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Theory and Problems of

ELECTRIC CIRCUITS

Fourth Edition

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PREFACE

This book is designed for use as a textbook for a first course in circuit analysis or as a supplement to standard texts and can be used by electrical engineering students as well as other engineereing and technology students. Emphasis is placed on the basic laws, theorems, and problem-solving techniques which are common to most courses.

The subject matter is divided into 17 chapters covering duly-recognized areas of theory and study. The chapters begin with statements of pertinent definitions, principles, and theorems together with illustrative examples. This is followed by sets of solved and supplementary problems. The problems cover a range of levels of difficulty. Some problems focus on fine points, which helps the student to better apply the basic principles correctly and confidently. The supplementary problems are generally more numerous and give the reader an opportunity to practice problem-solving skills. Answers are provided with each supplementary problem.

The book begins with fundamental definitions, circuit elements including dependent sources, circuit laws and theorems, and analysis techniques such as node voltage and mesh current methods. These theorems and methods are initially applied to DC-resistive circuits and then extended to RLC circuits by the use of impedance and complex frequency. Chapter 5 on amplifiers and op amp circuits is new. The op amp examples and problems are selected carefully to illustrate simple but practical cases which are of interest and importance in the student's future courses. The subject of waveforms and signals is also treated in a new chapter to increase the student's awareness of commonly used signal models.

Circuit behavior such as the steady state and transient response to steps, pulses, impulses, and exponential inputs is discussed for first-order circuits in Chapter 7 and then extended to circuits of higher order in Chapter 8, where the concept of complex frequency is introduced. Phasor analysis, sinuosidal steady state, power, power factor, and polyphase circuits are thoroughly covered. Network functions, frequency response, filters, series and parallel resonance, two-port networks, mutual inductance, and transformers are covered in detail. Application of Spice and PSpice in circuit analysis is introduced in Chapter 15. Circuit equations are solved using classical differential equations and the Laplace transform, which permits a convenient comparison. Fourier series and Fourier transforms and their use in circuit analysis are covered in Chapter 17. Finally, two appendixes provide a useful summary of the complex number system, and matrices and determinants.

This book is dedicated to our students from whom we have learned to teach well. To a large degree it is they who have made possible our satisfying and rewarding teaching careers. And finally, we wish to thank our wives, *Zahra Nahvi* and *Nina Edminister* for their continuing support, and for whom all these efforts were happily made.

MAHMOOD NAHVI
JOSEPH A. EDMINISTER

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CHAPTER 2

Circuit Concepts

2.1 PASSIVE AND ACTIVE ELEMENTS

An electrical device is represented by a *circuit diagram* or *network* constructed from series and parallel arrangements of two-terminal elements. The analysis of the circuit diagram predicts the performance of the actual device. A two-terminal element in general form is shown in Fig. 2-1, with a single device represented by the rectangular symbol and two perfectly conducting leads ending at connecting points A and B. *Active* elements are voltage or current sources which are able to supply energy to the network. Resistors, inductors, and capacitors are *passive* elements which take energy from the sources and either convert it to another form or store it in an electric or magnetic field.

Fig. 2-1

Figure 2-2 illustrates seven basic circuit elements. Elements (a) and (b) are voltage sources and (c) and (d) are current sources. A voltage source that is not affected by changes in the connected circuit is an *independent* source, illustrated by the circle in Fig. 2-2(a). A *dependent* voltage source which changes in some described manner with the conditions on the connected circuit is shown by the diamond-shaped symbol in Fig. 2-2(b). Current sources may also be either independent or dependent and the corresponding symbols are shown in (c) and (d). The three passive circuit elements are shown in Fig. 2-2(e), (f), and (g).

The circuit diagrams presented here are termed *lumped-parameter* circuits, since a single element in one location is used to represent a distributed resistance, inductance, or capacitance. For example, a coil consisting of a large number of turns of insulated wire has resistance throughout the entire length of the wire. Nevertheless, a single resistance *lumped* at one place as in Fig. 2-3(b) or (c) represents the distributed resistance. The inductance is likewise lumped at one place, either in series with the resistance as in (b) or in parallel as in (c).

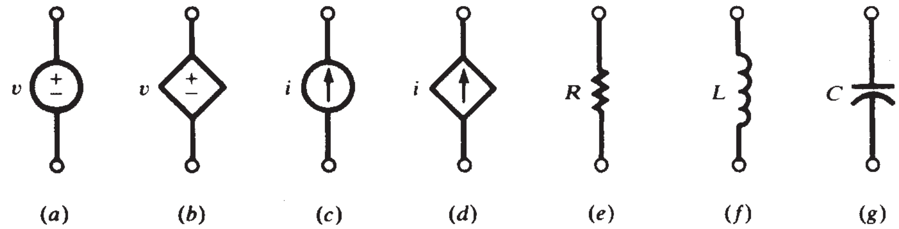


Fig. 2-2

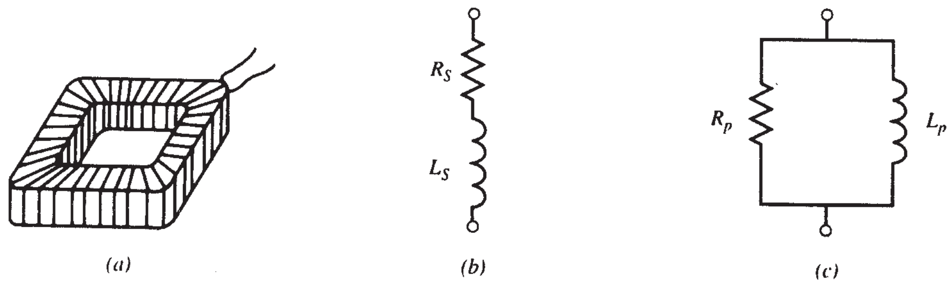


Fig. 2-3

In general, a coil can be represented by either a series or a parallel arrangement of circuit elements. The frequency of the applied voltage may require that one or the other be used to represent the device.

2.2 SIGN CONVENTIONS

A voltage function and a polarity must be specified to completely describe a voltage source. The polarity marks, + and −, are placed near the conductors of the symbol that identifies the voltage source. If, for example, $v = 10.0 \sin \omega t$ in Fig. 2-4(*a*), terminal *A* is positive with respect to *B* for $0 > \omega t > \pi$, and *B* is positive with respect to *A* for $\pi > \omega t > 2\pi$ for the first cycle of the sine function.

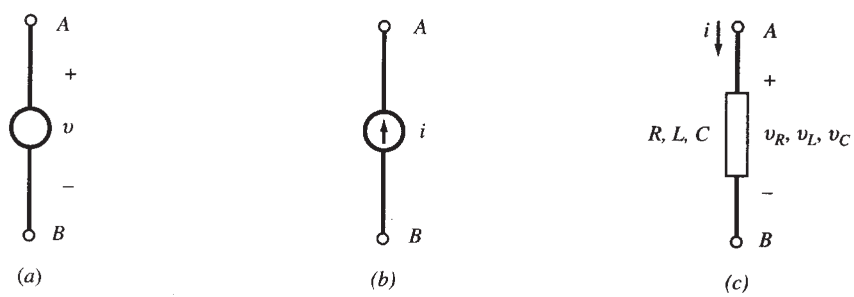


Fig. 2-4

Similarly, a current source requires that a direction be indicated, as well as the function, as shown in Fig. 2-4(*b*). For passive circuit elements *R*, *L*, and *C*, shown in Fig. 2-4(*c*), the terminal where the current enters is generally treated as positive with respect to the terminal where the current leaves.

The sign on power is illustrated by the dc circuit of Fig. 2-5(*a*) with constant voltage sources $V_A = 20.0$ V and $V_B = 5.0$ V and a single 5-Ω resistor. The resulting current of 3.0 A is in the clockwise direction. Considering now Fig. 2-5(*b*), power is absorbed by an element when the current enters the element at the positive terminal. Power, computed by VI or I^2R, is therefore absorbed by both the resistor and the V_B source, 45.0 W and 15 W respectively. Since the current enters V_A at the negative terminal, this element is the power source for the circuit. $P = VI = 60.0$ W confirms that the power absorbed by the resistor and the source V_B is provided by the source V_A.

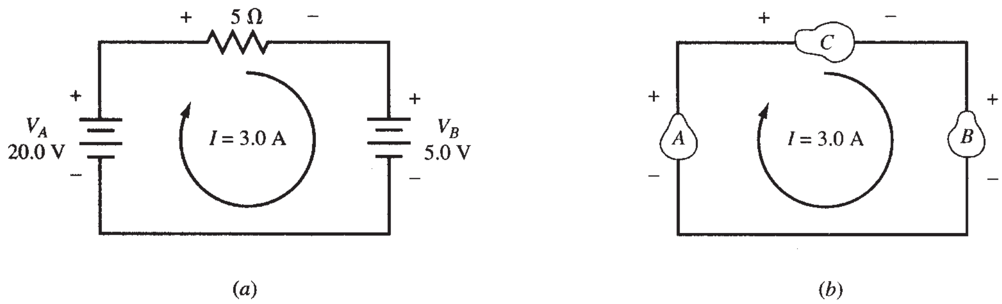
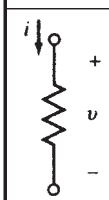
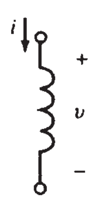
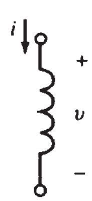


Fig. 2-5

2.3 VOLTAGE-CURRENT RELATIONS

The passive circuit elements resistance R, inductance L, and capacitance C are defined by the manner in which the voltage and current are related for the individual element. For example, if the voltage v and current i for a single element are related by a constant, then the element is a resistance, R is the constant of proportionality, and $v = Ri$. Similarly, if the voltage is the time derivative of the current, then the element is an inductance, L is the constant of proportionality, and $v = L\,di/dt$. Finally, if the current in the element is the time derivative of the voltage, then the element is a capacitance, C is the constant of proportionality, and $i = C\,dv/dt$. Table 2-1 summarizes these relationships for the three passive circuit elements. Note the current directions and the corresponding polarity of the voltages.

