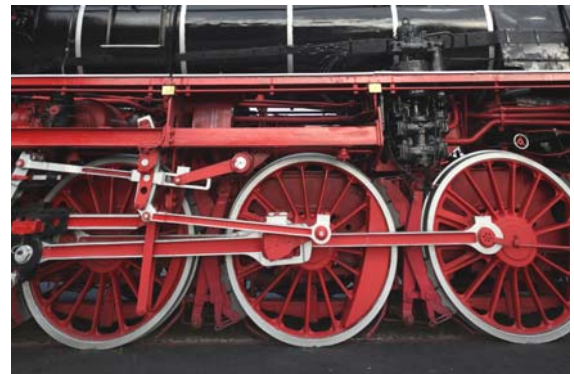
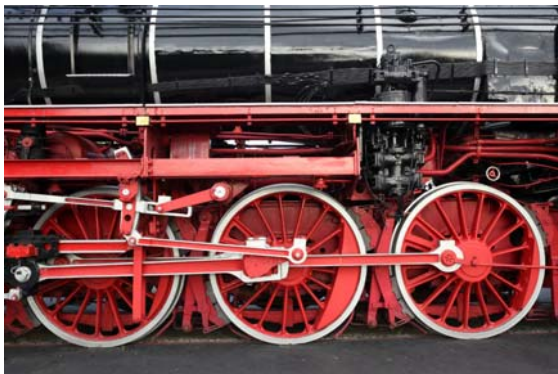


Imaging function and angle scale factor for a fisheye lens producing a non-format-filling circular image. This type of projection is called “**equidistant**”, since the image path of an angle interval is equal throughout the image. Lenses of this type were developed mainly for scientific applications for the measurement of angles in the object space by measuring the distances in the image.



*Two shots taken from the same point of view with a 24mm-shiftlens. For the image on the left hand side the lens had been shifted to the left, while it was shifted to the right for the shot on the right hand. To achieve about the same field of view in both cases the camera swivelled accordingly, while the tripod position was identical.*

*Thus on the left the angle between the image plane and the longitudinal direction of the locomotive is larger. This causes stronger convergence of horizontal lines. At the same time the front wheel is imaged in the range of larger field angles of the lens, so that more radial elongation applies to this side of the object. That is the reason for different shapes of the wheels: while the rear ones are seen as a vertical ellipse due to the oblique view, the radial elongation distorts the front wheel into a horizontally extended ellipse. This is really a matter of taste...*



*Another pair of shots with the 24mm-shiftlens from the same point of view. On the left hand side the lens is in the normal centre position and the camera is parallel to the front surface of the engine. On the right hand side the lens has been shifted downwards and the camera was tilted upwards to see again about the same field of view. Three effects are evident: 1) the total object field size has decreased a bit, 2) due to the oblique viewing direction and due to the increasing radial elongation towards the lower frame edge the wheels are distorted into ellipses and their axes are no longer in the centre, 3) the white bands on the kettle and the axes of the elliptic shapes of the left and the right wheel exhibit convergence. Nobody would believe that this engine could roll ...*

## Correction by Software

We have now learned from several examples that lens distortion cannot be perfectly described by a single number. Instead we have to understand a curve or a kind of map. This sounds like plenty of numbers.

Fortunately things are not that bad, since the distortion of many lenses can be described by just two numbers with a sufficient accuracy. All distortion functions which we have seen above follow a quite simple equation:

$$D = a \cdot u^2 + b \cdot u^4$$

In the simplest cases we have  $b=0$  and then the radial distortion  $D$  is proportional to the square of the off-axis distance. If  $b$  is clearly different from zero and  $a$  and  $b$  have different sign then we have the case of a wave-type distortion. The absolute error of the distorted image point position is given by: (known as the 5<sup>th</sup>-order approximation)

$$\Delta r = a \cdot u^3 + b \cdot u^5$$

The two constant parameters  $a$  and  $b$  in this equation tell us everything about the distortion of this lens – at a particular taking distance. When distance and scale factor are changed  $a$  and  $b$  change as well.

When the radial distortion curve of a lens is known, e.g. from the maker's data sheet, then  $a$  and  $b$  can be calculated from just two points on the curve.

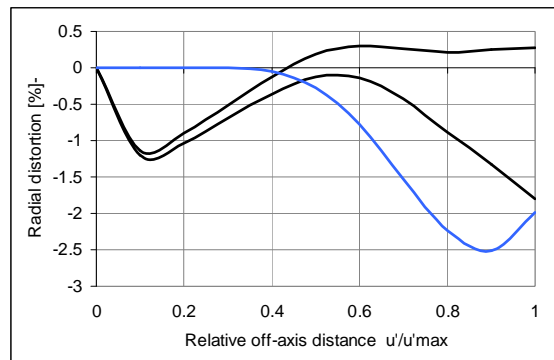
The parameters can also be calculated from image data: three points on an image line close to the frame edge are basically enough. In practice one uses more points, to account for small errors of co-ordinate reading and camera orientation. And of course one has to assume that the line in the target is absolutely straight.