Table 2-1

Circuit element	Units	Voltage	Current	Power
 **Resistance**	ohms (Ω)	$v = Ri$ **(Ohms's law)**	$i = \frac{v}{R}$	$p = vi = i^2 R$
 **Inductance**	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
 **Capacitance**	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$

2.4 RESISTANCE

All electrical devices that consume energy must have a resistor (also called a *resistance*) in their circuit model. Inductors and capacitors may store energy but over time return that energy to the source or to another circuit element. Power in the resistor, given by $p = vi = i^2R = v^2/R$, is always positive as illustrated in Example 2.1 below. Energy is then determined as the integral of the instantaneous power

$$w = \int_{t_1}^{t_2} p\,dt = R\int_{t_1}^{t_2} i^2\,dt = \frac{1}{R}\int_{t_1}^{t_2} v^2\,dt$$

EXAMPLE 2.1. A 4.0-Ω resistor has a current $i = 2.5\sin\omega t$ (A). Find the voltage, power, and energy over one cycle. $\omega = 500\,\text{rad/s}$.

$$v = Ri = 10.0\sin\omega t\ \text{(V)}$$
$$p = vi = i^2R = 25.0\sin^2\omega t\ \text{(W)}$$
$$w = \int_0^t p\,dt = 25.0\left[\frac{t}{2} - \frac{\sin 2\omega t}{4\omega}\right]\ \text{(J)}$$

The plots of i, p, and w shown in Fig. 2-6 illustrate that p is always positive and that the energy w, although a function of time, is always increasing. This is the energy absorbed by the resistor.

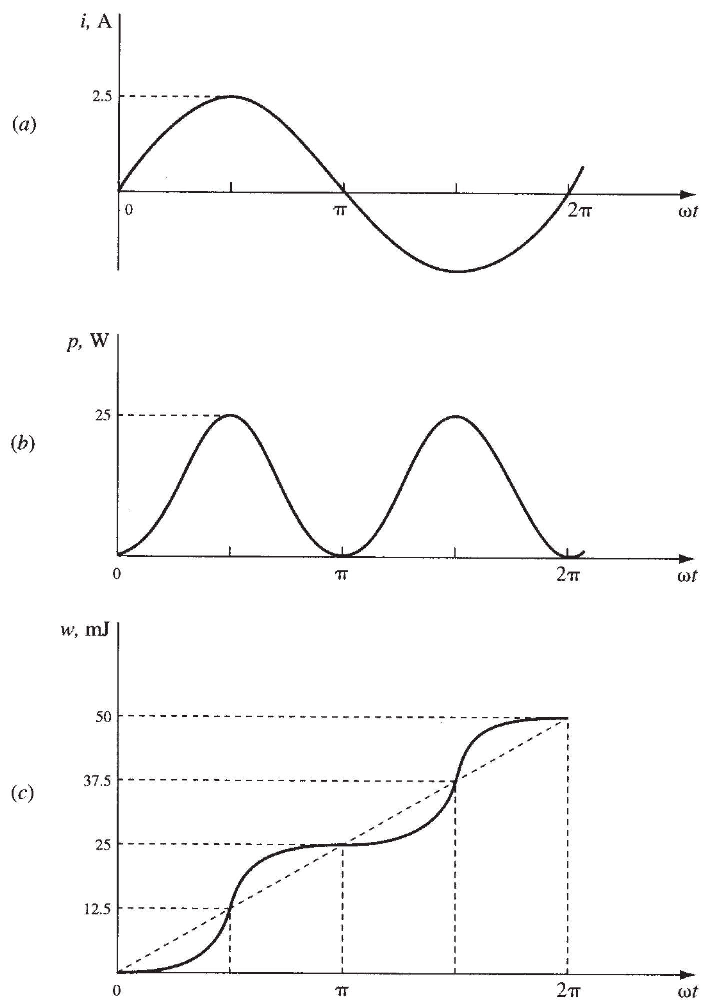


Fig. 2-6

2.5 INDUCTANCE

The circuit element that stores energy in a magnetic field is an inductor (also called an *inductance*). With time-variable current, the energy is generally stored during some parts of the cycle and then returned to the source during others. When the inductance is removed from the source, the magnetic field will collapse; in other words, no energy is stored without a connected source. Coils found in electric motors, transformers, and similar devices can be expected to have inductances in their circuit models. Even a set of parallel conductors exhibits inductance that must be considered at most frequencies. The power and energy relationships are as follows.

$$p = vi = L\frac{di}{dt}\,i = \frac{d}{dt}\left[\frac{1}{2}Li^2\right]$$

$$w_L = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} Li\,dt = \frac{1}{2}L[i_2^2 - i_1^2]$$

Energy stored in the magnetic field of an inductance is $w_L = \frac{1}{2}Li^2$.

EXAMPLE 2.2. In the interval $0 > t > (\pi/50)$ s a 30-mH inductance has a current $i = 10.0\sin 50t$ (A). Obtain the voltage, power, and energy for the inductance.

$$v = L\frac{di}{dt} = 15.0\cos 50t\ \text{(V)} \qquad p = vi = 75.0\sin 100t\ \text{(W)} \qquad w_L = \int_0^t p\,dt = 0.75(1-\cos 100t)\ \text{(J)}$$

As shown in Fig. 2-7, the energy is zero at $t = 0$ and $t = (\pi/50)$ s. Thus, while energy transfer did occur over the interval, this energy was first stored and later returned to the source.

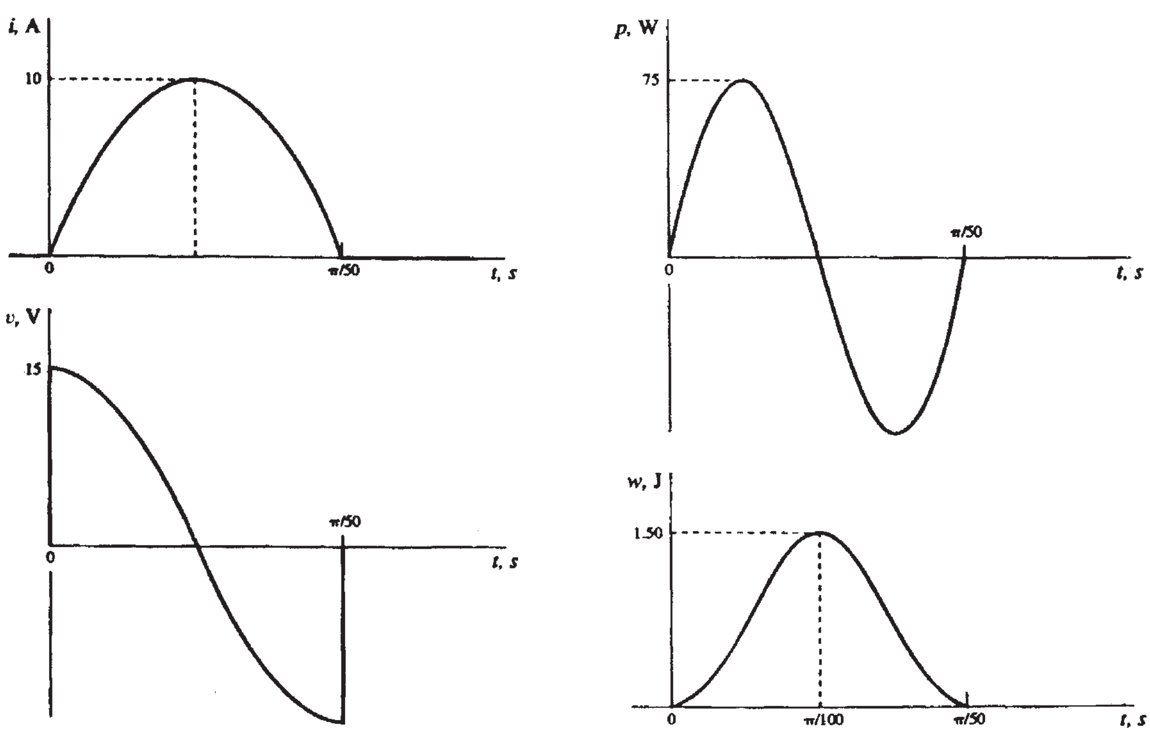


Fig. 2-7

2.6 CAPACITANCE

The circuit element that stores energy in an electric field is a *capacitor* (also called *capacitance*). When the voltage is variable over a cycle, energy will be stored during one part of the cycle and returned in the next. While an inductance cannot retain energy after removal of the source because the magnetic field collapses, the capacitor retains the charge and the electric field can remain after the source is removed. This charged condition can remain until a discharge path is provided, at which time the energy is released. The charge, $q = Cv$, on a capacitor results in an electric field in the dielectric which is the mechanism of the energy storage. In the simple parallel-plate capacitor there is an excess of charge on one plate and a deficiency on the other. It is the equalization of these charges that takes place when the capacitor is discharged. The power and energy relationships for the capacitance are as follows.

$$p = vi = Cv\frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2}Cv^2\right]$$

$$w_C = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} Cv\,dv = \frac{1}{2}C[v_2^2 - v_1^2]$$

The energy stored in the electric field of capacitance is $w_C = \frac{1}{2}Cv^2$.

EXAMPLE 2.3. In the interval $0 > t > 5\pi$ ms, a 20-μF capacitance has a voltage $v = 50.0 \sin 200t$ (V). Obtain the charge, power, and energy. Plot w_C assuming $w = 0$ at $t = 0$.

$$q = Cv = 1000 \sin 200t \ (\mu\text{C})$$

$$i = C\frac{dv}{dt} = 0.20 \cos 200t \ (\text{A})$$

$$p = vi = 5.0 \sin 400t \ (\text{W})$$

$$w_C = \int_{t_1}^{t_2} p\,dt = 12.5[1 - \cos 400t] \ (\text{mJ})$$

In the interval $0 > t > 2.5\pi$ ms the voltage and charge increase from zero to 50.0 V and 1000 μC, respectively. Figure 2-8 shows that the stored energy increases to a value of 25 mJ, after which it returns to zero as the energy is returned to the source.

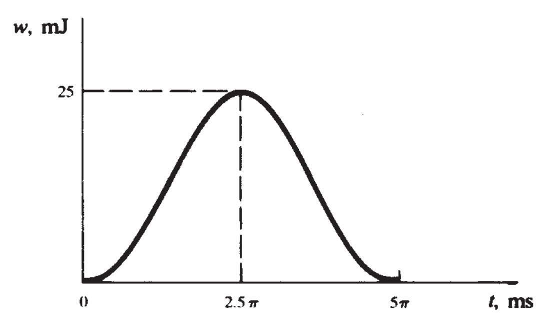


Fig. 2-8

2.7 CIRCUIT DIAGRAMS

Every circuit diagram can be constructed in a variety of ways which may look different but are in fact identical. The diagram presented in a problem may not suggest the best of several methods of solution. Consequently, a diagram should be examined before a solution is started and redrawn if necessary to show more clearly how the elements are interconnected. An extreme example is illustrated in Fig. 2-9, where the three circuits are actually identical. In Fig. 2-9(a) the three "junctions" labeled A

CHAPTER 12

Frequency Response, Filters, and Resonance

12.1 FREQUENCY RESPONSE

The response of linear circuits to a sinusoidal input is also a sinusoid, with the same frequency but possibly a different amplitude and phase angle. This response is a function of the frequency. We have already seen that a sinusoid can be represented by a phasor which shows its magnitude and phase. The *frequency response* is defined as the ratio of the output phasor to the input phasor. It is a real function of $j\omega$ and is given by

$$\mathbf{H}(j\omega) = \text{Re}[\mathbf{H}] + j\,\text{Im}[\mathbf{H}] = |\mathbf{H}|e^{j\theta} \tag{1a}$$

where Re [**H**] and Im [**H**] are the real and imaginary parts of $\mathbf{H}(j\omega)$ and $|\mathbf{H}|$ and θ are its magnitude and phase angle. Re [**H**], Im [**H**], $|\mathbf{H}|$, and θ are, in general, functions of ω. They are related by

$$|\mathbf{H}|^2 = |\mathbf{H}(j\omega)|^2 = \text{Re}^2[\mathbf{H}] + \text{Im}^2[\mathbf{H}] \tag{1b}$$

$$\theta = \underline{/\mathbf{H}(j\omega)} = \tan^{-1}\frac{\text{Im}[\mathbf{H}]}{\text{Re}[\mathbf{H}]} \tag{1c}$$

The frequency response, therefore, depends on the choice of input and output variables. For example, if a current source is connected across the network of Fig. 12-1(*a*), the terminal current is the input and the terminal voltage may be taken as the output. In this case, the input impedance $\mathbf{Z} = \mathbf{V}_1/\mathbf{I}_1$ constitutes the frequency response. Conversely, if a voltage source is applied to the input and

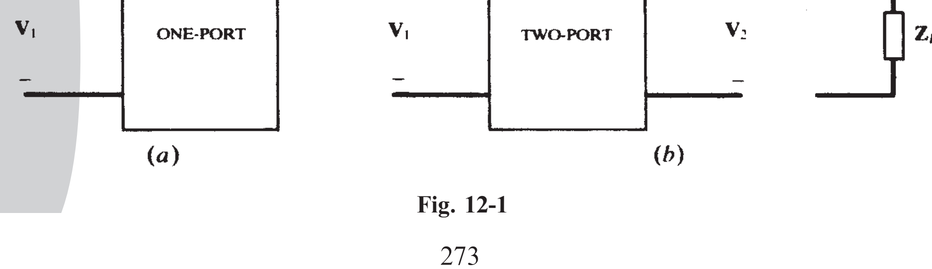


Fig. 12-1

the terminal current is measured, the input admittance $\mathbf{Y} = \mathbf{I}_1/\mathbf{V}_1 = 1/\mathbf{Z}$ represents the frequency response.

For the two-port network of Fig. 12-1(*b*), the following frequency responses are defined:

Input impedance $\mathbf{Z}_{\text{in}}(j\omega) = \mathbf{V}_1/\mathbf{I}_1$
Input admittance $\mathbf{Y}_{\text{in}}(j\omega) = 1/\mathbf{Z}_{\text{in}}(j\omega) = \mathbf{I}_1/\mathbf{V}_1$
Voltage transfer ratio $\mathbf{H}_v(j\omega) = \mathbf{V}_2/\mathbf{V}_1$
Current transfer ratio $\mathbf{H}_i(j\omega) = \mathbf{I}_2/\mathbf{I}_1$
Transfer impedances $\mathbf{V}_2/\mathbf{I}_1$ and $\mathbf{V}_1/\mathbf{I}_2$

EXAMPLE 12.1 Find the frequency response $\mathbf{V}_2/\mathbf{V}_1$ for the two-port circuit shown in Fig. 12-2.

Let $\mathbf{Y}_{RC}$ be the admittance of the parallel RC combination. Then, $\mathbf{Y}_{RC} = 10^{-6}j\omega + 1/1250$. $\mathbf{V}_2/\mathbf{V}_1$ is obtained by dividing $\mathbf{V}_1$ between $\mathbf{Z}_{RC}$ and the 5-kΩ resistor.

$$\mathbf{H}(j\omega) = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{Z}_{RC}}{\mathbf{Z}_{RC} + 5000} = \frac{1}{1 + 5000\mathbf{Y}_{RC}} = \frac{1}{5(1 + 10^{-3}j\omega)} \tag{2a}$$

$$|\mathbf{H}| = \frac{1}{5\sqrt{1 + 10^{-6}\omega^2}} \theta = -\tan^{-1}(10^{-3}\omega) \tag{2b}$$

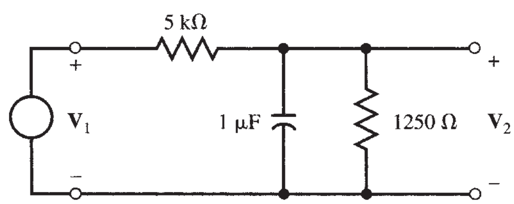


Fig. 12-2

Alternative solution: First we find the Thévenin equivalent of the resistive part of the circuit, $V_{\text{Th}} = V_1/5$ and $R_{\text{Th}} = 1\ \text{k}\Omega$, and then divide V_{Th} between R_{Th} and the 1-μF capacitor to obtain (*2a*).

12.2 HIGH-PASS AND LOW-PASS NETWORKS

A resistive voltage divider under a no-load condition is shown in Fig. 12-3, with the standard two-port voltages and currents. The voltage transfer function and input impedance are

$$\mathbf{H}_{v\infty}(\omega) = \frac{R_2}{R_1 + R_2} \qquad \mathbf{H}_{z\infty}(\omega) = R_1 + R_2$$

The ∞ in subscripts indicates no-load conditions. Both $\mathbf{H}_{v\infty}$ and $\mathbf{H}_{z\infty}$ are real constants, independent of frequency, since no reactive elements are present. If the network contains either an inductance or a capacitance, then $\mathbf{H}_{v\infty}$ and $\mathbf{H}_{z\infty}$ will be complex and will vary with frequency. If $|\mathbf{H}_{v\infty}|$ decreases as

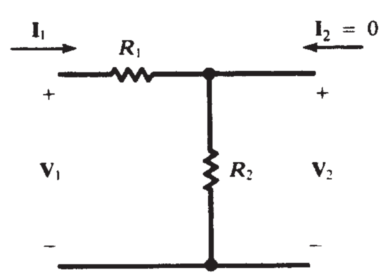
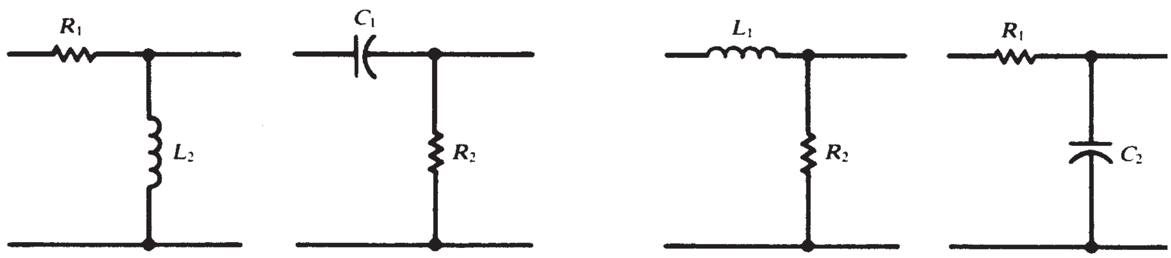


Fig. 12-3

frequency increases, the performance is called *high-frequency roll-off* and the circuit is a *low-pass network*. On the contrary, a *high-pass network* will have low-frequency roll-off, with $|\mathbf{H}_{v\infty}|$ decreasing as the frequency decreases. Four two-element circuits are shown in Fig. 12-4, two high-pass and two low-pass.