If one knows the numbers  $a$  and  $b$ , one knows the distortion error in each position of the frame. And then this error can be cancelled when the image is available in terms of digital data.

Everybody is familiar with the change of the digital image size, for instance to match the file to a certain print size. Interpolation algorithms are used for that purpose. Distortion correction works in a similar way, the size changes are just variable within the frame.

In addition the correction based on the simple radial function assumes that the distortion error is rotationally symmetric. Real lenses have however also tangential distortion, and the radial distortion is not constant on a circle in the image. But these effects are small and only of importance for highly accurate measurement procedures. They can be neglected in normal photography.

Sometimes there exist distortion functions which don't follow above simple equations and then many correction tools fail:



*Distortion of two lenses for mobile phone cameras; the black curves are for two different taking distances. Lenses with strongly aspheric surfaces exhibit complex distortion, which cannot be expressed as a 5<sup>th</sup>-order approximation.*

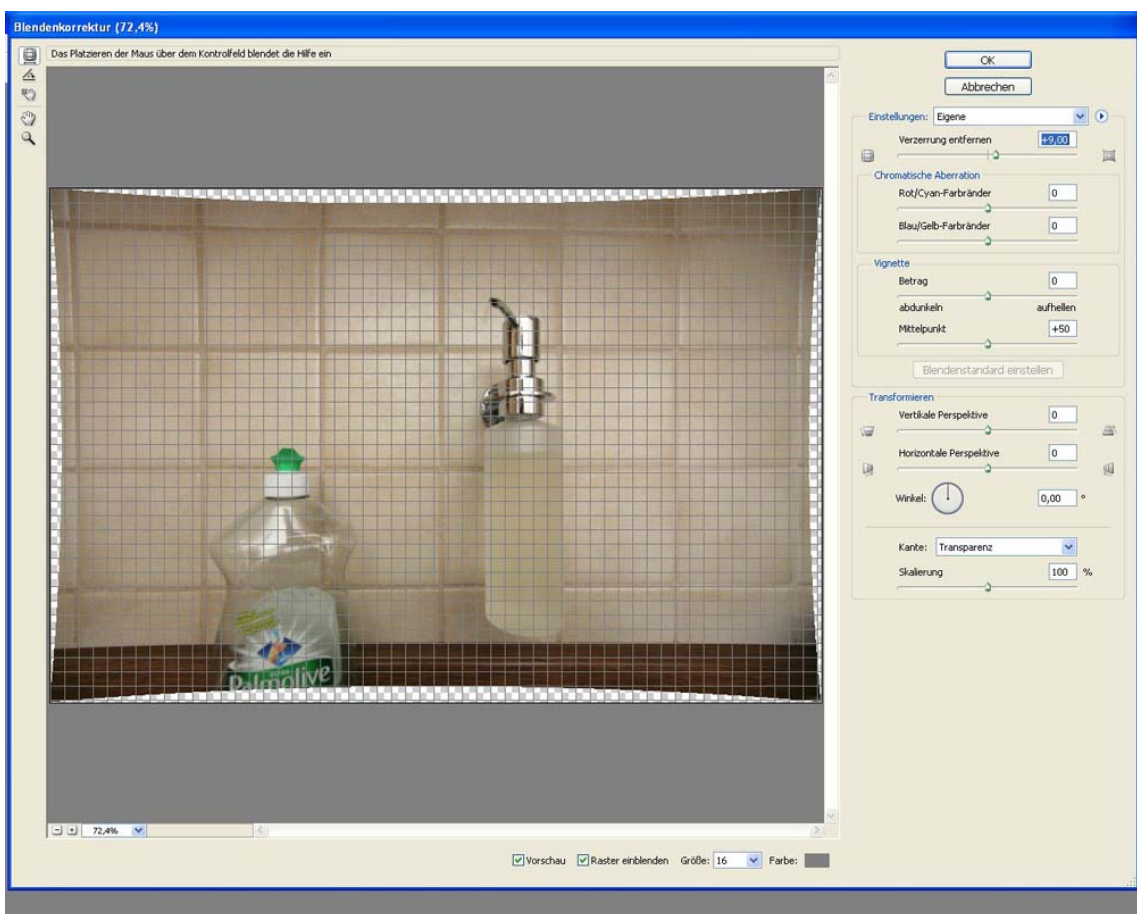
Simple correction programs can only handle the case  $b=0$ , pure barrel or pincushion type of distortion. They cannot correct moustache distortion perfectly.

Converging lines and curvilinear distortion in digital image files can be treated by suitable software modules. Annoying errors of this type can be cancelled semi-automatically or manually. Some RAW conversion programs (e.g. Canon Digital Photo Professional, Nikon Capture NX2) offer an automatic correction of distortion based on the EXIF data of the lens.

Independent software for image improvement (e.g. DXO Tools, Acolens, PTLens) allows an individual definition of correction profiles for various lenses.

**Photoshop** and **Photoshop Elements** offer simple but quite effective correction of normal barrel or pincushion distortion with help of control sliders.

We plan to deal with these correction procedures in more detail in one of the future issues of CLN.



*Manual distortion correction in Photoshop*