(*a*) High-Pass (*b*) Low-Pass

Fig. 12-4

The *RL* high-pass circuit shown in Fig. 12-5 is open-circuited or under no-load. The input impedance frequency response is determined by plotting the magnitude and phase angle of

$$\mathbf{H}_{z\infty}(\omega) = R_1 + j\omega L_2 \equiv |\mathbf{H}_z| \underline{/\theta_{\mathbf{H}}}$$

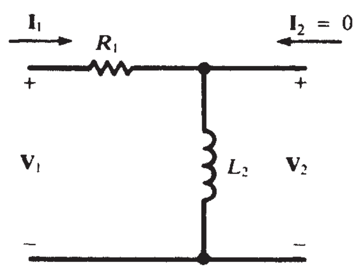


Fig. 12-5

or, normalizing and writing $\omega_x \equiv R_1/L_2$,

$$\frac{\mathbf{H}_{z\infty}(\omega)}{R_1} = 1 + j(\omega/\omega_x) = \sqrt{1 + (\omega/\omega_x)^2}\, \underline{/\tan^{-1}(\omega/\omega_x)}$$

Five values of ω provide sufficient data to plot $|\mathbf{H}_z|/R_1$ and $\theta_{\mathbf{H}}$, as shown in Fig. 12-6. The magnitude approaches infinity with increasing frequency, and so, at very high frequencies, the network current $\mathbf{I}_1$ will be zero.

In a similar manner, the frequency response of the output-to-input voltage ratio can be obtained. Voltage division under no-load gives

$$\mathbf{H}_{v\infty}(\omega) = \frac{j\omega L_2}{R_1 + j\omega L_2} = \frac{1}{1 - j(\omega_x/\omega)}$$

so that

$$|\mathbf{H}_v| = \frac{1}{\sqrt{1 + (\omega_x/\omega)^2}} \qquad \text{and} \qquad \theta_{\mathbf{H}} = \tan^{-1}(\omega_x/\omega)$$

ω	$\lvert\mathbf{H}_z\rvert/R_1$	$\theta_{\mathbf{H}}$
0	1	0°
$0.5\omega_x$	$0.5\sqrt{5}$	26.6°
ω_x	$\sqrt{2}$	45°
$2\omega_x$	$\sqrt{5}$	63.4°
∞	∞	90°

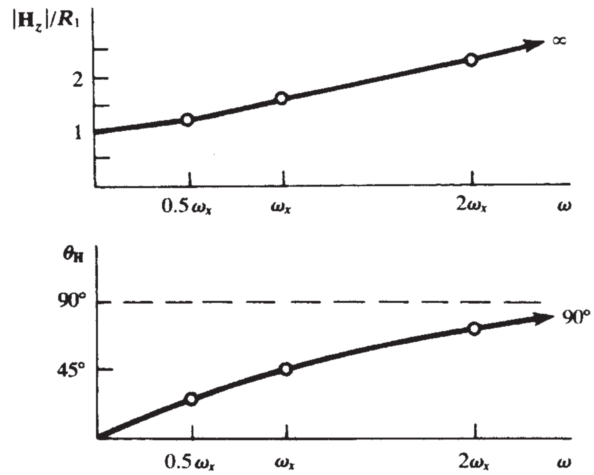


Fig. 12-6

The magnitude and angle are plotted in Fig. 12-7. This transfer function approaches unity at high frequency, where the output voltage is the same as the input. Hence the description "low-frequency roll-off" and the name "high-pass."

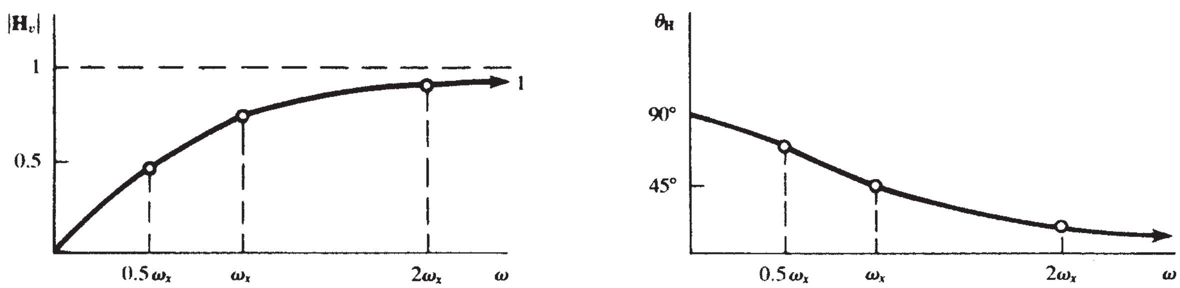


Fig. 12-7

A transfer impedance of the RL high-pass circuit under no-load is

$$\mathbf{H}_\infty(\omega) = \frac{\mathbf{V}_2}{\mathbf{I}_1} = j\omega L_2 \qquad \text{or} \qquad \frac{\mathbf{H}_\infty(\omega)}{R_1} = j\,\frac{\omega}{\omega_x}$$

The angle is constant at 90°; the graph of magnitude versus ω is a straight line, similar to a reactance plot of ωL versus ω. See Fig. 12-8.

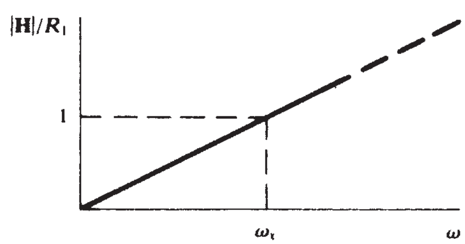


Fig. 12-8

Interchanging the positions of R and L results in a low-pass network with high-frequency roll-off (Fig. 12-9). For the open-circuit condition

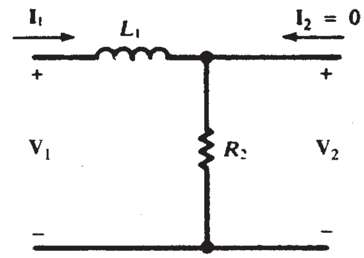


Fig. 12-9

$$\mathbf{H}_{v\infty}(\omega) = \frac{R_2}{R_2 + j\omega L_1} = \frac{1}{1 + j(\omega/\omega_x)}$$

with $\omega_x \equiv R_2/L_1$; that is,

$$|\mathbf{H}_v| = \frac{1}{\sqrt{1 + (\omega/\omega_x)^2}} \qquad \text{and} \qquad \theta_{\mathbf{H}} = \tan^{-1}(-\omega/\omega_x)$$

The magnitude and angle plots are shown in Fig. 12-10. The voltage transfer function $\mathbf{H}_{v\infty}$ approaches zero at high frequencies and unity at $\omega = 0$. Hence the name "low-pass."

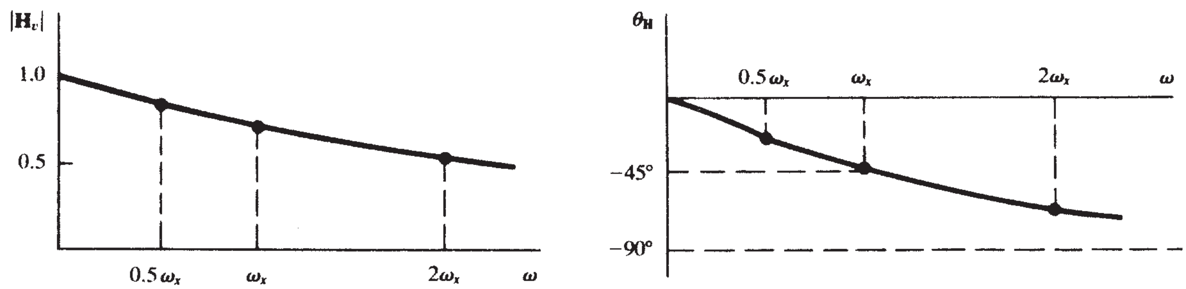


Fig. 12-10

The other network functions of this low-pass network are obtained in the Solved Problems.

EXAMPLE 12.2 Obtain the voltage transfer function $\mathbf{H}_{v\infty}$ for the open circuit shown in Fig. 12-11. At what frequency, in hertz, does $|\mathbf{H}_v| = 1/\sqrt{2}$ if (*a*) $C_2 = 10$ nF, (*b*) $C_2 = 1$ nF?

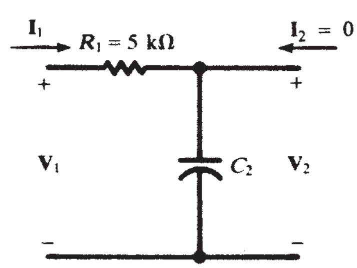


Fig. 12-11

$$\mathbf{H}_{v\infty}(\omega) = \frac{1/j\omega C_2}{R_1 + (1/j\omega C_2)} = \frac{1}{1 + j(\omega/\omega_x)} \qquad \text{where} \qquad \omega_x \equiv \frac{1}{R_1 C_2} = \frac{2 \times 10^{-4}}{C_2} \quad \text{(rad/s)}$$

(*a*)

$$|\mathbf{H}_v| = \frac{1}{\sqrt{1 + (\omega/\omega_x)^2}}$$

and so $|\mathbf{H}_v| = 1/\sqrt{2}$ when

$$\omega = \omega_x = \frac{2 \times 10^{-4}}{10 \times 10^{-9}} = 2 \times 10^4 \text{ rad/s}$$

or when $f = (2 \times 10^4)/2\pi = 3.18$ kHz.

(*b*)

$$f = \frac{10}{1}(3.18) = 31.8 \text{ kHz}$$

Comparing (*a*) and (*b*), it is seen that the greater the value of C_2, the lower is the frequency at which $|\mathbf{H_v}|$ drops to 0.707 of its peak value, 1; in other words, the more is the graph of $|\mathbf{H_v}|$, shown in Fig. 12-10, shifted to the left. Consequently, any stray shunting capacitance, in parallel with C_2, serves to reduce the response of the circuit.

12.3 HALF-POWER FREQUENCIES

The frequency ω_x calculated in Example 12.2, the frequency at which

$$|\mathbf{H}_v| = 0.707|\mathbf{H}_v|_{\max}$$

is called the *half-power frequency*. In this case, the name is justified by Problem 12.5, which shows that the power input into the circuit of Fig. 12-11 will be half-maximum when

$$\left|\frac{1}{j\omega C_2}\right| = R_1$$

that is, when $\omega = \omega_x$.

Quite generally, any nonconstant network function $\mathbf{H}(\omega)$ will attain its greatest absolute value at some unique frequency ω_x. We shall call a frequency at which

$$|\mathbf{H}(\omega)| = 0.707|\mathbf{H}(\omega_x)|$$

a *half-power frequency* (or *half-power point*), whether or not this frequency actually corresponds to 50 percent power. In most cases, $0 < \omega_x < \infty$, so that there are two half-power frequencies, one above and one below the peak frequency. These are called the *upper* and *lower* half-power frequencies (points), and their separation, the *bandwidth*, serves as a measure of the sharpness of the peak.

12.4 GENERALIZED TWO-PORT, TWO-ELEMENT NETWORKS

The basic *RL* or *RC* network of the type examined in Section 12.2 can be generalized with $\mathbf{Z}_1$ and $\mathbf{Z}_2$, as shown in Fig. 12-12; the load impedance $\mathbf{Z}_L$ is connected at the output port.

By voltage division,

$$\mathbf{V}_2 = \frac{\mathbf{Z}'}{\mathbf{Z}_1 + \mathbf{Z}'}\mathbf{V}_1 \qquad \text{or} \qquad \mathbf{H}_v = \frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{\mathbf{Z}'}{\mathbf{Z}_1 + \mathbf{Z}'}$$

where $\mathbf{Z}' = \mathbf{Z}_2\mathbf{Z}_L/(\mathbf{Z}_2 + \mathbf{Z}_L)$, the equivalent impedance of $\mathbf{Z}_2$ and $\mathbf{Z}_L$ in parallel. The other transfer functions are calculated similarly, and are displayed in Table 12-1.

Ans. $\mathbf{A} = 1 - 10^{-9}\omega^2 + j10^{-9}\omega$, $\mathbf{B} = 10^{-3}(1 + j\omega)$, $\mathbf{C} = 10^{-6}j\omega$, and $\mathbf{D} = 1$. At $\omega = 1$ rad/s, $\mathbf{A} = 1$, $\mathbf{B} = 10^{-3}(1 + j)$, $\mathbf{C} = 10^{-6}j$, and $\mathbf{D} = 1$. At $\omega = 10^3$ rad/s, $\mathbf{A} \approx 1$, $\mathbf{B} \approx j$, $\mathbf{C} = 10^{-3}j$, and $D = 1$. At $\omega = 10^6$ rad/s, $\mathbf{A} \approx -10^3$, $\mathbf{B} \approx 10^3 j$, $\mathbf{C} = j$, and $\mathbf{D} = 1$

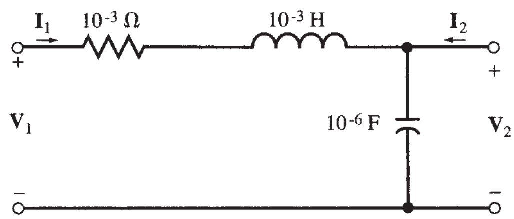


Fig. 13-36

13.35 A two-port network contains resistors, capacitors, and inductors only. With port #2 open [Fig. 13-37(*a*)], a unit step voltage $v_1 = u(t)$ produces $i_1 = e^{-t}u(t)$ (μA) and $v_2 = (1 - e^{-t})u(t)$ (V). With port #2 short-circuited [Fig. 13-37(*b*)], a unit step voltage $v_1 = u(t)$ delivers a current $i_1 = 0.5(1 + e^{-2t})u(t)$ (μA). Find i_2 and the T-equivalent network. *Ans.* $i_2 = 0.5(-1 + e^{-2t})u(t)$ [see Fig. 13-37(*c*)]

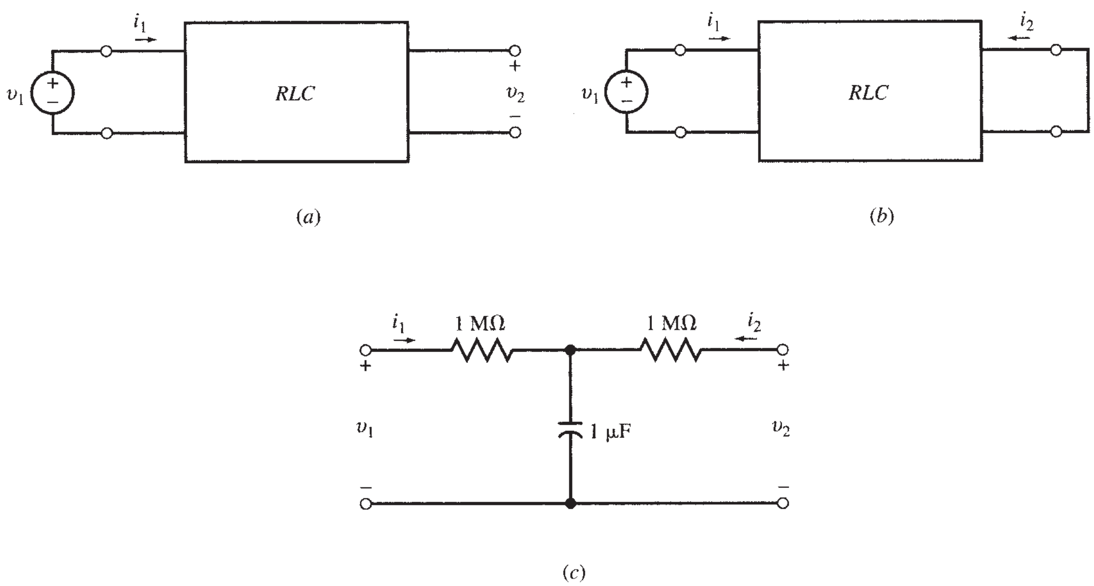


Fig. 13-37

13.36 The two-port network N in Fig. 13-38 is specified by $Z_{11} = 2$, $Z_{12} = Z_{21} = 1$, and $Z_{22} = 4$. Find I_1, I_2, and I_3. *Ans.* $I_1 = 24$ A, $I_2 = 1.5$ A, and $I_3 = 6.5$ A

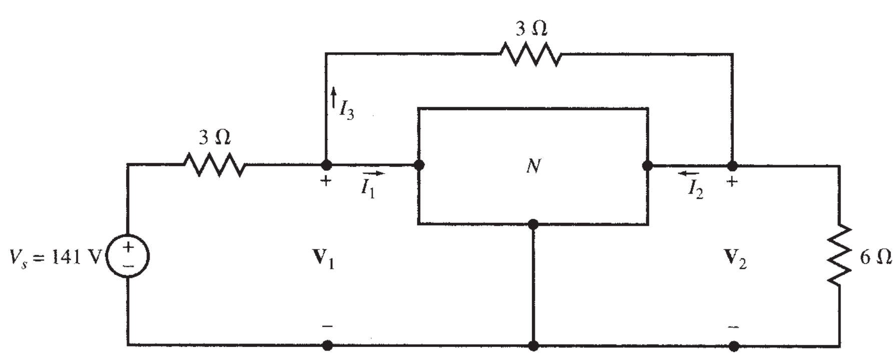


Fig. 13-38

CHAPTER 14

Mutual Inductance and Transformers

14.1 MUTUAL INDUCTANCE

The *total magnetic flux linkage* λ in a linear inductor made of a coil is proportional to the current passing through it; that is, $\lambda = Li$ (see Fig. 14-1). By Faraday's law, the voltage across the inductor is equal to the time derivative of the total influx linkage; that is,

$$v = \frac{d\lambda}{dt} = L\frac{di}{dt}$$

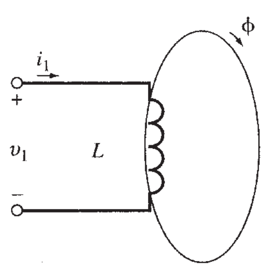


Fig. 14-1

The coefficient L, in H, is called the *self-inductance* of the coil.

Two conductors from different circuits in close proximity to each other are magnetically coupled to a degree that depends upon the physical arrangement and the rates of change of the currents. This coupling is increased when one coil is wound over another. If, in addition, a soft-iron core provides a path for the magnetic flux, the coupling is maximized. (However, the presence of iron can introduce nonlinearity.)

To find the voltage-current relation at the terminals of the two coupled coils shown in Fig. 14-2, we observe that the total magnetic flux linkage in each coil is produced by currents i_1 and i_2 and the mutual linkage effect between the two coils is symmetrical.

$$\begin{aligned}\lambda_1 &= L_1 i_1 + M i_2 \\ \lambda_2 &= M i_1 + L_2 i_2\end{aligned} \tag{1}$$

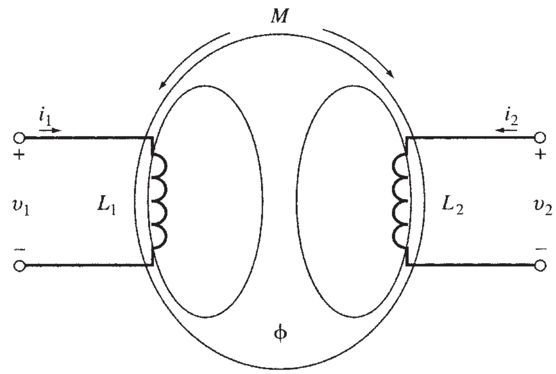


Fig. 14-2

where M is the *mutual inductance* (in H).

The terminal voltages are time derivatives of the flux linkages.

$$v_1(t) = \frac{d\lambda_1}{dt} = L_1\,\frac{di_1}{dt} + M\,\frac{di_2}{dt}$$
$$v_2(t) = \frac{d\lambda_2}{dt} = M\,\frac{di_1}{dt} + L_2\,\frac{di_2}{dt} \tag{2}$$

The coupled coils constitute a special case of a two-port network discussed in Chapter 13. The terminal characteristics (*2*) may also be expressed in the frequency domain or in the **s**-domain as follows.

Frequency Domain

$$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 + j\omega M\mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M\mathbf{I}_1 + j\omega L_2\mathbf{I}_2 \tag{3}$$

s-*Domain*

$$\mathbf{V}_1 = L_1\mathbf{s}\mathbf{I}_1 + M\mathbf{s}\mathbf{I}_2$$
$$\mathbf{V}_2 = M\mathbf{s}\mathbf{I}_1 + L_2\mathbf{s}\mathbf{I}_2 \tag{4}$$

The coupling coefficient M is discussed in Section 14.2. The frequency domain equations (*3*) deal with the sinusoidal steady state. The **s**-domain equations (*4*) assume exponential sources with complex frequency **s**.

EXAMPLE 14.1 Given $L_1 = 0.1$ H, $L_2 = 0.5$ H, and $i_1(t) = i_2(t) = \sin\omega t$ in the coupled coils of Fig. 14-2. Find $v_1(t)$ and $v_2(t)$ for (*a*) $M = 0.01$ H, (*b*) $M = 0.2$ H, and (*c*) $M = -0.2$ H.

(*a*)
$$v_1(t) = 0.1\,\omega\cos\omega t + 0.01\,\omega\cos\omega t = 0.11\,\omega\cos\omega t \quad \text{(V)}$$
$$v_2(t) = 0.01\,\omega\cos\omega t + 0.5\,\omega\cos\omega t = 0.51\,\omega\cos\omega t \quad \text{(V)}$$

(*b*)
$$v_1(t) = 0.1\,\omega\cos\omega t + 0.2\,\omega\cos\omega t = 0.3\,\omega\cos\omega t \quad \text{(V)}$$
$$v_2(t) = 0.2\,\omega\cos\omega t + 0.5\,\omega\cos\omega t = 0.7\,\omega\cos\omega t \quad \text{(V)}$$

(*c*)
$$v_1(t) = 0.1\,\omega\cos\omega t - 0.2\,\omega\cos\omega t = -0.1\,\omega\cos\omega t \quad \text{(V)}$$
$$v_2(t) = -0.2\,\omega\cos\omega t + 0.5\,\omega\cos\omega t = 0.3\,\omega\cos\omega t \quad \text{(V)}$$

14.2 COUPLING COEFFICIENT

A coil containing N turns with magnetic flux ϕ linking each turn has total magnetic flux linkage $\lambda = N\phi$. By Faraday's law, the induced *emf* (voltage) in the coil is $e = d\lambda/dt = N(d\phi/dt)$. A negative sign is frequently included in this equation to signal that the voltage polarity is established according to Lenz's law. By definition of self-inductance this voltage is also given by $L(di/dt)$; hence,

$$L\,\frac{di}{dt} = N\,\frac{d\phi}{dt} \qquad \text{or} \qquad L = N\,\frac{d\phi}{di} \tag{5a}$$

The unit of ϕ being the *weber*, where $1\ \text{Wb} = 1\ \text{V} \cdot \text{s}$, it follows from the above relation that $1\ \text{H} = 1\ \text{Wb/A}$. Throughout this book it has been assumed that ϕ and i are proportional to each other, making

$$L = N\,\frac{\phi}{i} = \text{constant} \tag{5b}$$

In Fig. 14-3, the total flux ϕ_1 resulting from current i_1 through the turns N_1 consists of *leakage flux*, ϕ_{11}, and *coupling* or *linking flux*, ϕ_{12}. The induced emf in the coupled coil is given by $N_2(d\phi_{12}/dt)$. This same voltage can be written using the mutual inductance M:

$$e = M\,\frac{di_1}{dt} = N_2\,\frac{d\phi_{12}}{dt} \qquad \text{or} \qquad M = N_2\,\frac{d\phi_{12}}{di_1} \tag{6}$$

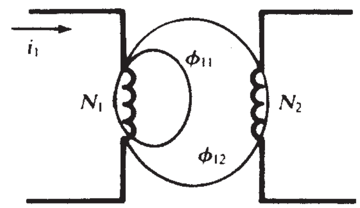


Fig. 14-3

Also, as the coupling is bilateral,

$$M = N_1\,\frac{d\phi_{21}}{di_2} \tag{7}$$

The *coupling coefficient*, k, is defined as the ratio of linking flux to total flux:

$$k \equiv \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

where $0 \le k \le 1$. Taking the product of (*6*) and (*7*) and assuming that k depends only on the geometry of the system,

$$M^2 = \left(N_2\,\frac{d\phi_{12}}{di_1}\right)\left(N_1\,\frac{d\phi_{21}}{di_2}\right) = \left(N_2\,\frac{d(k\phi_1)}{di_1}\right)\left(N_1\,\frac{d(k\phi_2)}{di_2}\right) = k^2\left(N_1\,\frac{d\phi_1}{di_1}\right)\left(N_2\,\frac{d\phi_2}{di_2}\right) = k^2 L_1 L_2$$

from which

$$M = k\sqrt{L_1 L_2} \qquad \text{or} \qquad X_M = k\sqrt{X_1 X_2} \tag{8}$$

Note that (*8*) implies that $M \le \sqrt{L_1 L_2}$, a bound that may be independently derived by an energy argument.

If all of the flux links the coils without any leakage flux, then $k = 1$. On the other extreme, the coil axes may be oriented such that no flux from one can induce a voltage in the other, which results in $k = 0$. The term *close coupling* is used to describe the case where most of the flux links the coils, either by way of a magnetic core to contain the flux or by interleaving the turns of the coils directly over one another. Coils placed side-by-side without a core are loosely coupled and have correspondingly low values of k.

14.3 ANALYSIS OF COUPLED COILS

Polarities in Close Coupling

In Fig. 14-4, two coils are shown on a common core which channels the magnetic flux ϕ. This arrangement results in *close coupling*, which was mentioned in Section 14.2. To determine the proper signs on the voltages of mutual inductance, apply the right-hand rule to each coil: If the fingers wrap

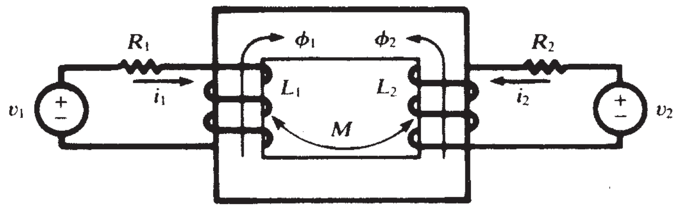


Fig. 14-4

around in the direction of the assumed current, the thumb points in the direction of the flux. Resulting positive directions for ϕ_1 and ϕ_2 are shown on the figure. If fluxes ϕ_1 and ϕ_2 aid one another, then the signs on the voltages of mutual inductance are the same as the signs on the voltages of self-inductance. Thus, the plus sign would be written in all four equations (*2*) and (*3*). In Fig. 14-4, ϕ_1 and ϕ_2 oppose each other; consequently, the equations (*2*) and (*3*) would be written with the minus sign.

Natural Current

Further understanding of coupled coils is achieved from consideration of a passive second loop as shown in Fig. 14-5. Source v_1 drives a current i_1, with a corresponding flux ϕ_1 as shown. Now *Lenz's law* implies that the polarity of the induced voltage in the second circuit is such that if the circuit is completed, a current will pass through the second coil in such a direction as to create a flux opposing the main flux established by i_1. That is, when the switch is closed in Fig. 14-5, flux ϕ_2 will have the direction shown. The right-hand rule, with the thumb pointing in the direction of ϕ_2, provides the direction of the *natural current* i_2. The induced voltage is the driving voltage for the second circuit, as suggested in Fig. 14-6; this voltage is present whether or not the circuit is closed. When the switch is closed, current i_2 is established, with a positive direction as shown.

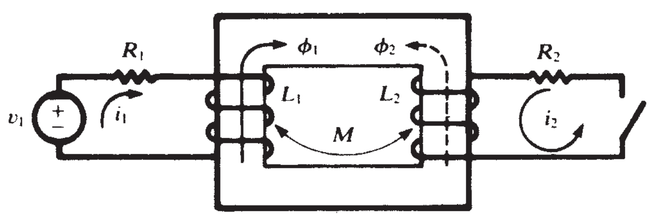


Fig. 14-5

EXAMPLE 14.2 Suppose the switch in the passive loop to be closed at an instant ($t = 0$) when $i_1 = 0$. For $t > 0$, the sequence of the passive loop is (see Fig. 14-6).

$$R_2 i_2 + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} = 0$$

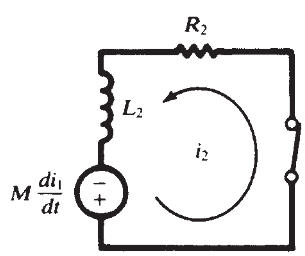


Fig. 14-6

while that of the active loop is

$$R_1 i_1 + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = v_1$$

Writing the above equations in the **s**-domain with the initial conditions $i_1(0^+) = i_2(0^+) = 0$ and eliminating $\mathbf{I}_1(\mathbf{s})$, we find

$$\mathbf{H}(\mathbf{s}) \equiv \frac{\text{response}}{\text{excitation}} = \frac{\mathbf{I}_2(\mathbf{s})}{\mathbf{V}_1(\mathbf{s})} = \frac{M\mathbf{s}}{(L_1 L_2 - M^2)\mathbf{s}^2 + (R_1 L_2 + R_2 L_1)\mathbf{s} + R_1 R_2}$$

and from the poles of $\mathbf{H}(\mathbf{s})$ we have the natural frequencies of i_2.

14.4 DOT RULE

The sign on a voltage of mutual inductance can be determined if the winding sense is shown on the circuit diagram, as in Figs. 14-4 and 14-5. To simplify the problem of obtaining the correct sign, the coils are marked with dots at the terminals which are instantaneously of the same polarity.

To assign the dots to a pair of coupled coils, select a current direction in one coil and place a dot at the terminal where this current *enters* the winding. Determine the corresponding flux by application of the right-hand rule [see Fig. 14-7(*a*)]. The flux of the other winding, according to Lenz's law, opposes the first flux. Use the right-hand rule to find the natural current direction corresponding to this second flux [see Fig. 14-7(*b*)]. Now place a dot at the terminal of the second winding where the natural current *leaves* the winding. This terminal is positive simultaneously with the terminal of the first coil where the initial current entered. With the instantaneous polarity of the coupled coils given by the dots, the pictorial representation of the core with its winding sense is no longer needed, and the coupled coils may be illustrated as in Fig. 14-7(*c*). The following *dot rule* may now be used:

(1) when the assumed currents both enter or both leave a pair of coupled coils by the dotted terminals, the signs on the M-terms will be the same as the signs on the L-terms; but

(2) if one current enters by a dotted terminal while the other leaves by a dotted terminal, the signs on the M-terms will be opposite to the signs on the L-terms.

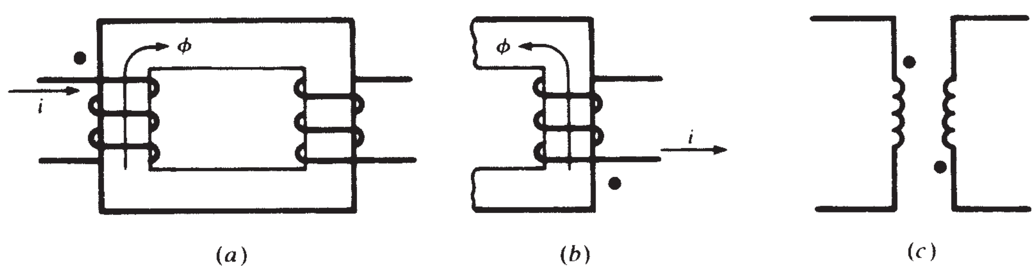
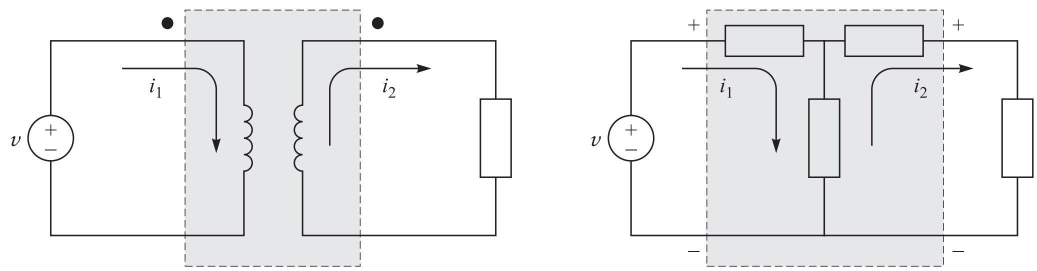


Fig. 14-7

EXAMPLE 14.3 The current directions chosen in Fig. 14-8(*a*) are such that the signs on the M-terms are opposite to the signs on the L-terms and the dots indicate the terminals with the same instantaneous polarity. Compare this to the conductively coupled circuit of Fig. 14-8(*b*), in which the two mesh currents pass through the common element in opposite directions, and in which the polarity markings are the same as the dots in the magnetically coupled circuit. The similarity becomes more apparent when we allow the shading to suggest two black boxes.

14.5 ENERGY IN A PAIR OF COUPLED COILS

The energy stored in a single inductor L carrying current i is $0.5Li^2$ J. The energy stored in a pair of coupled coils is given by



(a) (b)

Fig. 14-8

$$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 \quad \text{(J)} \tag{9}$$

where L_1 and L_2 are the inductances of the two coils and M is their mutual inductance. The term Mi_1i_2 in (9) represents the energy due to the effect of the mutual inductance. The sign of this term is (a) positive if both currents i_1 and i_2 enter either at the dotted or undotted terminals, or (b) negative if one of the currents enters at the dotted terminal and the other enters the undotted end.

EXAMPLE 14.4 In a pair of coils, with $L_1 = 0.1$ H and $L_2 = 0.2$ H, at a certain moment, $i_1 = 4$ A and $i_2 = 10$ A. Find the total energy in the coils if the coupling coefficient M is (a) 0.1 H, (b) $\sqrt{2}/10$ H, (c) -0.1 H, and (d) $-\sqrt{2}/10$ H.

From (9),

(a) $W = (0.5)(0.1)4^2 + (0.5)(0.2)10^2 + (0.1)(10)(4) = 14.8$ J

(b) $W = 16.46$ J

(c) $W = 6.8$ J

(d) $W = 5.14$ J

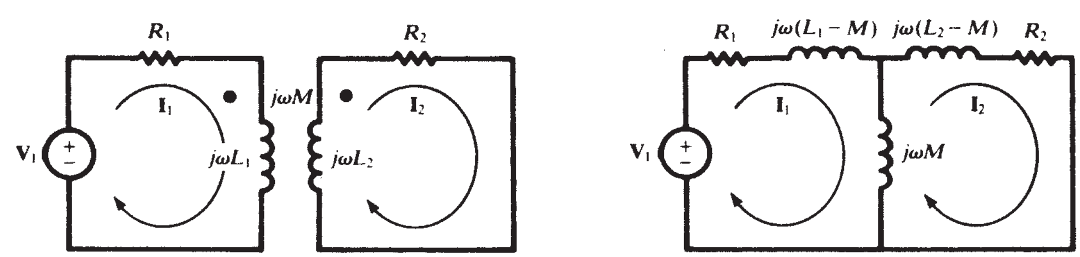
The maximum and minimum energies occur in conjunction with perfect positive coupling ($M = \sqrt{2}/10$) and perfect negative coupling ($M = -\sqrt{2}/10$).

14.6 CONDUCTIVELY COUPLED EQUIVALENT CIRCUITS

From the mesh current equations written for magnetically coupled coils, a conductively coupled equivalent circuit can be constructed. Consider the sinusoidal steady-state circuit of Fig. 14-9(a), with the mesh currents as shown. The corresponding equations in matrix form are

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ 0 \end{bmatrix}$$

In Fig. 14-9(b), an inductive reactance, $X_M = \omega M$, carries the two mesh currents in opposite directions, whence



(a) (b)

Fig. 14-9

$$\mathbf{Z}_{12} = \mathbf{Z}_{21} = -j\omega M$$

in the **Z**-matrix. If now an inductance $L_1 - M$ is placed in the first loop, the mesh current equation for this loop will be

$$(R_1 + j\omega L_1)\mathbf{I}_1 - j\omega M\mathbf{I}_2 = \mathbf{V}_1$$

Similarly, $L_2 - M$ in the second loop results in the same mesh current equation as for the coupled-coil circuit. Thus, the two circuits are equivalent. The dot rule is not needed in the conductively coupled circuit, and familiar circuit techniques can be applied.

14.7 LINEAR TRANSFORMER

A *transformer* is a device for introducing mutual coupling between two or more electric circuits. The term *iron-core transformer* identifies the coupled coils which are wound on a magnetic core of laminated specialty steel to confine the flux and maximize the coupling. *Air-core* transformers are found in electronic and communications applications. A third group consists of coils wound over one another on a nonmetallic form, with a movable slug of magnetic material within the center for varying the coupling.

Attention here is directed to iron-core transformers where the permeability μ of the iron is assumed to be constant over the operating range of voltage and current. The development is restricted to two-winding transformers, although three and more windings on the same core are not uncommon.

In Fig. 14-10, the *primary winding*, of N_1 turns, is connected to the source voltage $\mathbf{V}_1$, and the *secondary winding*, of N_2 turns, is connected to the load impedance $\mathbf{Z}_L$. The coil resistances are shown by lumped parameters R_1 and R_2. Natural current $\mathbf{I}_2$ produces flux $\phi_2 = \phi_{21} + \phi_{22}$, while $\mathbf{I}_1$ produces $\phi_1 = \phi_{12} + \phi_{11}$. In terms of the coupling coefficient k,

$$\phi_{11} = (1-k)\phi_1 \qquad \phi_{22} = (1-k)\phi_2$$

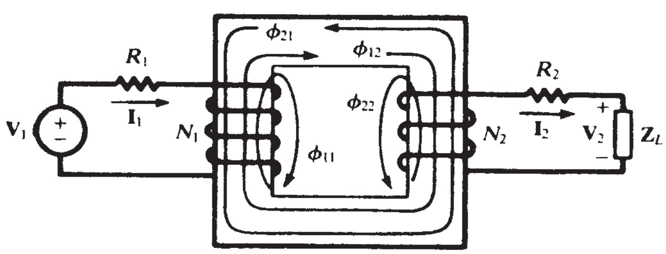


Fig. 14-10

From these flux relationships, *leakage inductances* can be related to the self-inductances:

$$L_{11} \equiv (1-k)L_1 \qquad L_{22} \equiv (1-k)L_2$$

The corresponding leakage reactances are:

$$X_{11} \equiv (1-k)X_1 \qquad X_{22} \equiv (1-k)X_2$$

It can be shown that the inductance L of an N-turn coil is proportional to N^2. Hence, for two coils wound on the same core,

$$\frac{L_1}{L_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{10}$$

The flux common to both windings in Fig. 14-10 is the *mutual flux*, $\phi_m = \phi_{12} - \phi_{21}$. This flux induces the coil emfs by Faraday's law,

$$e_1 = N_1 \frac{d\phi_m}{dt} \qquad e_2 = N_2 \frac{d\phi_m}{dt}$$

Defining the *turns ratio*, $a \equiv N_1/N_2$, we obtain from these the basic equation of the linear transformer:

$$\frac{e_1}{e_2} = a \tag{11}$$

In the frequency domain, $\mathbf{E}_1/\mathbf{E}_2 = a$.

The relationship between the mutual flux and the mutual inductance can be developed by analysis of the secondary induced emf, as follows:

$$e_2 = N_2 \frac{d\phi_m}{dt} = N_2 \frac{d\phi_{12}}{dt} - N_2 \frac{d\phi_{21}}{dt} = N_2 \frac{d\phi_{12}}{dt} - N_2 \frac{d(k\phi_2)}{dt}$$

By use of (*6*) and (*5a*), this may be rewritten as

$$e_2 = M \frac{di_1}{dt} - kL_2 \frac{di_2}{dt} = M \frac{di_1}{dt} - \frac{M}{a} \frac{di_2}{dt}$$

where the last step involved (*8*) and (*10*):

$$M = k\sqrt{(a^2 L_2)(L_2)} = kaL_2$$

Now, defining the *magnetizing current* i_ϕ by the equation

$$i_1 = \frac{i_2}{a} + i_\phi \qquad \text{or} \qquad \mathbf{I}_1 = \frac{\mathbf{I}_2}{a} + \mathbf{I}_\phi \tag{12}$$

we have

$$e_2 = M \frac{di_\phi}{dt} \qquad \text{or} \qquad \mathbf{E}_2 = jX_M \mathbf{I}_\phi \tag{13}$$

According to (*13*), the magnetizing current may be considered to set up the mutual flux ϕ_m in the core.

In terms of coil emfs and leakage reactances, an equivalent circuit for the linear transformer may be drawn, in which the primary and secondary are effectively decoupled. This is shown in Fig. 14-11(*a*); for comparison, the dotted equivalent circuit is shown in Fig. 14-11(*b*).

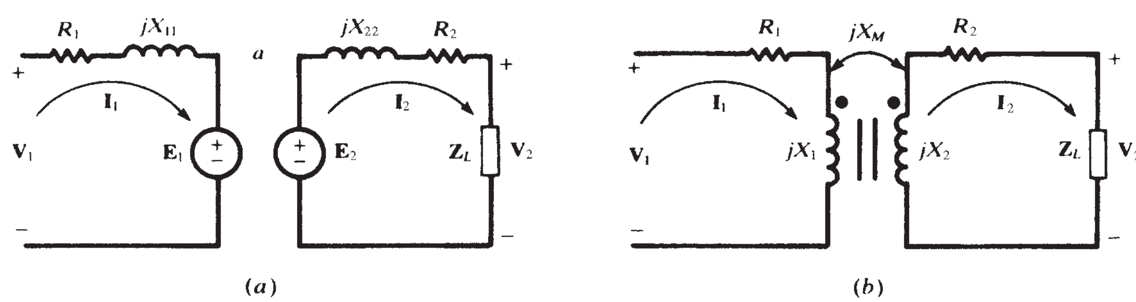


Fig. 14-11

EXAMPLE 14.5 Draw the voltage-current phasor diagram corresponding to Fig. 14-11(*a*), and from it derive the input impedance of the transformer.

The diagram is given in Fig. 14-12, in which θ_L denotes the phase angle of $\mathbf{Z}_L$. Note that, in accordance with (*13*), the induced emfs $\mathbf{E}_1$ and $\mathbf{E}_2$ lead the magnetizing current $\mathbf{I}_\phi$ by 90°. The diagram yields the three phasor equations

$$\mathbf{V}_1 = ajX_M\mathbf{I}_\phi + (R_1 + jX_{11})\mathbf{I}_1$$
$$jX_M\mathbf{I}_\phi = (\mathbf{Z}_L + R_2 + jX_{22})\mathbf{I}_2$$
$$\mathbf{I}_1 = \frac{1}{a}\mathbf{I}_2 + \mathbf{I}_\phi$$

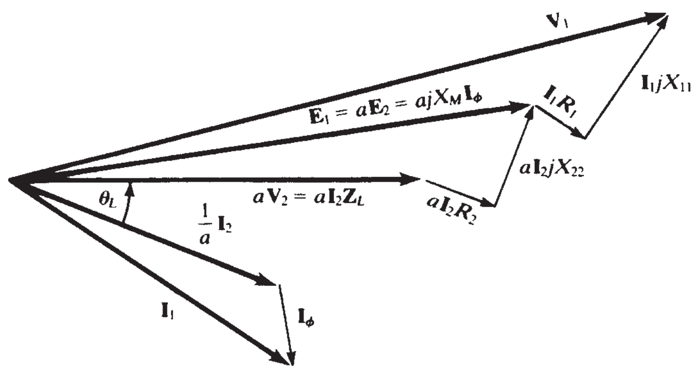


Fig. 14-12

Elimination of $\mathbf{I}_2$ and $\mathbf{I}_\phi$ among these equations results in

$$\frac{\mathbf{V}_1}{\mathbf{I}_1} \equiv \mathbf{Z}_{\text{in}} = (R_1 + jX_{11}) + a^2 \frac{(jX_M/a)(R_2 + jX_{22} + \mathbf{Z}_L)}{(jX_M/a) + (R_2 + jX_{22} + \mathbf{Z}_L)} \qquad (14a)$$

If, instead, the mesh current equations for Fig. 14-11(*b*) are used to derive $\mathbf{Z}_{\text{in}}$, the result is

$$\mathbf{Z}_{\text{in}} = R_1 + jX_1 + \frac{X_M^2}{R_2 + jX_2 + \mathbf{Z}_L} \qquad (14b)$$

The reader may verify the equivalence of (*14a*) and (*14b*)—see Problem 14.36.

14.8 IDEAL TRANSFORMER

An *ideal transformer* is a hypothetical transformer in which there are no losses and the core has infinite permeability, resulting in perfect coupling with no leakage flux. In large power transformers the losses are so small relative to the power transferred that the relationships obtained from the ideal transformer can be very useful in engineering applications.

Referring to Fig. 14-13, the lossless condition is expressed by

$$\tfrac{1}{2}\mathbf{V}_1\mathbf{I}_1^* = \tfrac{1}{2}\mathbf{V}_2\mathbf{I}_2^*$$

(see Section 10.7). But

$$\mathbf{V}_1 = \mathbf{E}_1 = a\mathbf{E}_2 = a\mathbf{V}_2$$

and so, a being real,

$$\frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\mathbf{I}_1} = a \qquad (15)$$

The input impedance is readily obtained from relations (*15*):

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{a\mathbf{V}_2}{\mathbf{I}_2/a} = a^2\,\frac{\mathbf{V}_2}{\mathbf{I}_2} = a^2\mathbf{Z}_L \qquad (16)$$

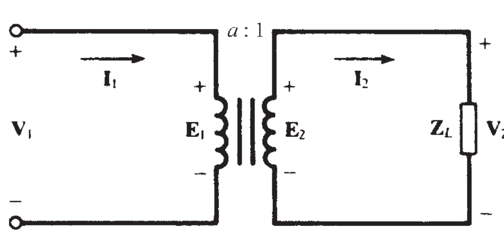


Fig. 14-13